On the Effects of Design Parameters on Quadruped Robot Gaits

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Abstract—In this work, we link the various parameters that characterize a quadruped robot to the gait that it eventually opts for the steady state part of its locomotion. We then introduce an appropriate class of motion patterns, and provide some evidence that certain parameter values tend to favor specific pattern subclasses. This offers a new robot walking behavior qualitative view and some novel guidelines for legged systems design, in regard to the gait-effects of each design parameter. To accomplish the above, we adopt the use of an underactuated quadruped robot, and the Hildebrand diagrams paradigm for the succinct portraiture of its gaits, as well as the use of two distinct definitions regarding the motion stride.

I. INTRODUCTION

In the past years, a large number of research activities have taken place in regard to legged robots. This is mainly due to the advantages legged robots have in unstructured or unprepared terrains, in comparison to wheeled locomotion [1], [2]. A great number of these activities concern quadruped robots, since they achieve a good trade-off between performance and design and control complexity.

Many interesting quadruped legged robots have been developed, including the BigDog and Cheetah of Boston Dynamics, which are able to perform a number of different tasks [3]. The way quadruped robots locomote affects many aspects of their function, like energy efficiency, dynamic stability and disturbance robustness. Thus, it is important to be able to describe concisely the way a quadruped robot moves. But how can one achieve this? One of the possible ways, is the Hildebrand diagram [4]. Another way to do this are the support sequences of Muybridge [5], [4]. Both of these techniques were first used in Biology, for the portraiture of the gaits of legged animals.

In this paper, we consider the problem of characterizing the impact the various quadruped robot parameters can carry on its locomotion behavior. To do this, (a) we employ two different robot models which differ in the existence of leg mass, and implement them so as to be able to conduct comparisons. Then, (b) we devise a novel method for computing the Hildebrand diagram pertaining to any given simulated robot motion. Our approach (c) employs two different definitions for the motion stride, the classical and the alternative one [6], [4]. Next, (d) we explore the effects of various robot parameters to the gait the robot eventually employs, by using the simulation environment and the procedure for computing Hildebrand diagrams. Finally, (e) we attempt to provide some intuition about the steady state part (i.e., the periodic part) of the robot motion, by employing a new family of abstract patterns which are able to characterize robot motions that are not composed of a single gait only, but rather by a periodic sequence of gaits.

In general, a gait is defined such that at each stride the motion repeats itself in all detail [4]. However, there are cases at which the robot does a few strides, which are all different among them, but after this number of strides the robot is at the same condition as it was at the beginning of these strides. Here, we cannot speak of a gait, but for steady state patterns in which the robot locomotes. We search for these patterns in a mechanistic way—by exploiting a number of mathematical relations that emerge between the computed Hildebrand diagram components, for both the two stride definitions, and the structure of these patterns. In this paper, we expand our previous preliminary work [7] so as to include both the examination of more robot parameters, and the introduction and computation of the new aforementioned patterns. We also provide some novel results about the connection between gravity acceleration, leg rest length, as well as leg mass, and the gait that is eventually selected by the robot in its steady state.

II. GAITS AND HILDEBRAND DIAGRAMS

In the study of legged animals or robots, the concepts of stride and gait are very important. Stride is the part of the robot motion that is defined by any two successive touchdowns of its left hind leg. The gait is a way to categorize the stride of a given robot locomotion, by incorporating its main features about the placement of each leg and their respective touch-down and lift-off timings.

Strides and gaits can be described and analysed using Hildebrand diagrams and/ or gait graphs [4]. The format of a Hildebrand diagram, see Fig. 1, is as follows: On the vertical axis, there are four black horizontal line segments, each corresponding to a different leg. When a leg touches the ground, the respective line segment is depicted to be thick, whereas when it is on the air, this segment is depicted to be dotted. On the horizontal axis appears the % percentage of the stride duration that is completed.
The quantities HDF, FDF and PR that appear in the Hildebrand diagrams are respectively: the hind (back) and fore (front) duty factors, and the phase relationship. Each duty factor informs us about the fraction of the stride duration the hind or fore legs touch the ground. The phase relationship informs us about the time difference, as a fraction of stride duration, between the touch-downs of the hind and fore legs.

