MEEG331 Incompressible Fluids Laboratory

3. Flow From an Orifice in the Side of a Tank (Bernoulli’s Equation)

Objective and Apparatus

The purpose of this experiment is to test the validity of the Bernoulli equation (energy equation without any loss) applied to the flow through a small orifice in the side of a tank. The apparatus consists of a hydraulic bench, an orifice & jet apparatus, a CCD (Charge-Coupled Device) camera, a computer equipped with a frame-grabber, a ruler, a stopwatch, and a measuring cylinder. The cross-sectional area of the tank (main tank area + overflow section area - overflow pipe area) is 160 cm$^2$, and the diameter of the orifice is 3 mm. In the experiment, the trajectory of the jet flowing out of the orifice, and the rate of fall of the liquid surface in the tank are recorded and compared with theory.

Theory

The steady state Bernoulli equation predicts that the horizontal jet velocity leaving the orifice at the vena contracta$^1$ is

$$ u = \sqrt{2gh} \quad (1) $$

where $g$ is the gravitational acceleration and $h$ is the water elevation above the center of the orifice. With energy losses present, the discharge velocity is modified by a velocity coefficient $C_v$

$$ u = C_v \sqrt{2gh} \quad (2) $$

After exiting the orifice the jet drops in free fall under the influence of gravity; each particle of fluid comprising the jet executes the same trajectory. Elementary mechanics tells us that this trajectory is a parabola:

$$ y = a + bx + cx^2 \quad (3) $$

where $y$ is the vertical coordinate and $x$ is the horizontal coordinate. Obviously, $x = ut$ and $y = \frac{1}{2}gt^2$. You will record the jet trajectory using a B&W CCD camera hooked up to a computer via a frame-grabber, and perform a parabolic fit to the jet in (almost) real time. If you set your origin ($x = 0, y = 0$) at the vena contracta, then $a = 0$ and $b = 0$ (you are asked to prove this in your report). Applying this result to Equation 3 and eliminating $t$, you can show that:

$$ u = \sqrt{\frac{g}{2c}}. \quad (4) $$

Knowing $u$ and $h$, you may solve for $C_v$ from Equation 2.

$^1$Even after emerging from the orifice, the streamlines in the jet are still converging! They continue to converge until a short distance from the orifice, the streamlines become parallel and horizontal, and the area of the jet becomes a minimum. The section is called the vena contracta.
The actual discharge (volume flux) $Q$ from the orifice is the product of the actual velocity and the area of the jet at the vena contracta. The area at the vena contracta is $C_c a$, where $a$ is the area of the orifice and $C_c$ is the coefficient of contraction.

$$Q = C_v C_c a \sqrt{2gh}$$

(5)

It is customary to combine the two coefficients into a discharge coefficient $C_d$:

$$Q = C_d a \sqrt{2gh}.$$  

(6)

On the other hand, conservation of mass in the tank yields

$$A \frac{dh}{dt} = -C_d a \sqrt{2gh}$$

(7)

where $A$ and $a$ are respectively the cross-sectional area of the tank and the orifice, $C_d$ is the discharge coefficient, and $t$ is time. Solving the above ordinary differential equation with the given initial condition, $h = h(0)$ at $t = 0$, gives

$$\sqrt{\frac{h(t)}{h(0)}} = 1 - \frac{\sqrt{2} C_d a}{2A} \sqrt{\frac{g}{h(0)}} t$$

(8)

$C_d$ can now be determined in two ways: (i) by measuring the discharge $Q$ for a constant head, based on Equation 6; and (ii) by recording the fall of the liquid in the tank (variable head based on Equation 8).

**Procedure**

The procedure consists of two parts.

1. To determine the velocity coefficient $C_v$ from Eq. (3) by recording the jet trajectory with the CCD camera and frame-grabber.

Perform the following steps:

(a) Fix the CCD camera to the base of the orifice & jet apparatus such that it captures the entire jet trajectory. Adjust the focus and contrast for optimal images.

