A Simple Controller for Quadrupedal Bounding

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1 Motivation

In this work, the dynamics of quadrupedal bounding is studied via the reduced-order sagittal-plane model of Fig.1. The model is composed of a rigid torso and two massless springy prismatic legs. Albeit simple, models in this family have been found useful in uncovering many important properties of bounding. For example, it has been shown in [1] that a variety of bounding gaits can be generated passively, suggesting that such motions represent natural interactions between a quadruped and its environment.

According to the modeling and control hierarchy introduced in [2], reductive locomotion models can be used to inform the control of more accurate high-dimensional representations of robots (or animals) by suggesting suitable coordination mechanisms. However, passive reduced-order models such as the one in [1] cannot be used directly as control targets for more complete robot models due to their limited ability in rejecting perturbations. For example, the majority of the passively generated fixed points in [1] are unstable; stable (within a total energy level) fixed points only exist when the forward velocity is in a range that is not realizable by most robotic quadrupeds. Even for these stable fixed points, the domain of attraction is not practical.

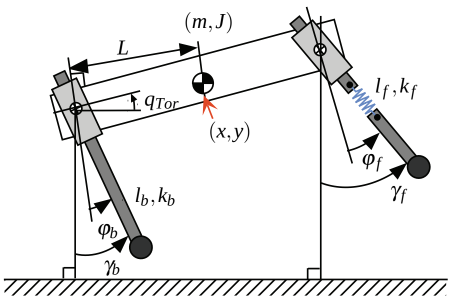

Figure 1: A sagittal-plane bounding model.

In this work, a controller for the bounding model of Fig. 1 is described. The proposed control law manipulates the touchdown angles of the front leg based on feedback of the liftoff angle of the back leg to stabilize a large number of passively generated periodic bounding gaits within a constant energy level. To reject perturbations that alter the total energy, actuation is introduced at the hip joints during the back and front leg stance phases, and a continuous-time controller is proposed that stabilizes the energy of the system at a desired level. It is shown in simulation that this two-layer controller design is capable of rejecting sufficiently large perturbations without requiring excessive inputs.

2 State of the art

A large number of controllers have been proposed to stabilize bounding motions on quadrupedal robots. Among them, Raibert's three-part controller [3] demonstrated high performance running motions over natural terrain. Variations of this controller have been used to control electrically actuated quadrupedal robots in [1]. A different paradigm for controller design that uses principles from neurobiology has been employed by [4] to induce dynamically stable locomotion on the quadrupedal robot Tekken.

To investigate the dynamics of legged systems, formal reductive models—termed *templates* in [2]—have been introduced in an effort to resolve complexity in locomotion. Among these models, the Spring Loaded Inverted Pendulum (SLIP) stands out as a canonical model of the center-of-mass (COM) dynamics of running robots (and animals), prompting a variety of control laws for stabilizing SLIP running. Most relevant to our work is the controller proposed in [5], which uses symmetries to stabilize periodic running orbits through an adaptive touchdown angle update law. Clearly though, point-mass hoppers like the SLIP cannot capture the torso pitch dynamics of bounding, which calls for non-trivial modifications of SLIP-based control ideas. Proposing such controllers for bounding is the subject of this work.

3 Our approach

The reduced-order model of Fig.1 is used to study bounding. In this model, the legs are assumed massless and actuation is introduced at the hips during the front and back leg stance phases to develop non-conservative corrective action. The geometric and inertia parameters are taken from [1] and bounding without double stance is considered.

3.1 Touchdown angle control policy

It has been observed in [1] that passively generated bounding motions have the following property

$$\overline{\gamma}^{f}_{TD} = -\overline{\gamma}^{b}_{LO}, \tag{1}$$

where $\overline{\gamma}^{f}_{TD}$ and $\overline{\gamma}^{b}_{LO}$ are the front leg touchdown and the back leg liftoff angles at a desired fixed point, respectively. Motivated by [5], the front leg touchdown angle γ^{f}_{TD} at the $(n+1)$-th stride is updated based on feedback of the back leg liftoff angle γ^b_{LO} at the n-th stride according to

$$\gamma^f_{TD}[n+1] = \bar{\gamma}^f_{TD} + c(\bar{\gamma}^b_{LO} - \gamma^b_{LO}[n]), \tag{2}$$

where c is a constant gain. To investigate the local stability of the system in closed loop with (2), the eigenvalues of the linearization of the corresponding Poincaré return map are computed for a variety of values for c, as shown in Fig. 2. It can be seen that for $c > 1.1$ all the eigenvalues enter the unit disc, except one eigenvalue that remains at one reflecting the conservative nature of the system.

