MEEG346 Thermal Laboratory

2. Heat Transfer from an extended surface

Objective

A cylindrical aluminum rod (12.7 mm diameter) is heated at its base and placed in a cross-flow created by a wind-tunnel. Under steady conditions, the heat *conducted* through the body of the rod is *convected* away by the oncoming stream of air.

- Measure the temperature distribution along the length of the rod (using thermocouples), for different wind speeds (measured using a Pitot-tube and inclined tube manometer).
- Next, determine (i) Nusselt number, Nu; (ii) heat transfer coefficient, h; and (iii) heat loss, Q using your experimental data and compare it with empirical correlations found in the textbook.
- Finally, determine the variation of h and Q as a function of air speed V.

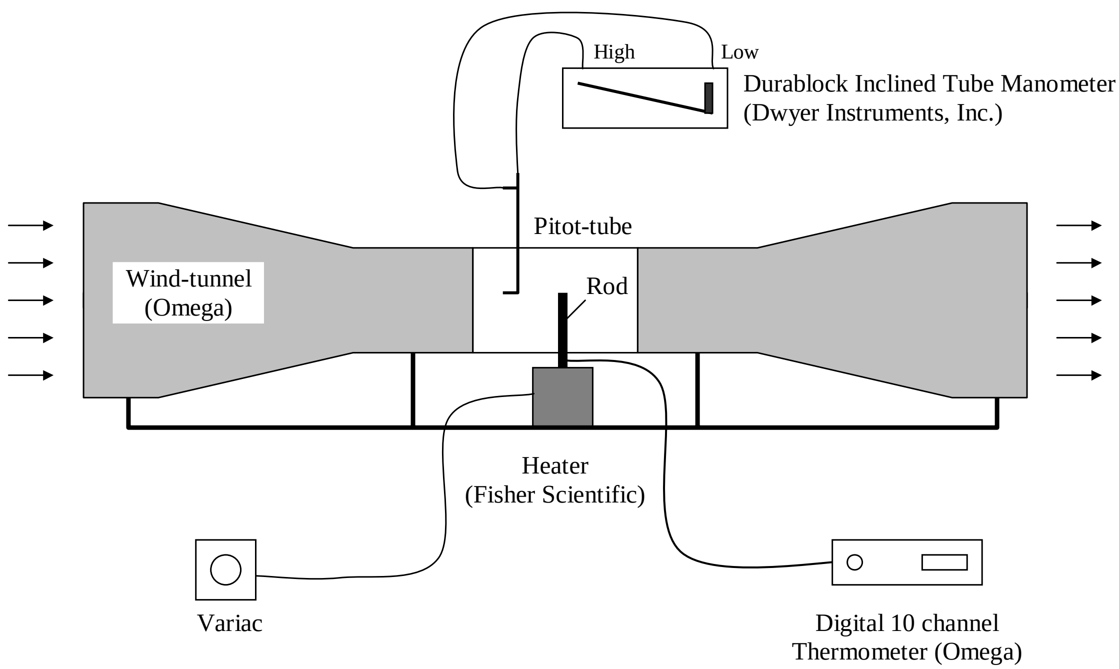

Figure 1: Schematic of heated fin in cross-flow

Theoretical Considerations

Conduction and Convection Heat Transfer

As shown in the Figure 1, a cylindrical aluminum fin is heated at its base by an electrical heater. The main part of the fin extends into a subsonic wind-tunnel and is cooled by a cross-flow of air. The fin is instrumented with thermocouples to obtain the temperature T as a function of distance along the fin x.

It is possible to relate $T(x)$ to the base temperature T_o, the temperature of the fluid stream T_f, the fin geometry and conductivity, and the heat transfer coefficient h. Each quantity in this relation can be directly measured or looked-up in a handbook, except h. Our goal is to use theory to derive an expression for the temperature distribution, compare the experimentally measured distribution with the theoretical one, and thus extract the heat transfer coefficient.

Let us now derive an expression for the temperature profile in the rod using the Fourier heat conduction law as a starting point.

$$Q(x) = -k_s A \frac{dT}{dx} \tag{1}$$

where $Q(x)$ is the heat conducted across the rod cross-section at x, A is the cross-sectional area, and k_s is the thermal conductivity of the rod material. The minus sign indicates that for $Q(x)$ to be positive, i.e., for it to flow from right to left, the temperature gradient has to be *negative* (heat flows from hot $\rightarrow$ cold).

