Decision Making in a Sensor Network with Poisson Process Observations

Chetan D. Pahlajani, Ioannis Poulakakis *Member, IEEE*, Herbert G. Tanner *Senior Member, IEEE*

***Abstract*— This paper addresses a detection problem where several spatially distributed sensors independently observe a time-inhomogeneous stochastic process. The task is to decide at the end of a fixed time interval between two hypotheses regarding the statistics of the observed process. In the proposed method, each of the sensors transmits once to a fusion center a locally processed summary of its information in the form of a likelihood ratio. The fusion center then combines these messages to arrive at an optimal decision in the Neyman-Pearson framework. The approach is motivated by applications arising in the detection of mobile radioactive sources, and it serves as a first step toward the development of novel fixed-interval detection algorithms that combine decentralized processing with optimal centralized decision making.**

I. INTRODUCTION

In applications where one is interested in measuring a physical process which varies spatially and temporally, the use of a *network* of sensors offers advantages over single-device solutions. Sensor networks, both mobile and static, have been developed for a wide range of applications from environmental monitoring [1]–[3], intruder detection [4], area coverage [5]–[8], and source localization [9]–[12], to mapping of spatially distributed physical quantities [13], [14]. In certain cases, the physical quantities of interest are generated by random processes characterized by events that are highly localized in time and/or space. Examples include customers to and from a service facility in queueing theory [15], [16], electron emission from a photodetector in an optical communications system [17], generation of electrical pulses in neurons [18], and nuclear measurement [19]–[21].

The mathematical framework for the modeling and analysis of these discrete random processes is provided by the theory of *point processes* [15], [22]–[24]. A realization of a point process is a random sequence of points, each point representing the time and/or spatial location of an event. The rate at which events occur is called the *intensity* of the point process. If one knows *a priori* that the point process intensity belongs to a set of finitely many alternatives, how should the local information gathered at individual sensors be processed and/or communicated to reach a global decision? In the context of detecting illicit radioactive material, such questions are of paramount importance to national security and nuclear nonproliferation. Since the detection of (shielded) nuclear material in transit, is very challenging [20], the option of deploying networks of radiation detectors is proposed in an effort to improve the efficiency of a monitoring system [19]–[21].

Classical approaches to network-based decision making involve sensors relaying the entirety of their observations to a central processing unit which analyzes the data and issues a global decision. While this *centralized* approach has the advantage of using all the information available, it does impose significant communication overhead. Alternatively, a *decentralized* or *distributed* decision making scheme [25]–[28] can be used; in this setting the sensors process measurement information and transmit a compressed version of it—typically in the form of a message with values in a finite alphabet—to a fusion center, which then provides a decision.

For the particular case of point processes, the problem of decision making between two alternative hypotheses ("all clear" versus "alarm") has been addressed in [15], [29]–[33]. The principal idea underlying the solution involves computing the correct "likelihood ratio" and comparing it against a threshold. The form of the likelihood ratio for a fairly large class of point processes is identified in [29]. In the specific context of radiation detection networks, related work is based on adaptations of the likelihood ratio test (LRT), using either sequential or fixed-interval testing theory [34], [35], and Bayesian or Neyman-Pearson approaches [12], [19], [36]. With the exception of [35], sensor observations have been assumed independent identically distributed (i.i.d.).

In the case where there is relative motion between the source and the sensors, as in this work, the point processes describing the arrival of rays at the detectors are time inhomogeneous. This paper develops an optimal decision making scheme for such time-inhomogeneous point processes, assuming that the motion of the suspected source is deterministic and known. In our setting, the sensors communicate processed information in the form of locally-computed likelihood ratios to the fusion center. The fusion center then combines these messages to arrive at a decision, without the need of any additional information such as the location or the raw data of individual sensors. With respect to the majority of the existing approaches that involve transmitting either raw data or (highly compressed) binary decisions to the fusion center, our method combines decentralized processing with centralized decision making.

The structure of the paper is as follows. Section II states formally the problem considered. Section III constructs the decision strategy and establishes its optimality in the context of the Neyman-Pearson Lemma. Section IV illustrates how threshold bounds for the likelihood ratio test can be

C. Pahlajani is with the Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, USA; e-mail: chetan@math.udel.edu

I. Poulakakis and H. G. Tanner are with the Department of Mechanical Engineering, University of Delaware, Newark, DE 19716, USA; e-mail: {poulakas, btanner}@udel.edu.

conservatively estimated, and these results are applied to a concrete nuclear detection problem in Section V. Section VI concludes the paper.

