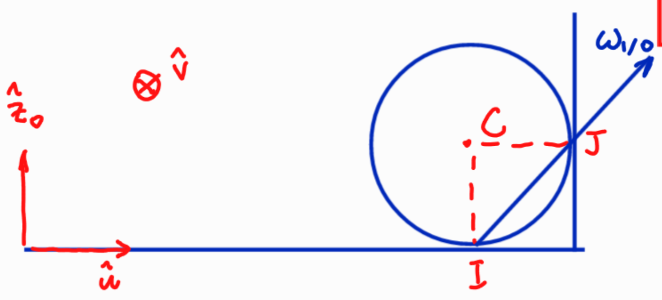
Exam 2



Q.1 Position of C is given by

$$\underline{r}_{OC} = R\,\hat{u}$$

Take $\left.\frac{d}{dt}\right|_0$ to find $\underline{v}_{C/0}$, using

$$\frac{d}{dt}\hat{u} = \dot{\theta}\,\hat{v} \Rightarrow \boxed{\underline{v}_{C/0} = R\dot{\theta}\,\hat{v}} \quad (1)$$

We also know that $\omega_{1/0} = \omega(\hat{u} + \hat{z}_0)$ (see sketch) since instantaneous axis must pass through both I and J (no slip is imposed)
This give another way to find $\underline{v}_{C/0}$ to relate ω to $\dot{\theta}$:

$$\underline{v}_{C/0} = \underbrace{\underline{v}_{I\in 1/0}}_{0} + \omega(\hat{u}+\hat{z}_0) \times \underbrace{r\,\hat{z}_0}_{\underline{r}_{IC}} = -r\omega\,\hat{v} \quad (2)$$

Compare (1) and (2) to find $\omega = -\frac{R}{r}\dot{\theta}$.
We conclude with the expression of the kinematic screw of body 1:

$$\boxed{\{\mathcal{V}_{1/0}\} = \left\{\begin{matrix} -\frac{R}{r}\dot{\theta}(\hat{u}+\hat{z}_0) \\ R\dot{\theta}\,\hat{v} \end{matrix}\right\}_C} \quad \text{(resolved @ C)}$$

Q.2 We take $\left.\frac{d}{dt}\right|_0$ of $\underline{v}_{C/0} = R\dot{\theta}\,\hat{v}$ to find $\boxed{\underline{a}_{C/0} = R(\ddot{\theta}\,\hat{v} - \dot{\theta}^2\,\hat{u})}$ (3)
where we use $\frac{d\hat{v}}{dt} = -\dot{\theta}\,\hat{u}$.
Also $\underline{\alpha}_{1/0} = \left.\frac{d}{dt}\right|_0 \underline{\omega}_{1/0} = \boxed{-\frac{R}{r}\ddot{\theta}(\hat{u}+\hat{z}_0) - \frac{R}{r}\dot{\theta}^2\,\hat{v} = \underline{\alpha}_{1/0}}$ (4)

Q.3 We have linear momentum $m\underline{v}_{C/0} = mR\dot{\theta}\,\hat{v}$.
To find angular momentum about C, we use $\underline{H}_C = \mathcal{I}_C(\omega_{1/0})$.
We do not need a basis to find this expression (for a sphere)
We get $\underline{H}_C = \frac{2}{5}mr^2\,\underline{\omega}_{1/0} = -\frac{2}{5}mrR\,\dot{\theta}(\hat{u}+\hat{z}_0)$.
We conclude with the expression of $\{\mathcal{H}_{1/0}\}$:

$$\boxed{\{\mathcal{H}_{1/0}\} = \left\{\begin{matrix} mR\dot{\theta}\,\hat{v} \\ -\frac{2}{5}mrR\,\dot{\theta}(\hat{u}+\hat{z}_0) \end{matrix}\right\}_C} \quad \text{(resolved about C)}$$