Convection heat transfer describes the mode by which heat is carried away from the rod surface to the free stream.

$$dQ_c = hPdx[T(x) - T_f] \tag{2}$$

With reference to Figure 2, dQ_c is the elemental heat convected away by the free stream from an elemental rod length of dx, and P is the perimeter (i.e. circumference) of the rod. h is the heat transfer coefficient. You can perhaps imagine that h depends on the speed with which the fluid is flowing over the rod. You could also guess that h will increase with air speed.

Refer back to Figure 2 and consider an elemental piece of the rod of length dx. Under steady state conditions (rate of accumulation of heat in the control volume is zero):

The heat conducting into the rod from the left face, $Q(x)$	$=$	The heat conducting out of the rod from the right face, $Q(x+dx)$	$+$	The heat convected away to the free stream, dQ_c

The first and third terms above are given by Equations 1 and 2 respectively. The second term be obtained by applying a Taylor series approximation to $Q(x)$:

$$Q(x+dx) = Q(x) + \frac{dQ}{dx}dx + \text{H.O.T}$$

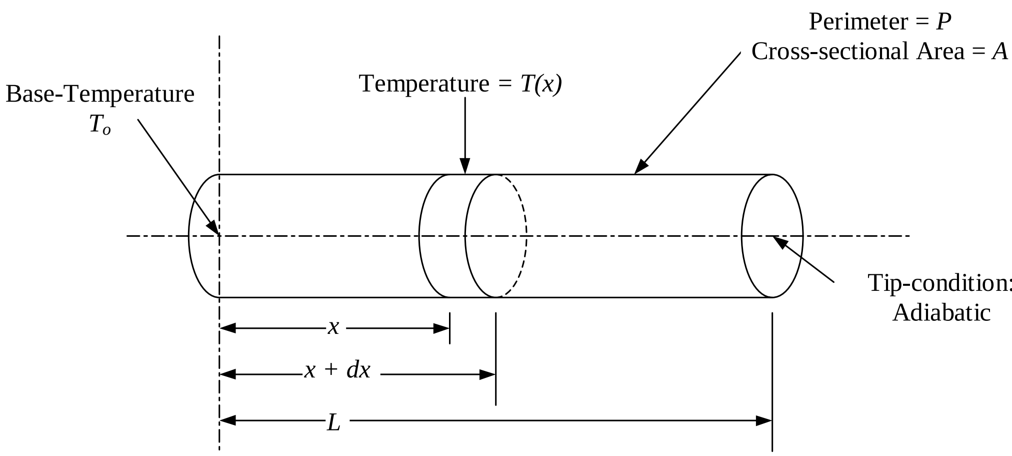

Figure 2: Schematic of heated fin in cross-flow

Putting all three terms together (and ignoring the Higher Order Terms, H.O.T.) we can arrive at the following differential equation.

$$\frac{d^2T(x)}{dx^2} = m^2[T(x) - T_f] \tag{3}$$

where $m^2 = hP/k_sA$. Two boundary conditions are required to solve this equation. We will apply the first one at the fin-base, or more precisely, at the x-location of the first thermocouple, which we will treat henceforth as our virtual fin-base ($x = 0$): $T(0) = T_o$. The second boundary condition must be applied at the fin-tip ($x = L$). Four possibilities exist for the fin-tip: prescribed heat-transfer coefficient, adiabatic tip, prescribed tip temperature, and infinitely long fin. Experimentally, it is easiest to enforce an adiabatic tip, which we have done by gluing a cylindrical styrofoam stub to the fin-tip. Thus, the second boundary condition is $dT(L)/dx = 0$. The solution to Equation 3 is:

$$\frac{T(x) - T_f}{T_o - T_f} = \frac{\cosh m(L-x)}{\cosh mL} \tag{4}$$

The total heat (Q) transferred to the air by the fin must equal the heat conducted across the fin at $x = 0$! Therefore, we can use Equation 4 in conjunction with Equation 1 to obtain:

$$Q = k_sA(T_o - T_f)m \tanh mL \tag{5}$$

Air-Speed Measurements

Air speed is measured using the Pitot-tube. Using Bernoulli's theorem:

$$V = \sqrt{\frac{2(P_T - P_S)}{\rho_{air}}} \tag{6}$$

where P_T and P_S are the total (or stagnation) and static pressures respectively. Further, using the principle of an inclined tube manometer:

$$P_T - P_S = \rho_w \; g \; \Delta h$$

where ρ_w is the density of water and Δh is the manometer reading. Note that though the manometric fluid is red oil, the scale directly reads *inches of water.* For your convenience, the lower red scale on the manometer reads in fpm (1 m/s $\approx$ 200 fpm).