II. Problem Statement and Assumptions

Consider a collection of sensors observing a point process generated by some physical phenomenon. The goal is to decide between two hypotheses regarding the state of the environment. To this end, each sensor communicates a processed version of its observations to a fusion center, which combines all received messages to a binary decision (Fig. 1). The decision must be made within a fixed time interval.

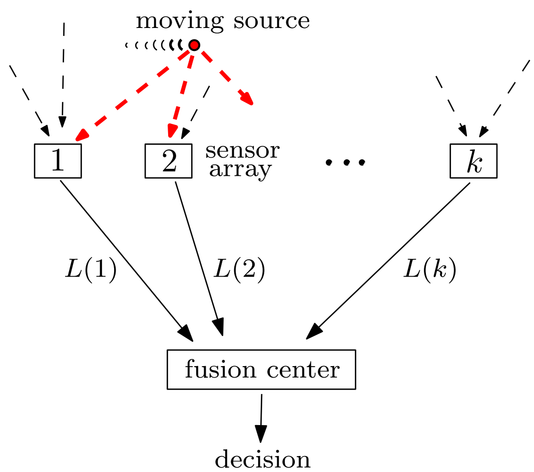


Fig. 1. The overall architecture of the detection scheme. There is a network of sensors each counting the total number of rays that have arrived at them. Thick arrows represent rays emitted from a moving radioactive source and thin arrows mark rays from background radiation. Based on the number and timing of arrival of those counts, each sensor computes a likelihood ratio $L(i)$ which is then transmitted to a fusion center. The fusion center combines this information to make a decision regarding the presence of the radioactivity in the target moving in front of the sensors.

The problem is formulated as a binary hypothesis test based on measurements from an array of k sensors connected in a parallel network architecture, as shown in Fig. 1. Let H_0 and H_1 denote the two hypotheses regarding the state of the environment; for example, the absence or presence of a radioactive source on the moving target.

Assumption 1: The reading at sensor i, where $1 \le i \le k$, is a non-decreasing $\mathbb{Z}^+$-valued piecewise constant, right-continuous function, which increases in steps of size one.

Assumption 2: Conditioned on hypothesis H_j, $j \in \{0,1\}$, the observations at distinct sensors are independent.

Sensor observations are modeled as inhomogeneous Poisson processes, whose intensities are deterministic functions of time. In particular, for $1 \le i \le k$, sensor i observes a Poisson process, whose time-dependent intensity under H_0 is $b_i(\cdot)$, while under H_1 is $b_i(\cdot) + R_i(\cdot)$. The explicit dependence of the intensities on time arises from the known motion of the target, which is a potential "source" of the point process with intensity R_i.

The detection problem can now be summarized as follows: Suppose that $T > 0$ is the *decision time*; that is, the time by which a decision must be made. Given a single realization of a k-dimensional vector of Poisson processes over the time horizon $[0, T]$ (the k components corresponding to the k sensors), decide whether the intensities are given by the collection $b_i(\cdot)$ or by the collection $b_i(\cdot) + R_i(\cdot)$, $1 \le i \le k$.

Let us scale the physical time $\tau \in [0, T]$, by introducing a rescaled time parameter $t \in [0, 1]$, which is defined by

$$t \triangleq \frac{\tau}{T} \ . \tag{1}$$

There are some analytical benefits in working with the dimensionless time t. The intensities of the corresponding Poisson processes must also be rescaled with respect to t. For $1 \le i \le k$, the intensity at sensor i at time $t \in [0, 1]$ under H_0 becomes $T\beta_i(t)$ (see Appendix) and under H_1 becomes $T(\beta_i(t) + \nu_i(t))$, where

$$\beta_i(t) \triangleq b_i(T \cdot t) \text{ and } \nu_i(t) \triangleq R_i(T \cdot t) \ . \tag{2}$$

As defined, functions β_i and ν_i depend on the decision time T, but to simplify notation we drop this dependence. Finally, the following assumptions will be imposed on β_i and ν_i.