An alternative form is the gait graph, see Fig. 2, in which each gait is depicted as a dot. The vertical axis shows the mean value of the hind and fore duty factors (or mean factor). The phase relationship (or phase) and the horizontal the mean fraction of stride duration, between the touch-downs of the hind and fore legs.

There is a vast variety of different possible gait forms. For example, whenever the hind or fore pair of legs operate in phase the gait that results is called bound. In the case where all the legs operate in phase, we have pronk [8].

III. THE QUADRUPED ROBOT

A planar quadruped robot consists of a body, the hind and fore compliant virtual legs, or VLegs [1], and two rotational joints. A graphical illustration is depicted in Fig. 3.

**Modeling.** For the modeling of the quadruped robot [9], [10], [11], we employed the Euler–Lagrange method [12], [2]. Let $U$ denote the potential energy of the robot, $T$ its kinetic energy and let $t$ denote time. Let, also, $\dot{q} = \frac{dq}{dt}$, for any modeling quantity $q$. Using the Lagrangian

$$L = U - T$$

and the energy losses

$$\Delta = \frac{1}{2} b_i^2 + \frac{1}{2} b_f^2$$

and, denoting by $\tau_{z,h}$ and $\tau_{z,f}$ the torques exerted at the hips of the VLegs, the power input

$$\Pi = \dot{\phi}_h \tau_{z,h} + \dot{\phi}_f \tau_{z,f}$$

the equations of motion are given by

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\mu}} \right) = \frac{\partial L}{\partial \mu} + \frac{\partial \Pi}{\partial \dot{\mu}} - \frac{\partial \Delta}{\partial \dot{\mu}}$$

where $\mu$ ranges over the appropriate, each time, set of degrees of freedom (DOFs) since this process is implemented for two different models of the robot — according to whether the legs are considered to have mass or not.

**Model A.** [DOFs: $x, y, \theta$] The Lagrangian for a robot with negligible leg mass is (see Fig. 3a):

$$L_A = \frac{1}{2} m_0 (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} J_0 \dot{\theta}^2 - \frac{1}{2} k (l_0 - l_f)^2$$

$$- \frac{1}{2} \left( \frac{l_0 - l_f}{2} - m_g \cos(\phi) y - m_b \sin(\phi) x \right)$$

**Model B.** [DOFs: $x, y, \theta, \phi_h, \phi_f$] The Lagrangian for a robot with legs with significant leg mass is (see Figs. 3a and 3b):

$$L_B = \frac{1}{2} m_0 (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} J_0 \dot{\theta}^2 - m_g \sin(\phi) x$$

$$- \frac{1}{2} m_0 \cos(\phi) y + \frac{1}{2} m_0 (x_h^2 + y_h^2) + \frac{1}{2} J_h \dot{\gamma}_h^2$$

$$- \frac{1}{2} k (l_0 - l_h)^2 - m_b \sin(\phi) x_h - m_g \cos(\phi) y_h$$

$$+ \frac{1}{2} m_f (\dot{x}_f^2 + \dot{y}_f^2) + \frac{1}{2} J_f \dot{\gamma}_f^2 - \frac{1}{2} k (l_0 - l_f)^2$$

$$- m_g \sin(\phi) x_f - m_g \cos(\phi) y_f.$$
In the flight phase of the robot described by model $B$, the conservation of the angular momentum is taken into account, as it provides an integral of motion. Note that the torques exerted at the hips are quantities bounded by some function defined on foot–ground friction — according to criteria regarding foot slipping and loss of balance [13].

**Dynamical Phases.** The dynamical phases of our modeled robot are four: flight, hind stance, double stance and fore stance. In flight, every leg is on the air. In double stance, every leg touches the ground. In hind stance, only the hind VLeg touches the ground. Similarly, we define fore stance.