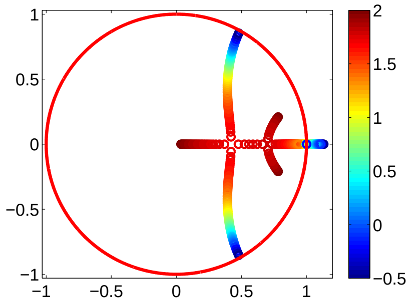

Figure 2: Locus of the eigenvalues as c varies in $[-0.5, 2]$.

3.2 Energy controller

The conservation of energy precludes the existence of asymptotically stable fixed points; for, perturbations altering the total energy of the system cannot be rejected. To incorporate non-conservative forces to the model of Fig. 1, actuation is introduced at the hip joint. The hip actuators are active only during the stance phase of the corresponding leg, developing torques according to the prescription

$$\tau = -K\,\frac{E - \bar{E}}{\Delta\bar{\varphi}}, \tag{3}$$

where $\Delta\bar{\varphi}$ is the change of the stance leg angle φ relative to the torso at the desired fixed point, E is the total energy and $\bar{E}$ its nominal value at the fixed point, and $K \in [0, 1]$.

3.3 Controller evaluation in simulation

The performance of the proposed two-level controller is evaluated in simulation. The passively generated fixed point $y = 0.35$m, $q_{\mathrm{Tor}} = 0$deg, $\dot{x} = 3$m/s, $\dot{q}_{\mathrm{Tor}} = 150$deg/s (states refer at the apex height) is used, and (2) and (3) for $c = 1.3$ and $K = 0.2$ are employed. Figures 3(a) and 3(b) demonstrate convergence to the nominal orbit after a perturbation 0.4m/s is introduced in the forward velocity at apex. Note that despite the relatively large size of the perturbation, Fig. 3(b) shows that the torque required is small.

4 Discussion outline

This work aims at enhancing the stability properties of periodic bounding orbits passively generated via the reduced-order sagittal plane model of Fig. 1. As in the modeling and control hierarchy proposed in [2], this model can serve as a behavioral control target for more complete models of quadrupedal robots. To achieve this objective, there are a number of issues that require further attention. In particular, the choice of the controller constant c in (2) has a strong influence on the ability of the controller to stabilize passively generated fixed points. We are currently in the process of establishing analytical criteria for the choice of this parameter. Furthermore, to make these results relevant to the control of quadrupedal robots, we plan to devise control strategies in the spirit of the Hybrid Zero Dynamics (HZD) method [6]. These strategies coordinate the joints of higher-dimensional—more realistic—models of legged robots so that their behavior is governed by lower-dimensional control targets that encode the desired task; the model of Fig. 1 can serve as such a control target for bounding.

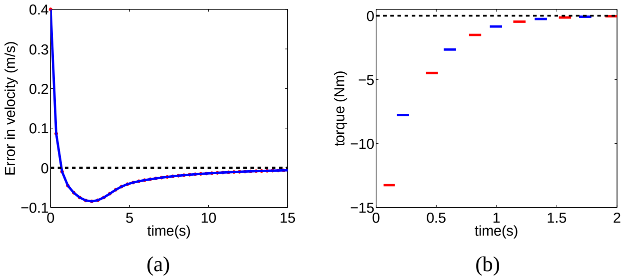

Figure 3: (a) Error in forward velocity at apex. (b) Back leg (red) and front leg (blue) hip actuator torques. The energy converges to its nominal value faster than the velocity.

5 Acknowledgment

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