Assumption 3: For $1 \le i \le k$, $\beta_i : [0, 1] \to [\beta_{\mathsf{min}}, \beta_{\mathsf{max}}]$ is a bounded, continuous function with $0 < \beta_{\mathsf{min}} < \beta_{\mathsf{max}} < \infty$, β_{min}, β_{max} independent of $i \in \{1, 2, \dots, k\}$.

Assumption 4: For $1 \le i \le k$, $\nu_i : [0, 1] \to [\nu_{\mathsf{min}}, \nu_{\mathsf{max}}]$ is a bounded, continuous function with $0 < \nu_{\mathsf{min}} < \nu_{\mathsf{max}} < \infty$, ν_{min}, ν_{max} independent of $i \in \{1, 2, \dots, k\}$.

III. Main Result

We start with a measurable space $(\Omega, \mathscr{F})$ supporting a k-dimensional vector of counting processes $N_t = (N_t(1), \dots, N_t(k))$, $t \in [0, 1]$. The *sample space* Ω is the set of all possible outcomes ω of a random experiment, and $\mathscr{F}$ is the σ-field (or σ-algebra) of *events*. Intuitively, $N_t(i)$ is the number of counts registered at sensor $i \in \{1, 2, \dots, k\}$ up to (and including) time $t \in [0, 1]$. Hypotheses H_0 and H_1 correspond to probability measures $\mathbb{P}_0$ and $\mathbb{P}_1$ on $(\Omega, \mathscr{F})$, with respect to which $N_t(i)$, $1 \le i \le k$, are independent Poisson processes with intensities $T\beta_i(t)$ and $T(\beta_i(t) + \nu_i(t))$, respectively. The decision problem is thus one of identifying the correct probability measure ($\mathbb{P}_0$ versus $\mathbb{P}_1$) on $(\Omega, \mathscr{F})$, based on a realization of the k-dimensional process $N_t = (N_t(1), \dots, N_t(k))$.

We will keep track of the flow of information using the *filtration*[1] $(\mathscr{F}_t^N : 0 \le t \le 1)$ generated by the process N_t. For any event $A \in \mathscr{F}_t^N$, an observer of the sample path $s \mapsto N_s$, $0 \le s \le t$, knows at time t whether or not the event A has occurred. If one uses the test $A_1 \in \mathscr{F}_1^N$ (i.e., decides H_1 if the outcome $\omega \in A_1$, or H_0 if the outcome $\omega \in \Omega \setminus A_1$), then two types of errors might occur. A *false alarm* occurs when $\omega \in A_1$ (i.e., one decides H_1) while H_0 is the correct hypothesis. A *miss* occurs when $\omega \in \Omega \setminus A_1$ (i.e., one decides H_0) while H_1 is the correct hypothesis. Clearly, the probability of false alarm is $\mathbb{P}_0(A_1)$, while the probability of

[1] Definition: A filtration $(\mathscr{F}_t : t \ge 0)$ is an increasing family of sub-σ-fields of $\mathscr{F}$, i.e. $\mathscr{F}_t \subset \mathscr{F}$ for all t, and $s \le t$ implies $\mathscr{F}_s \subset \mathscr{F}_t$. The σ-field $\mathscr{F}_t$ represents the information available at time t. $\mathscr{F}_t^N = \sigma(N_s : 0 \le s \le t)$ is the smallest σ-field on Ω with respect to which all the (k-dimensional) random variables N_s, $0 \le s \le t$ are measurable.

a miss is $\mathbb{P}_1(\Omega \setminus A_1)$. The probability of detection is given by $\mathbb{P}_1(A_1) = 1 - \mathbb{P}_1(\Omega \setminus A_1)$.

The next theorem characterizes an optimal policy in the Neyman-Pearson paradigm [15], [37], [38] for our decision-making problem. The optimal policy[2] is a threshold test for an explicitly computable likelihood ratio which maximizes the probability of detection among all tests with probability of false alarm less than or equal to a pre-specified $\alpha \in (0,1)$.