**Control Philosophy.** In order for our quadruped to locomote, there should be a way to sustain the energy amount required for its motion. This is accomplished by compensating the inevitable energy losses occurring at each stance phase [10]. In each stance, the hip joint motors are activated to compress the leg springs push forward the robot body — and thus to add energy to the system. So, each time the robot is on the air, the control system rotates the legs in order for these to land safely in the upcoming stride. In this way, the gaits that result are the same to those obtained by a passive robot, i.e., robot motion without energy losses — although here all losses are compensated for by power provided by the controlled hip motors. We note that the controller takes as inputs the desired CoM apex height and the CoM forward velocity values, thus, we can be sure that the leg tip will not collide with the ground—if one adjusts appropriately the desired CoM apex height value.

**IV. HILDEBRAND DIAGRAMS COMPUTATION**

To generate a Hildebrand diagram, we first isolate the periodic part of the robot motion, and then we compute the quantities HDF, FDF and PR that regard the steady state part of its motion.

*Transition Time Instance.* We define as steady state the part of the robot motion that is periodic. The complement of the steady state, with respect to the total motion duration, is called transient state. Let $t_{TIS}$ (i.e., *Transient to Steady*) be the time instance that separates the transient from the steady state part. We present an example about the nature of the time instance $t_{TIS}$, in Fig. 4.

**Components Calculation.** The quantities that compose a Hildebrand diagram, are the quantities HDF, FDF and PR, see Figs. 1 and 2. We will employ two different definitions about the stride [4], [6]. The first is the classical (subscript $C$) definition in which the stride is defined as the part of the robot locomotion located in between of two successive touch-downs of the hind VLeg. The alternative (subscript $A$) definition, is that in which the stride is perceived as the part of the robot locomotion contained between two successive touch-downs of the fore VLeg. Let $Q \in \{HDF, FDF, PR\}$. We denote as $Q_j$, with $j \in \{C, A\}$, the vectors that hold the values of the quantity $Q$ computed according to the classical and the alternative stride definition, respectively, for all the steady state strides. For a vector $V$, we denote by $V[i]$ its $i^{th}$ entry. The need for using two stride definitions comes from the fact that when using only the classical one, and the robot lands with its fore legs after a flight phase, for two consecutive flight phases, we get unrealistically large ($\geq 50\%$) PR value results, see Fig. 5, in which, for the $i^{th}$ steady state stride, say, we get

$$PR_C[i] = \frac{\alpha}{\beta} \approx 0.95\%$$

$$PR_A[i] = \frac{\gamma}{\delta} \approx 0.05\%$$

On the contrary, the values of HDF and FDF do not differ by much ($< 5\%$) when using separately the two stride definitions. For these quantities, we use only the classical.

![Fig. 4: Transition time instance example.](image-url)

![Fig. 5: Unrealistically large PR value computation when using the classical definition.](image-url)
and, for every \( j \in \{ C, A \} \), we also define
\[
Q_j^\text{Small} = \text{def the subvector of } Q_j,
\]
consisting of its Small values only. \(10\)

Finally, if we denote by "\( \cup \)" the vector-union operation, that allows element-repetitions, we set
\[
\begin{align*}
\text{HDF} & = \text{def mean (HDF}_C) \approx \text{mean (HDF}_A) , \\
\text{FDF} & = \text{def mean (FDF}_C) \approx \text{mean (FDF}_A) , \\
\text{PR} & = \text{def mean (PR}_C^\text{Small} \cup \text{PR}_A^\text{Small}) . 
\end{align*}
\]

\textbf{Steady State Patterns.} We define a class of patterns that are more abstract from the concept of gait. Here, we are concerned only about the first VLeg to touch the ground, after a flight phase. For example, by "HFF" we denote the pattern that, according to which, the steady state is composed after a flight phase. For example, by "Steady State Patterns."

\textbf{Pattern Model.} The respective subpattern when encountering a Big value in PR\(_C\), and a "HF" or a "HH" subpattern when encountering a Small value in PR\(_C\). But identifying "HH" is impossible since there are no Big values in PR\(_A\). All together, give the pattern "HFF." In a similar way, one can justify the conditions that lead to the rest of the patterns.