(b) The first step is to calibrate the CCD camera. The CCD camera’s output is in pixel units, so we will need to determine how many mm in the jet plane corresponds to one pixel. Align the two middle needles using the Plexiglas calibration jig (TA will assist). Using the IMAQ software, snap an image of the needle tips. Store your calibration image under C:\Students\meeg33\group_name, as say, “calib.bmp”. You will use this file later in post-processing. (Refer to the separate sheet on how to use the images.) Once calibration is complete, DO NOT TOUCH the camera again!
(c) Raise overflow pipe to a suitable level, release water into the tank and control
the flow until the water is just spilling into the overflow.

(d) Record the constant head $h$ on the scale.

(e) Assess the position of the *vena contracta* visually and note the horizontal distance
from the orifice (it will probably be very close to the jet exit itself).

(f) Exactly as with the calibration image, use the B&W CCD camera to grab an im-
age of the jet on the computer monitor. Save the image as C\Students\meeg331
\jet#.bmp. You will use this image later in post-processing. It might be useful
to incorporate the water height in mm into the image name, say jet312.bmp.

(g) Repeat (c) – (f) for five different values of $h$ by moving the overflow pipe.

2. To determine the discharge coefficient $C_d$ using Equations 6 and 8.

Perform the following steps:

(a) **Constant Head**

i. raise overflow pipe to a suitable level, release water into the tank and control
the flow until the water is just spilling into the overflow

ii. read the constant head $h$ on the scale and record

iii. measure the flow rate by intercepting the jet with a measuring cylinder

iv. repeat (i) - (iii) for at least five different water levels

(b) **Variable Head**

i. raise the overflow pipe to a maximum

ii. fill the tank to the overflow level and close the inlet

iii. start a stopwatch when the water level reaches a convenient initial depth
$h(0)$

iv. take a reading of the head $h(t)$ at 15 seconds intervals

**Data analysis**

1. In the first part of the experiment, plot $u$ versus $\sqrt{2gh}$. The slope of the graph will
yield $C_v$.

2. In the second part of the experiment:

(a) compute and tabulate $Q$; plot $Q$ against $a \sqrt{2gh}$ and obtain $C_d$ from the slope
of this graph

(b) plot $\sqrt{\frac{h(t)}{h(0)}}$ versus $\frac{\sqrt{2a}}{4} \sqrt{\frac{g}{h(0)}} t$ and obtain $C_d$ from the slope of this graph.

If the values of $C_d$ obtained by the two methods differ, which method is more reliable
and why? Discuss which measurements may have caused large errors.
3. Compute the value of $C_c = C_d/C_v$.

4. Compare the values of $C_v$ and $C_d$ with values reported in the textbook and discuss any difference.

5. In the theory section of your lab report, complete the derivations of:
   
   - Equation 1
   - Equation 4. Why are $a$ and $b$ zero in Equation 3? Think about the two boundary conditions that need to be applied to determine the values of $a$ and $b$.
   - Equation 8

**Error analysis**

Estimate the random error in:

1. $C_v$ (Equation 2)
2. $C_d$ (Equation 6)
3. $C_d$ (Equation 8)

The following example will indicate how to proceed. Let us consider how to evaluate the error in $C_d$ using Equation 6. Rearranging, we have

$$C_d = \frac{Q}{a\sqrt{2gh}}$$

As far as the error analysis is concerned, $a$ and $g$ are given constants. Therefore, the error in $C_d$ is determined solely by the error in measuring $Q$ and $h$. Using the propagation of statistical errors (see the Error analysis hand-out), prove the following:

$$\left(\frac{\Delta C_d}{C_d}\right) = \sqrt{\left(\frac{\Delta Q}{Q}\right)^2 + \left(\frac{\Delta h}{2h}\right)^2} \quad (9)$$

Furthermore, $Q = V/t$, where $V$ is the volume collected in time $t$. Show that the error $\Delta Q$ is obtained similarly as:

$$\left(\frac{\Delta Q}{Q}\right) = \sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta t}{t}\right)^2} \quad (10)$$

Obtain $\Delta Q$ using reasonable estimates for $\Delta V$ and $\Delta t$. Substitute the result from Equation 10 into Equation 9, and use a reasonable estimate for $\Delta h$ to obtain the error in $C_d$.

Repeat the above analysis for the other two cases.