Theorem 1 (Main Result): Suppose $(\Omega, \mathscr{F}, \mathbb{P}_0)$ is a probability space, on which $N_t = (N_t(1), \dots, N_t(k))$, $t \in [0,1]$, is a vector of independent Poisson processes whose components $N_t(i)$ admit intensities $T\beta_i(t)$, $1 \le i \le k$, with $\beta_i(t)$ satisfying Assumption 3. Let $(\tau_n(i) : n \ge 1)$ denote the jump times of $N_t(i)$, $1 \le i \le k$ and for $t \in [0,1]$, define the process L_t by

$$L_t \triangleq \prod_{i=1}^{k} L_t(i) \tag{3}$$

with

$$L_t(i) \triangleq \exp\left(-T\int_0^t \nu_i(s)ds\right) \prod_{n=1}^{N_t(i)} \left(1 + \frac{\nu_i(\tau_n(i))}{\beta_i(\tau_n(i))}\right) \tag{4}$$

where $\nu_i(\cdot)$ satisfies Assumption 4. By convention, $\prod_{n=1}^{0}(\cdot) = 1$.

1) A probability measure $\mathbb{P}_1$ on $(\Omega, \mathscr{F})$, absolutely continuous with respect to $\mathbb{P}_0$, can be defined by

$$\frac{d\mathbb{P}_1}{d\mathbb{P}_0} \triangleq L_1 \ , \tag{5}$$

where L_1 is given by (3) and (4) with $t = 1$. Moreover, with respect to $\mathbb{P}_1$, $N_t(i)$ for $1 \le i \le k$, are independent Poisson processes over $t \in [0,1]$ with intensities $T(\beta_i(t) + \nu_i(t))$.

2) Suppose for $\alpha \in (0,1)$, there exists $\sigma > 0$ such that

$$\mathbb{P}_0(L_1 \ge \sigma) = \alpha \ . \tag{6}$$

Then, the test $A_1^* = \{L_1 \ge \sigma\}$ is optimal for $\mathscr{F}_1^N$-observations in the sense that for any $A_1 \in \mathscr{F}_1^N$ with $\mathbb{P}_0(A_1) \le \alpha$, we have $\mathbb{P}_1(A_1^*) \ge \mathbb{P}_1(A_1)$.

Proof: By a modification of the proof in [15, Theorem VI.2.T4], it can be shown that the process $(L_t : t \in [0,1])$ is a nonnegative $(\mathbb{P}_0, \mathscr{F}_t^N)$-martingale with $\mathbb{E}_0[L_t] = \mathbb{E}_0[L_0] = 1$ for all $t \in [0,1]$. Hence, $\mathbb{P}_1$ defined by (5) is a probability measure on $(\Omega, \mathscr{F})$, which is absolutely continuous with respect to $\mathbb{P}_0$. The first part of Theorem 1 now follows from [15, Theorem VI.2.T3] with $\lambda_t(i) \equiv T\beta_i(t)$, $\mu_t(i) = 1 + \nu_i(t)/\beta_i(t)$ for $1 \le i \le k$, and $\mathscr{F}_t \equiv \mathscr{F}_t^N$. We next prove the second part of Theorem 1. Since L_1 is a nonnegative, $\mathscr{F}_1^N$-measurable random variable which satisfies $\mathbb{P}_1(A) = \int_A L_1(\omega)\mathbb{P}_0(d\omega)$ for $A \in \mathscr{F}_1^N$, it follows that L_1 is the (unique) Radon-Nikodym derivative of $\mathbb{P}_1$ with respect to $\mathbb{P}_0$, both measures restricted to $\mathscr{F}_1^N$. The result is now a direct consequence of the Neyman-Pearson Lemma [15, Theorem VI.1.T1]. ■

[2]We restrict attention here to tests without randomization; see [37], [38].

Remark 2: Theorem 1 provides a solution to our detection problem that combines *lossless* decentralized processing with centralized decision making. In particular, each sensor collects information and forms a likelihood ratio through *local* processing. Then, at the decision time T all sensors communicate their likelihood ratios to the fusion center that combines them optimally in the Neyman-Pearson sense to provide the final decision. Note that this is different from the decentralized decision setting [39], [40], in that the sensors do not send decisions to the fusion center; rather they send a processed version of their individual observations.

Remark 3: The importance of the assumption of independent observations is that a sizable portion of the computation pertinent to decision making can be localized at the individual sensors, leading to significant savings in communication costs.

IV. Performance Analysis

Here we provide a lower bound on the probability of detection and an upper bound on the probability of false alarm when the proposed detection scheme is used. It turns out that bounds on both these probabilities involve the tails of (different) Poisson distributions. To compactly describe our results, we follow the notation of [41].