\textbf{V. SIMULATION RESULTS AND ANALYSIS}

Using the two proposed robot models, i.e., model A, see eq. (5), and model B, see eq. (6), the developed methodology for computing the Hildebrand diagrams, and the set of initial conditions and nominal parameter values of Table III, that are directly related to the NTUA Quadruped [11], we explore the effects of various robot parameters to the characteristic measures of the gaits followed by the robot.

\textbf{TABLE III} 
\textbf{Nominal Parameter Values}

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_b)</td>
<td>Body mass</td>
<td>9.25</td>
<td>kg</td>
</tr>
<tr>
<td>(m_l)</td>
<td>Leg mass (in Model B only)</td>
<td>0.61</td>
<td>kg</td>
</tr>
<tr>
<td>(d)</td>
<td>Hip joint distance</td>
<td>0.25</td>
<td>m</td>
</tr>
<tr>
<td>(l_0)</td>
<td>Lag rest length</td>
<td>0.30</td>
<td>m</td>
</tr>
<tr>
<td>(k)</td>
<td>Leg stiffness</td>
<td>680000</td>
<td>N/m</td>
</tr>
<tr>
<td>(b)</td>
<td>Leg dissipation coefficient</td>
<td>10.00</td>
<td>Ns/m</td>
</tr>
<tr>
<td>(g)</td>
<td>Gravity acceleration</td>
<td>9.81</td>
<td>m/s(^2)</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Ground inclination</td>
<td>0.00</td>
<td>rad</td>
</tr>
<tr>
<td>(x_{des})</td>
<td>Desired body CoM forward speed</td>
<td>1.00</td>
<td>m/s</td>
</tr>
<tr>
<td>(h_{des})</td>
<td>Desired body CoM apex height</td>
<td>0.32</td>
<td>m</td>
</tr>
<tr>
<td>(x_0)</td>
<td>Initial body CoM height</td>
<td>0.35</td>
<td>m</td>
</tr>
<tr>
<td>(x_{fo})</td>
<td>Initial body CoM forward speed</td>
<td>0.60</td>
<td>m/s</td>
</tr>
<tr>
<td>(y_0)</td>
<td>Initial body CoM vertical speed</td>
<td>0.00</td>
<td>m/s</td>
</tr>
<tr>
<td>(\theta_0)</td>
<td>Initial body CoM pitch angle</td>
<td>0.60</td>
<td>m/s</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Robot-specific parameter used in pattern computation</td>
<td>0.35</td>
<td>—</td>
</tr>
</tbody>
</table>

\textbf{TABLE II} 
\textbf{Pattern Computation and Portaiture}

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Model A/B</th>
<th>PR(_C)/PR(_A)</th>
<th>Equivalent Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>(\bigotimes)(\bigotimes) (\bigotimes)(\bigotimes)</td>
<td>s/B</td>
<td>(\text{min}(PR}_A) \times \text{max}(PR}_C) + \tau</td>
</tr>
<tr>
<td>F</td>
<td>NO/NO</td>
<td>B/s</td>
<td>(\text{min}(PR}_C) &gt; \text{max}(PR}_A) + \tau</td>
</tr>
<tr>
<td>HF</td>
<td>NO/NO</td>
<td>s/s</td>
<td>(\text{tr}(PR}_C) = 0 &amp; \text{tr}(PR}_A) = 0</td>
</tr>
<tr>
<td>One F</td>
<td>(\bigotimes)(\bigotimes) (\bigotimes)</td>
<td>s/s</td>
<td>(\text{max}(PR}_C) &gt; \text{max}(PR}_A) + \tau</td>
</tr>
<tr>
<td>HFF</td>
<td>(\bigotimes)(\bigotimes) (\bigotimes)</td>
<td>s/s</td>
<td>(\text{tr}(PR}_C) = \text{pr}(PR}_C) - 1 &amp; \text{tr}(PR}_A) = 0</td>
</tr>
<tr>
<td>FHH</td>
<td>(\bigotimes)(\bigotimes) (\bigotimes)</td>
<td>s/s</td>
<td>(\text{tr}(PR}_A) = \text{pr}(PR}_A) - 1 &amp; \text{tr}(PR}_C) = 0</td>
</tr>
<tr>
<td>Random</td>
<td>(\bigotimes)(\bigotimes) (\bigotimes)</td>
<td>s/s</td>
<td>Applies if no other pattern does.</td>
</tr>
</tbody>
</table>

