MEEG331 Incompressible Fluids Laboratory

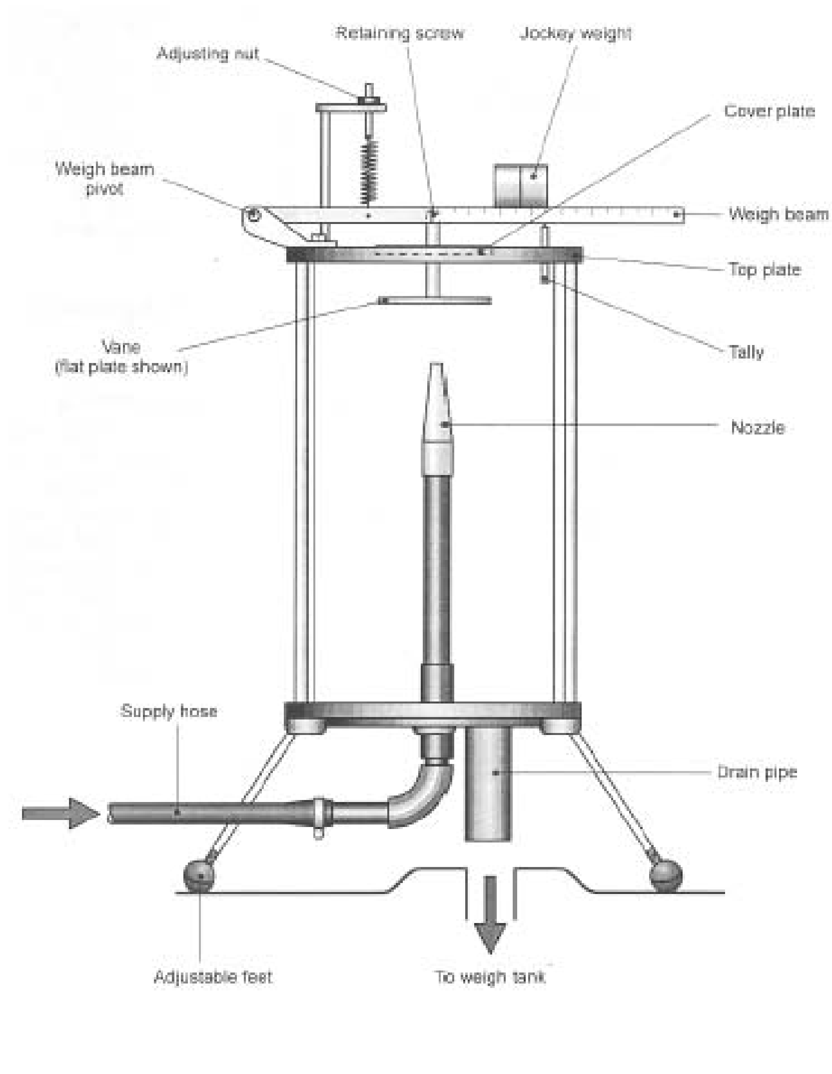
5. Conservation of Linear Momentum (Impact of a Jet)

Objective

This experiment demonstrates the principle of conservation of linear momentum by measuring the force generated on a circular target plate by an impinging water jet, and comparing it with the theoretical value.

Apparatus

As shown below, the water supply from the hydraulic bench is led to a vertical pipe, terminating in a tapered nozzle. This produces a water jet which impinges on a vane in the form of a flat plate. The nozzle and vane are contained in a transparent cylinder. An outlet at the base of the cylinder directs the flow to a catch-tank for measuring the flow rate.



The vane is attached to a pivoted beam which carries a jockey weight and is restrained by a light spring. The beam is initially balanced as follows: with the water jet turned off, place the jockey weight at its zero position and adjust the knurled knob above the spring until the beam is balanced (as indicated by the tally suspended from it). Subsequently, the force generated by the impact of the jet on the vane can be measured by moving the jockey weight along the beam until the tally shows that the beam has been restored to its original balanced position.

The following quantities will be required:

Diameter of nozzle	= 10 mm
Cross-sectional area of plate, A	= 78.5 mm^2
Mass of jockey weight	= 0.6 kg
Distance of vane center to pivot	= 0.15 m
Height of vane above nozzle-exit, s	= 35 mm

Theory

Using an appropriate control volume, you are asked to show that the force F_p exerted by the plate on the jet causes a change in the momentum of the jet given by:

$$F_p = -\dot{m} v_p \tag{1}$$

where $\dot{m}$ is the mass flux and v_p is the velocity of the jet impacting the plate. (The minus sign indicates that the force is acting downwards.) Obviously, an equal and opposite force F_j is exerted upward by the jet on the plate, i.e.,

$$F_j = -F_p = \dot{m} v_p. \tag{2}$$

Note that the velocity of the jet at the plate, v_p is somewhat smaller than the nozzle-exit velocity v_o due to the deceleration caused by gravity. Determine v_p, given v_o and the fact that the distance of the plate above the nozzle-exit (s) is 35 mm.

Sliding the jockey-weight along the lever by the correct distance, x (measured from the zero position) creates a clockwise moment about the pivot point that will exactly balance the counter-clockwise moment caused by the impact of the jet. Using the moment balance for the lever, and the quantities given in the list above, derive the following equation:

$$F = 4gx \tag{3}$$

where F is the force on the plate required to balance the lever. This measured value F determined from the moment balance should closely match the theoretical value F_j determined from Equation 2.

Procedure

1. Balance the lever (as indicated by the tally) with the jockey weight at the zero position and the jet turned off.

2. Admit water into the nozzle by adjusting the bench valve. Increase the flow rate to its maximum value; record the position of the jockey weight, and measure the flow rate with the catch-tank and stopwatch method.

3. Record a total of ten different jockey positions (x) for gradually decreasing flow rates (Q), such that the jockey weight is moved to the left in roughly equal steps.

Data analysis and discussion

1. Plot F (y-axis) vs. F_j (x-axis). Fit a least-squares line to the data. What is the slope? What do you expect it to be? What is the correlation coefficient?

2. If the slope of the above graph is different from what you expect, speculate on the possible causes.

   The theory leading to Equation 2 assumes that the jet velocity profile is uniform (plug flow) across its cross-section. However, a real jet has a velocity distribution which is zero at the edge and maximum at the center (though it is not exactly parabolic). Sketch what the real jet velocity profile might look like. What effect should a non-uniform velocity profile have on F_j? Will it increase or decrease F_j relative to $\dot{m}v_p$? Note that the *average* value of the velocity at nozzle-exit is still $u_o = Q/a$ where a is the nozzle-exit area.

   Hint: Read the section "Momentum-Flux Correction Factor" (page 155, White).

3. Does the linear fit to the data pass through the origin? If not, why not?

4. How would Equation 2 change if the circular-plate target were replaced with a hemispherical cup with its mouth facing downwards?

Error Analysis

1. What would be the systematic error caused by:

   - Jockey weight is in error by 1 g?
   - Distance from vane center to pivot point is in error by 1 mm?

2. Perform a propagation of statistical error analysis for F_j (Equation 2), and F (Equation 3).