Definition 1 (Poisson Tails): For $\lambda > 0$, $j \in \mathbb{Z}^+$, let $p(\lambda, j)$ denote the Poisson distribution

$$p(\lambda, j) = e^{-\lambda}\frac{\lambda^j}{j!} \ .$$

The left and right tail probabilities are defined by

$$P(\lambda, n) = \sum_{j=0}^{n} p(\lambda, j), \qquad \overline{P}(\lambda, n) = \sum_{j=n}^{\infty} p(\lambda, j) \ , \tag{7}$$

respectively. Note that $P(\lambda, n-1) + \overline{P}(\lambda, n) = 1$.

The following quantities are also of interest:

$$B = \sum_{i=1}^{k} \int_0^1 \beta_i(s)ds \ , \qquad J = \sum_{i=1}^{k} \int_0^1 \nu_i(s)ds \ . \tag{8}$$

It now follows from (3)-(4) that

$$L_1 = \exp(-JT) \prod_{i=1}^{k} \prod_{n=1}^{N_1(i)} \left(1 + \frac{\nu_i(\tau_n(i))}{\beta_i(\tau_n(i))}\right) \ .$$

In the sequel we will also use the integer ceiling function $\lceil \cdot \rceil$ which assigns to a real number x the smallest integer greater than or equal to x.

For $\sigma > 0$, consider the test $A_1^* = \{L_1 \ge \sigma\}$. A lower bound on the probability of detection $\mathbb{P}_1(L_1 \ge \sigma)$, and an upper bound on the probability of false alarm $\mathbb{P}_0(L_1 \ge \sigma)$ can now be obtained as follows. Recalling Assumptions 3 and 4, define

$$C = 1 + \frac{\nu_{\mathsf{min}}}{\beta_{\mathsf{max}}} \le \min_{1 \le i \le k} \min_{0 \le t \le 1} \left(1 + \frac{\nu_i(t)}{\beta_i(t)}\right), \tag{9a}$$

$$D = 1 + \frac{\nu_{\mathsf{max}}}{\beta_{\mathsf{min}}} \ge \max_{1 \le i \le k} \max_{0 \le t \le 1} \left(1 + \frac{\nu_i(t)}{\beta_i(t)}\right). \tag{9b}$$

Note that if one can find ℓ^-, ℓ^+ such that $\ell^- \leq L_1 \leq \ell^+$, then for $j \in \{0, 1\}$,

$$\mathbb{P}_j(\ell^- \geq \sigma) \leq \mathbb{P}_j(L_1 \geq \sigma) \leq \mathbb{P}_j(\ell^+ \geq \sigma) \ .$$

Letting

$$\ell^- = \exp(-JT) \prod_{i=1}^{k} C^{N_1(i)} \ , \quad \ell^+ = \exp(-JT) \prod_{i=1}^{k} D^{N_1(i)}$$

it can be verified that $\ell^- \leq L_1 \leq \ell^+$. Next, note that

$$\ell^- \geq \sigma \iff \sum_{i=1}^{k} N_1(i) \geq \frac{\log \sigma + JT}{\log C}$$
$$\ell^+ \geq \sigma \iff \sum_{i=1}^{k} N_1(i) \geq \frac{\log \sigma + JT}{\log D} \ .$$

Since $N_1(i)$ for $1 \leq i \leq k$ are independent Poisson random variables with parameters $T \int_0^1 \beta_i(s)ds$ with respect to $\mathbb{P}_0$, it follows that $\sum_{i=1}^{k} N_1(i)$ is a Poisson random variable with parameter $BT = (\sum_{i=1}^{k} \int_0^1 \beta_i(s)ds)T$ with respect to $\mathbb{P}_0$. Under the probability measure $\mathbb{P}_1$, $N_1(i)$ for $1 \leq i \leq k$ are independent Poisson random variables with parameters $T(\int_0^1 [\beta_i(s)+\nu_i(s)]ds)$. It follows that under $\mathbb{P}_1$, $\sum_{i=1}^{k} N_1(i)$ is a Poisson random variable with parameter $(J + B)T$. Hence,

$$\mathbb{P}_1(L_1 \geq \sigma) \geq \overline{P}\left((J+B)T, \left\lceil \frac{\log \sigma + JT}{\log C} \right\rceil\right) \tag{10a}$$
$$\mathbb{P}_0(L_1 \geq \sigma) \leq \overline{P}\left(BT, \left\lceil \frac{\log \sigma + JT}{\log D} \right\rceil\right). \tag{10b}$$

