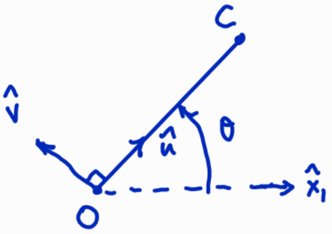
EXAM I: SOLUTION

RVR
SPRING 2024

① 


$\underline{r}_{OC} = R\hat u$ , $\left.\dfrac{d\hat u}{dt}\right|_1 = \dot\theta\,\hat v$

$$\Rightarrow \boxed{\underline{v}_{C/1} = R\dot\theta\,\hat v} \quad [1]$$

Next, we use $\left.\dfrac{d\hat v}{dt}\right|_1 = -\dot\theta\,\hat u$ & take $\left.\dfrac{d}{dt}\right|_1$ of [1] to find

$$\boxed{\underline{a}_{C/1} = R\ddot\theta\,\hat v - R\dot\theta^2\,\hat u}$$

This is the kinematics of a point whose path is a circle of center O and radius R.

② Both I & J satisfy $\underline{v}_{Q\in 2/1} = 0$ due to the <u>no-slip</u> condition. So $\Delta_{2/1}$ must pass through both I & J : this is an instantaneous axis of rotation for the motion 2/1.
So $\underline{\omega}_{2/1}$ must be directed along line IJ. A unit vector along this line is $\hat e = (\hat u + \hat z_1)/\sqrt2$ . This shows that $\underline{\omega}_{2/1}$ can be written in the form $\boxed{\underline{\omega}_{2/1} = \omega_2\,\hat e}$.

③ There are 2 ways of writing $\{\mathcal{V}_{2/1}\}$ :

- resolved @ I: $\begin{Bmatrix} \omega_2\hat e \\ \underline{0} \end{Bmatrix}_I$ (using $\underline{v}_{I\in 2/1} = 0$)
- resolved @ C: $\begin{Bmatrix} \omega_2\hat e \\ R\dot\theta\,\hat v \end{Bmatrix}_C$ (use the expression of $\underline{v}_{C/1}$ found in ①)

To make both expressions compatible, we write:

$$R\dot\theta\,\hat v = \underline{0} + \omega_2\frac{(\hat u + \hat z_1)}{\sqrt2} \times \cancel{\underline{r}_{IC}}^{\,r\hat z_1}$$

$$\Rightarrow R\dot\theta\,\hat v = -\frac{\omega_2}{\sqrt2}\, r\,\omega_2\,\hat v \Rightarrow \boxed{\omega_2 = -\frac{R}{r}\sqrt2\,\dot\theta}$$

Now we have $\boxed{\{\mathcal{V}_{2/1}\} = \begin{Bmatrix} -\frac{R}{r}\dot\theta\,(\hat u + \hat z_1) \\ R\dot\theta\,\hat v \end{Bmatrix}_C}$

④ If 1/0 is a rotation of axis $(O, \hat z_1)$ we can write $\{\mathcal{V}_{1/0}\} = \begin{Bmatrix} \omega_1\hat z_1 \\ \underline{0} \end{Bmatrix}_O$

Then we can write:

$$\{\mathcal{V}_{2/0}\} = \{\mathcal{V}_{2/1}\} + \{\mathcal{V}_{1/0}\} = \begin{Bmatrix} -\frac{R}{r}\dot\theta\,(\hat u + \hat z_1) \\ R\dot\theta\,\hat v \end{Bmatrix}_C + \begin{Bmatrix} \omega_1\hat z_1 \\ \underline{0} \end{Bmatrix}_O$$

$$\boxed{\{\mathcal{V}_{2/0}\} = \begin{Bmatrix} \omega_1\hat z_1 - \frac{R}{r}\dot\theta\,(\hat u + \hat z_1) \\ R(\dot\theta + \omega_1)\,\hat v \end{Bmatrix}_C}$$