Q.4 With the expression of the action screw

$$\{\mathcal{A}_{\overline{1}\to 1}\} = \underbrace{\left\{\begin{matrix} -mg\,\hat{z}_0 \\ \underline{0} \end{matrix}\right\}_C}_{\text{gravity}} + \underbrace{\left\{\begin{matrix} N_I\,\hat{z}_0 + F_{Iu}\,\hat{u} + F_{Iv}\,\hat{v} \\ \underline{0} \end{matrix}\right\}_I}_{\text{contact @ I}} + \underbrace{\left\{\begin{matrix} -N_J\,\hat{u} + F_J\,\hat{v} \\ \underline{0} \end{matrix}\right\}_J}_{\text{contact @ J}}$$

and the expression of dynamic screw

$$\{\mathcal{D}\ \ \} = \left\{\frac{d\mathcal{H}}{\ }\ \ \right\} = \left\{\begin{matrix} mR(\ddot{\theta}\,\hat{v} - \dot{\theta}^2\,\hat{u}) \\ \ \end{matrix}\right\}$$

$$\left\{\mathcal{D}_{1/0}\right\} = \left\{\frac{d}{dt}\mathcal{H}_{1/0}\right\}_0 = \left\{ \begin{array}{c} \cdots \\ -\frac{2}{5} m r R\left(\ddot{\theta}\,\hat{u} + \dot{\theta}\,\hat{z}_0 - \dot{\theta}^2\,\hat{v}\right) \end{array} \right\}_C$$

we can immediately apply the FTD to obtain $\underline{\underline{6}}$ equations.

Note that we have $\underline{\underline{6}}$ unknowns: $\theta(t), N_I, F_{Iu}, F_{Iv}, N_J, F_J$.

The resultant gives 3 equations on basis $(\hat{u}, \hat{v}, \hat{z}_0)$:

$$\boxed{\begin{aligned} -mR\dot{\theta}^2 &= F_{Iu} - N_J && (5)\\ mR\ddot{\theta} &= F_{Iv} + F_J && (6)\\ 0 &= N_I - mg && (7) \end{aligned}}$$

For the moment equation, we need to resolve the action screw about C:

$$\underline{M}_{C,\bar{1}\to 1} = \underline{0} + \underbrace{\underline{r}_{CI}}_{-r\,\hat{z}_0} \times \left(N_I\hat{z}_0 + F_{Iu}\hat{u} + F_{Iv}\hat{v}\right) + \underbrace{\underline{r}_{CJ}}_{r\,\hat{u}} \times \left(-N_J\hat{u} + F_J\hat{v}\right)$$

$$= -r F_{Iu}\,\hat{v} + r F_{Iv}\,\hat{u} + r F_J\,\hat{z}_0$$

This gives 3 additional equations:

$$\boxed{\begin{aligned} -\frac{2}{5} m r R\,\ddot{\theta} &= r F_{Iv} && (8)\\ \frac{2}{5} m r R\,\dot{\theta}^2 &= -r F_{Iu} && (9)\\ -\frac{2}{5} m r R\,\ddot{\theta} &= r F_J && (10) \end{aligned}}$$

Q.5

We take $(8) + (10)$ to find $F_{Iv} + F_J = -\frac{4}{5} m R\,\ddot{\theta}$

We also have (6): $F_{Iv} + F_J = m R\,\ddot{\theta}$

$$\Rightarrow \boxed{\ddot{\theta} = 0}$$

This shows that $\dot{\theta} = \dot{\theta}_0 \Rightarrow \boxed{F_{Iv} = F_J = 0}$

$(7) \Rightarrow \boxed{N_I = mg}$

$(9) \Rightarrow \boxed{F_{Iu} = -\frac{2}{5} m R\,\dot{\theta}_0^2}$

$(5) \Rightarrow N_J = F_{Iu} + mR\dot{\theta}^2 = \boxed{\frac{3}{5} m R\,\dot{\theta}^2 = N_J}$

Since $N_I > 0$ and $N_J > 0$ (if $\dot{\theta}_0 \neq 0$), the sphere never loses contact @ I and J.

Q.6

For no-slip to be maintained, we need to impose

$\ldots$ and $|F_{I\ldots}| \leq \mu N_I$

$$|F_J| < \mu N_J \quad \text{and} \quad |F_{Iu}| < \mu N_I$$

Since $F_J = 0$, the first condition is always satisfied.

Using $F_{Iu} = -\frac{2}{5} m R \dot{\theta}_0^2$ and $N_I = mg$, we find the condition

$$\frac{2}{5} m R \dot{\theta}_0^2 < \mu m g \quad \text{or} \quad \boxed{\dot{\theta}_0^2 < \frac{5}{2}\frac{\mu}{R}}$$