V. Example: Nuclear Detection

An example of a detection problem that fits this framework is that of nuclear detection using spatially distributed sensors. Radiation sensors always record background radiation (due to cosmic radiation and due to naturally occurring radioactive isotopes in the environment). In the absence of illicit nuclear material (hypothesis H_0 is true), the sensors simply measure background. If radioactive material is present (hypothesis H_1 is true), the sensors record the *sum* of the photons coming from background and the photons coming from the material. These two sources of radiation act independently, and one can treat each sensor as observing a single Poisson process whose intensity is the *sum of intensities* due to background and material (the source). The problem we face is to determine in a fixed amount of time whether a target passing in front of the sensors, is a source of radiation.

The specific assumptions for this problem are as follows: Our workspace is the horizontal plane, $\mathbb{R}^2$. We have k sensors uniformly spaced along the positive x-axis at locations $x = 0$, $x = \ell$, $x = 2\ell, \ldots, x = (k-1)\ell$, where $\ell = 10$ m. For simplicity here, we assume that the sensors are identical. Let us denote $\tau \in [0, T]$ the physical time elapsed between the instant the target is detected and count recording is initiated, and the final time T at which a decision regarding the existence of a source is to be made. Let $b_i(\tau)$ be the intensity of background radiation at the location of sensor i, $1 \leq i \leq k$, which does not have to be uniform and in general can be time-dependent. In this example we assume that background intensity is time-invariant, so $b_i(\tau) = \beta_i \in \mathbb{N}$, where β_i is assumed to be varying exponentially between locations, from a minimum of $\beta_{\mathsf{min}} = 2$ counts per second (cps) to a maximum of $\beta_{\mathsf{max}} = 8$ cps, with the maximum appearing at the first and last sensor and the minimum occurring at the sensor in the middle (Fig. 2(a)). We assume that a target will be passing at a distance $h = 20$ m from the x-axis, namely with a constant coordinate $y = h$, appearing first at some initial location $(x_0, h) = (-4, 20) \in \mathbb{R}^2$ and moving with constant speed $v = 20$ m/s in the direction of the positive x-axis.

To illustrate the derivation process, let us take $k = 10$, and for the sake of argument assume that the acceptable probability of false alarm in this scenario is $\alpha = 10^{-6}$ (see (10b)). With $r_i(\tau)$, $1 \leq i \leq k$, denoting the distance between sensor i and the potential source, the intensity $R_i(\tau)$ at sensor i due to the source is modeled in [19] by[3]

$$R_i(\tau) = \frac{\chi a}{r_i(\tau)^2} \ , \tag{11}$$

where $a > 0$ is the activity of the potential source and $\chi > 0$ is the sensors' cross-section coefficient.[4] We assume a numerical value for χa equal to what has been used in [19], but shielded in 9 cm of lead, dropping the source's perceived intensity by three orders of magnitude to $\chi a = 5.068$ cps. We also assume that no sensor is ever closer than distance h to the target, ensuring that $R_i(\tau)$ is always bounded.

Since the location of the potential source at time $\tau \in [0, T]$ is $(x_0 + v\tau, h)$, the distance $r_i(\tau)$ between the potential source and sensor i, $1 \leq i \leq k$ is given by $r_i(\tau) = \sqrt{(x_0 + v\tau - (i-1)\ell)^2 + h^2}$. Recalling (11) and (2) with

$$T = \frac{(k-1)\ell - 2x_0}{v} = 9.8 \text{ s}$$

for the decision time, we get for the scaled time variable $t = \frac{\tau}{T} \in [0, 1]$ (see (1))

$$\beta_i(t) = b_i(T \cdot t)$$
$$\nu_i(t) = R_i(T \cdot t) = \frac{\chi a}{(x_0 + vT \cdot t - (i-1)\ell)^2 + h^2} \ .$$

Since

$$\int_0^1 \nu_i(s)ds = \frac{\chi a}{hvT} \tan^{-1}\left(\frac{x_0 + vT - (i-1)\ell}{h}\right) - \frac{\chi a}{hvT} \tan^{-1}\left(\frac{x_0 - (i-1)\ell}{h}\right),$$