Empirical Nusselt Numbers

The literature contains several correlations for heat transfer from a cylinder in cross-flow by various researchers. A few of these are:

$$Nu_1 = (0.35 + 0.56 \; Re^{0.52}) \; Pr^{0.3} \tag{7}$$

$$Nu_2 = (0.4 \; Re^{0.5} + 0.06 \; Re^{2/3}) \; Pr^{0.4} \tag{8}$$

$$Nu_3 = 0.3 + \frac{0.62 \; Re^{1/2} \; Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re}{282,000}\right)^{5/8}\right]^{4/5} \tag{9}$$

According to the usual definitions,

$$Nu = \frac{h \; D}{k_a}$$

$$Pr = \frac{\mu_a \; C_p}{k_a}$$

$$Re = \frac{\rho_a \; V D}{\mu_a}$$

where k_a, μ_a and C_p are thermal conductivity, dynamic viscosity, and specific heat at constant pressure of air, and D is the rod diameter. You will notice that air properties are a function of temperature; usually the *average* of the fin and ambient air temperatures is used to calculate air properties (known as *film* temperature).

Procedure

1. Place the instrumented aluminum fin inside the wind-tunnel such that the thermocouple wires face downstream (to minimize flow disturbances). Record all thermocouple readings to check for uniformity and to establish room temperature. Turn on the fin heater (use $\approx$ 80 volts). Wait at least 30 minutes to let the heater reach steady state.

2. Set the tunnel for a low speed, say 4 m/s (about 20 V). Note the Pitot-tube reading after waiting at least 5 minutes for steady state. Eight thermocouples are attached at 2.54 cm increments along the length of the fin. As noted earlier, use $x = 0$ for the first thermocouple

as it is our virtual fin-base. Also, for an accurate value of L, note that the fin extends slightly beyond the last thermocouple. When the temperatures reach steady-state (in about 10 minutes) record all temperatures $T(x)$.

3. Repeat the above step for four additional velocities, taking care to exploit the full range of tunnel velocities, say 4, 8, 12, 16, and 20 m/s (avoid $V = 0$).

4. Record the barometric pressure (used to determine air properties).

Analysis

1. Plot $T(x)$ vs. x for all five trials. For each case, also plot the curve provided by Equation 4. The unknown value of m has to be determined for each trial such that the error between the experimental and theoretical temperature profiles is minimized in a least-squares sense.

2. Knowing m, determine h_{exp}, Nu_{exp}, and Q_{exp}. Plot Nu_1, Nu_2, Nu_3, and Nu_{exp} against Re.

3. Plot h_1, h_2, h_3, and h_{exp} vs. V.

4. Plot Q_{exp} vs. V.

Discussion

1. How well does the form of the temperature profile match the theoretical curve?

2. Discuss the variation of h with V. Does it increase linearly? The expressions for Nu_1, Nu_2, and Nu_3 will provide clues. Why does h increase with V? (Hint: boundary layers!)

3. How does Q vary with V? Why is the variation not very significant? How do you explain the fact that heat loss is pretty much the same at low and high wind speeds (i.e., even though h is varying substantially)?

4. What result would you expect if $V = 0$? Write a few lines about the nature of heat transfer from the fin at this extreme. Would you still expect Equation 4 and 5 to hold?

5. Suppose you were given a water-tunnel instead of a wind-tunnel. For a speed of 10 m/s, and a water temperature of 25° C, determine Nu_3 and h. Compare with the air values.

Error Analysis

1. Estimate the error in h from your experiments. Use the expression $m = \sqrt{hP/K_sA}$ to perform the propagation of errors. You will need to estimate the error in m from your curves (i.e., what is the error in the temperature measurement?).

2. What is the error in Nu_1 based on errors in Re and Pr? An error in V will lead to an error in Re. Note that air properties can vary with temperature, so this will also result in some uncertainty in Re and Pr.