Self-stable Bounding with a Flexible Torso

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1 Motivation

Various robotic quadrupeds have been introduced to investigate the realization of dynamically-stable running behaviors. The majority of these platforms involve rigid, non-deformable torsos, a feature that distinguishes them from their counterparts in the animal world, which owe much of their remarkable locomotion abilities to their flexible bodies. Only a few quadrupedal robots employ torso flexibility. An early example is the planar quadruped with articulated torso introduced in the MIT's Leg Lab, [1]. Contemporary robots with flexible torso include Canid [2] and the MIT Cheetah quadruped [3]. However, only limited information on how torso bending movements affect locomotion is available in the context of these platforms.

2 State of the art

To provide insight into the leg-torso coordination mechanisms that produce quadrupedal running in the presence of a segmented torso, models of varying complexity and different actuation schemes have been proposed. In particular, bounding motions have been investigated in [4, 5] using a sagittal-plane model composed of compliant legs and a two-segment torso. In [4], the torso spinal joint was actuated, and bounding was generated via PID control loops enforcing desired values on the relative angle between the two segments of the torso. On the other hand, in [5] the torso was unactuated and compliant, and bounding was achieved by keeping the torso joint angle constant when it reaches maximum flexion and extension. Our recent work in [6] provides a systematic analysis of the conditions under which cyclic bounding motions can be generated passively. In this work, rather than focusing on motion generation in the presence of torso flexibility, we turn our attention to the local stability properties of passively generated bounding motions.

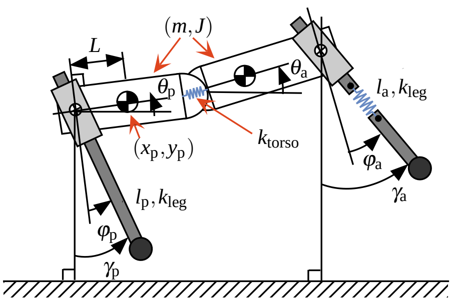


Figure 1: A sagittal-plane model with a segmented torso.

3 Our approach

The lower-dimensional, sagittal-plane model of Fig. 1 is the subject of our study. The torso comprises two identical rigid bodies connected via a torsional spring, intended to introduce flexibility. The posterior and anterior virtual legs are assumed to be massless prismatic springs.

3.1 Non-dimensional dynamics

In the non-dimensional setting, the six physical parameters $\{m, J, L, l_0, k_{torso}, k_{leg}\}$, which describe the morphology of the system, can be reduced to four dimensionless parameter groups, namely,

$$I := \frac{J}{mL^2}, \quad d := \frac{L}{l_0}, \quad \kappa_{leg} := \frac{k_{leg} l_0}{mg}, \quad \kappa_{torso} := \frac{k_{torso}}{mgl_0}. \qquad (1)$$

Note that l_0 corresponds to the uncompressed length of the leg spring. The non-dimensional form of the dynamics allows us to explore systematically a much larger fraction of the solution space for various combinations of the dimensionless parameters. We concentrate on the bounding gait described in Fig. 2, which includes extended and gathered flight phases, and apply Poincaré's method. The Poincaré section is selected as the apex height in the extended flight phase, and the resulting map is

$$z_f^*[k+1] = \mathscr{P}^*(z_f^*[k], \alpha_f^*[k]), \qquad (2)$$

where $z_f^* := (y_p^*, \theta_p^*, \theta_a^*, \dot{x}_p^*, \dot{\theta}_p^*, \dot{\theta}_a^*)'$ and α_f^* contains the absolute touchdown angles, i.e., $\alpha_f^* := (\gamma_a^{td*}, \gamma_p^{td*})'$; see Fig. 1.

3.2 Self-stable bounding motions

A large number of fixed points has been computed for suitable initial conditions and touchdown angles. To analyze local stability, we linearize (2) at a fixed point $(\bar{z}_f^*, \bar{\alpha}_f^*)$ and compute the eigenvalues of the Jacobian $A := \partial \mathscr{P}^* / \partial z_f^*$. One of the eigenvalues of A is located at 1 reflecting the conservative nature of the system. The location of the rest of the eigenvalues depends on the values of the dimensionless parameters. Of particular interest here are the combinations of the relative torso stiffness κ_{torso} and relative leg stiffness κ_{leg} that generate bounding motions.