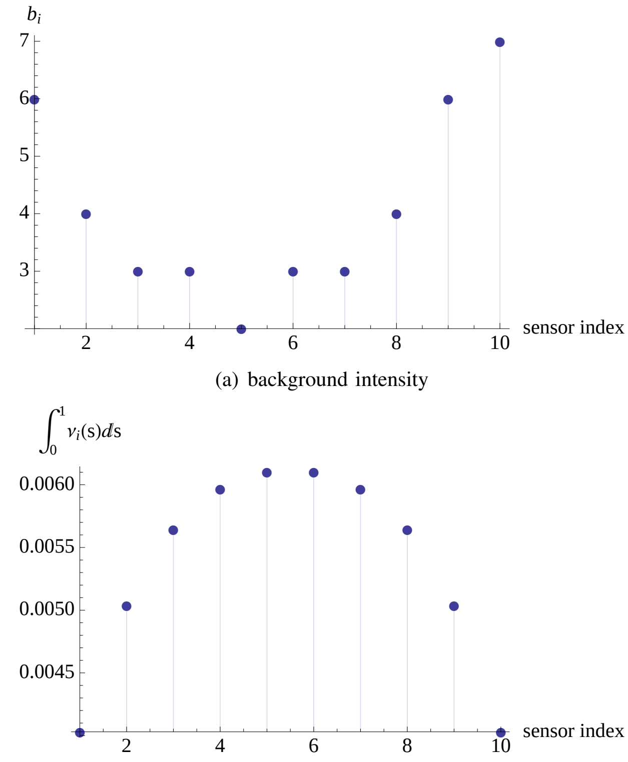
from (8) we get

$$J = \frac{\chi a}{hvT} \sum_{i=1}^{k} \left[\operatorname{atan}\left(\tfrac{x_0 + vT - (i-1)\ell}{h}\right) - \operatorname{atan}\left(\tfrac{x_0 - (i-1)\ell}{h}\right)\right]$$

and for the ten-sensor array we have $J = 0.05356$ cps/m^2.

[3] In fact, for a planar detection scenario the sold angle scales proportionally to $1/r_i$.

[4] For the case of heterogenous sensors, each will have its own χ_i.



(a) background intensity

(b) $\int_0^1 \nu_i(s)ds$

Fig. 2. Distribution of background intensity (up) and integrated perceived source intensity (bottom) over the considered ten-sensor array.

It is not hard to see that

$$r_{\mathsf{min}} = \min_{1\le i\le k} \min_{0\le\tau\le T} r_i(\tau) = h$$
$$r_{\mathsf{max}} = \max_{1\le i\le k} \max_{0\le\tau\le T} r_i(\tau) = \sqrt{(x_0+vT)^2+h^2}\ .$$

Thus, for the case of the ten-sensor array, (9) evaluates to

$$C = 1 + \frac{\chi a}{[(x_0+vT)^2+h^2]\beta_{\mathsf{max}}} = 1 + 6.859\times 10^{-5}$$
$$D = 1 + \frac{\chi a}{h^2\beta_{\mathsf{min}}} = 1 + 6.335\times 10^{-3}$$

in units of m^{-2}. With reference to Fig. 2(a), we have $B = \sum_{i=1}^{k} \beta_i = 41$ cps, and with this we turn to (10b) to numerically compute the threshold σ for the likelihood ratio test. Evaluating the right hand side of (10b) numerically for different values of σ, we see that for $\sigma = 14$, the probability of false alarm $\mathbb{P}_0$ falls below 10^{-6}, which is the acceptable error rate. The decision rule therefore is based on the test:

$$L_1 \ge 14 \tag{12}$$

which if true, suggests that the target is indeed a radioactive source. It should be mentioned that there is conservatism built in the bounds (9), which renders the probability of detection using (12) rather small for the given false alarm rate. Improving the bounds in (9) is part of ongoing work. Nevertheless, the analysis still gives insight into the effect of different parameters on the probability of detection. To illustrate that point, let us consider the possibility of using more sensors with the same spacing ℓ as before. Without changing the decision rule (12) (keeping the same threshold), the analysis shows (Fig. 3) how the upper bound on the probability of false alarm (PFA) estimated in (10b) falls monotonically with the addition of new sensors.

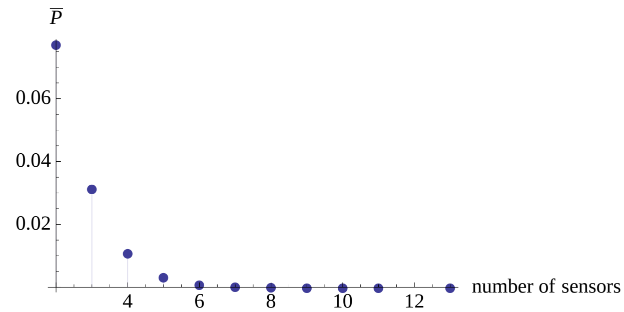


Fig. 3. Bound on the probability of false alarm (PFA) for $\sigma = 14$. Without resetting the threshold constant, we see that the upper bound on the PFA decreases exponentially with the addition of new sensors. For $k = 3$ sensors, $\sigma = 14$ yields a PFA of 3%, while for $k = 13$ sensors and for the same σ the bound falls to 1.5×10^{-9}.

VI. Conclusions

A network of sensors is employed to optimally decide between two hypotheses regarding the statistics of a time-inhomogeneous point process. The sensors collect their measurements over a fixed-time interval, at the end of which a processed summary is communicated to a fusion center. In particular, each sensor transmits a locally computed likelihood ratio to the fusion center, which then compares the product of the sensor-specific likelihood ratios against a threshold to arrive at a decision. The analysis is based on the Neyman-Pearson formulation. A set of conservative performance bounds on the error probabilities is provided and the framework is applied to the problem of detecting a moving radioactive source using an array of sensors.

Appendix

The effect of rescaling time on the intensity of a Poisson process is discussed by the following lemma.

Suppose $(\Omega, \mathscr{F}, \mathbb{P})$ is a probability space equipped with the filtration $(\mathscr{G}_u : u \ge 0)$. Let $\lambda(u)$ be a nonnegative, measurable function defined on $[0,\infty)$ with $\int_0^u \lambda(v)dv < \infty$ for all $u > 0$. Let $(X_u : u \ge 0)$ be a $(\mathbb{P}, \mathscr{G}_u)$-Poisson process with intensity $\lambda(u)$.

Lemma 1: Fix $T > 0$. Let $t = \frac{u}{T}$. Let $Y_t = X_u = X_{T\cdot t}$, $\mathscr{F}_t = \mathscr{G}_u = \mathscr{G}_{T\cdot t}$ for $t \ge 0$. Define $\hat{\lambda}(t)$ on $[0,\infty)$ by

$$\hat{\lambda}(t) \triangleq T\lambda(T\cdot t)$$

for $t \ge 0$. Then, Y_t is a $(\mathbb{P}, \mathscr{F}_t)$-Poisson process with intensity $\hat{\lambda}(t)$.

Proof: Note that $\hat{\lambda}(t)$ is nonnegative and measurable with $\int_0^t \hat{\lambda}(s)ds < \infty$ for all $t > 0$. Next, since $X_{T\cdot t}$ is $\mathscr{G}_{T\cdot t}$-measurable for all $t \ge 0$, it follows that Y_t is $\mathscr{F}_t$-adapted. To complete the proof, we need to show that for $0 \le s \le t$, $Y_t - Y_s$ is independent of $\mathscr{F}_s$ and is a Poisson random variable with parameter $\int_s^t \hat{\lambda}(\tau)d\tau$. Since $X_{T\cdot t} - X_{T\cdot s}$ is independent of $\mathscr{G}_{T\cdot s}$, it follows that $Y_t - Y_s$ is independent of $\mathscr{F}_s$. Finally, for $n \in \mathbb{Z}^+$, we have

$$\begin{aligned}\mathbb{P}(Y_t - Y_s = n) &= \mathbb{P}(X_{T\cdot t} - X_{T\cdot s} = n) \\ &= \exp\left(-\int_{T\cdot s}^{T\cdot t} \lambda(u)du\right) \frac{\left(\int_{T\cdot s}^{T\cdot t} \lambda(u)du\right)^n}{n!} \\ &= \exp\left(-\int_{s}^{t} \hat{\lambda}(\tau)d\tau\right) \frac{\left(\int_{s}^{t} \hat{\lambda}(\tau)d\tau\right)^n}{n!} \quad ,\end{aligned}$$

where the last equality follows by making the change of variables $\tau = u/T$. ∎

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