Fig. 3 shows how the spectral radius $\rho(A) := \max_i |\lambda_i|$ of A changes as a function of $(\kappa_{leg}, \kappa_{torso})$ keeping the rest of the dimensionless parameters constant. Note that the grey area in Fig. 3 represents the irrelevant fixed points that correspond to periodic motions with multiple torso oscillations

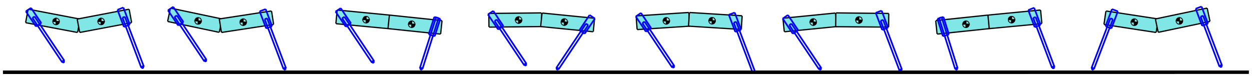


Figure 2: Snapshots of the model during one bounding cycle.

during one stride. Fig. 4(a) shows how the relative pitch $\theta_a^* - \theta_p^*$ evolves for such motions, which appear for small values of leg stiffness; clearly, a softer leg requires a relatively longer time period to go through a complete compression and decompression process during stance, allowing the torso to oscillate multiple times. We neglect such fixed points and focus on bounding motions with a single torso oscillation during one stride, as shown in Fig. 4(b). Such motions are represented by the colored-coded points of Fig. 3, in which we can see that for certain fixed points the spectral radius is equal to 1 implying that all but one of the eigenvalues are within the unit disc. This result shows that *self-stable* bounding can be generated for particular combinations of leg and torso stiffness values.

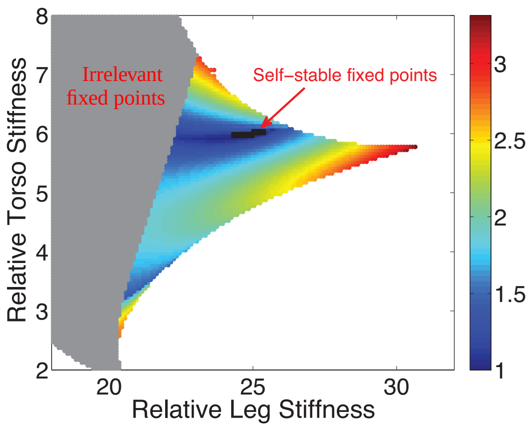
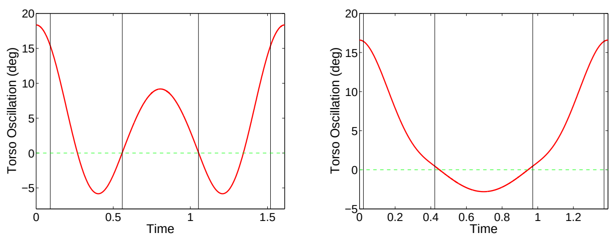


Figure 3: Fixed points computed for the same total energy, average speed and hopping height and for different values of dimensionless leg and torso stiffness. The color code corresponds to the values of the spectral radius of A.



(a) (b)

Figure 4: Torso oscillation in one stride. (a) An irrelevant fixed point. (b) A physically meaningful fixed point.

It should be emphasized that the emergence of self-stability in the presence of a flexible segmented torso is not an immediate consequence in view of the self-stable bounding orbits in quadrupeds with rigid torso as in [7]. The reason is that torso bending movements may cause divergent behavior when they are not properly coordinated with the hybrid oscillations of the legs. Also, it should be pointed out that the domain of attraction of the self-stable fixed points is small, and the system cannot tolerate large perturbations.

4 Discussion outline

In this work, the existence and stability of bounding running gaits were studied in the context of a reductive sagittal-plane model with a flexible segmented torso and compliant legs. The relationship between the leg and torso spring stiffness was discussed, showing that a range of possible leg-torso stiffness combinations can produce stable (within a constant energy level) bounding motions. However, the ability of the system to reject sufficiently large perturbations in an open-loop fashion is limited. Currently, we work on control laws that can enhance the stability of these self-stable motions. In addition, a more complete parametric study that includes I and d is conducted to further understand how physical parameters affect the resulting motions.

5 Acknowledgment

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