\textbf{But why does the condition} \(\text{tr} \ (PR}_C) = \text{pr}(PR}_C) - 1 \& \text{tr} \ (PR}_A) = 0 \) \textit{imply the pattern HFF}? Well, the values of the \(PR}_C\) are Small or Big in an alternating fashion since the number of transitions is the maximum possible, \(|PR}_C| - 1\). Can the values of PR\(_A\) be all Big? No, since every Big value in PR\(_C\) is matched to two Small values in PR\(_A\). Thus, PR\(_A\) contains only Small values. So we can identify a “FF” subpattern when encountering a Big value in PR\(_C\), and a "HF" or a "HH" subpattern when encountering a Small value in PR\(_C\). But identifying "HH" is impossible since there are no Big values in PR\(_A\). All together, give the pattern "HFF." In a similar way, one can justify the conditions that lead to the rest of the patterns.

\textbf{TABLE I} 
\textbf{PR\(_j\) Vectors Specifications, for \( j \in \{ A, C \} \)}

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>The respective PR(_j) vector contains Small values only.</td>
</tr>
<tr>
<td>B</td>
<td>The respective PR(_j) vector contains Big values only.</td>
</tr>
<tr>
<td>sB</td>
<td>The respective PR(_j) vector contains Small or Big values.</td>
</tr>
<tr>
<td>sBo</td>
<td>The respective PR(_j) vector contains Small or Big values in an specific, alternate, fashion.</td>
</tr>
</tbody>
</table>

In Table II, we present a set of conditions which, when held true, characterize the robot locomotion in terms of the pattern that abstractly describes it. Note that "NO" stands for \textit{not observed}, and that all the other combinations of PR vectors specifications are mathematically impossible. Note, also, that for a vector \( V \), \(|V|\) denotes the number of its elements.

\textbf{But why does the condition} \(\text{tr} \ (PR}_C) = \text{pr}(PR}_C) - 1 \& \text{tr} \ (PR}_A) = 0 \) \textit{imply the pattern HFF}? Well, the values of the \(PR}_C\) are Small or Big in an alternating fashion since the number of transitions is the maximum possible, \(|PR}_C| - 1\). Can the values of PR\(_A\) be all Big? No, since every Big value in PR\(_C\) is matched to two Small values in PR\(_A\). Thus, PR\(_A\) contains only Small values. So we can identify a “FF” subpattern when encountering a Big value in PR\(_C\), and a "HF" or a "HH" subpattern when encountering a Small value in PR\(_C\). But identifying "HH" is impossible since there are no Big values in PR\(_A\). All together, give the pattern "HFF." In a similar way, one can justify the conditions that lead to the rest of the patterns.

To explore the parameters effects, we create some multiple gait graphs, that is, simultaneously plotted gait graphs for many simulated motions which differ by a single value — the value of the parameter whose effects we want to examine regarding robot walking. The arrows in these figures indicate that the parameter under study is increasing along the arrow direction. The markers refer to the patterns of Table II.

\textbf{Design Parameters.} We investigate the significance of body mass, leg rest length and leg stiffness to the robot locomotion, using Fig. 6. We treat each parameter separately:

As shown in Fig. 6a, as the body mass increases, the mean factor is increasing from \(32\%–38\%\) to \(56\%–58\%\) for both models. The reason is that greater mass forces the legs to stay longer on the ground. However, as also shown in Fig. 6a, the phase does not change much.

Observing Fig. 6b, we have that as the leg rest length increases, the mean factor increases from \(20\%–25\%\) to \(40\%–45\%\) for both models. This result is somewhat surprising, since longer legs can lead to greater jump-steps, and thus less ground contact, but greater leg rest length can also lead
to more frequent ground contact, for equilibrium reasons, as it is the case here. From Fig. 6b we also deduce that phase does not get affected significantly.

Fig. 6c, shows that the increase in leg stiffness reduces the mean factor from 54%–58% to 28%–30% for both models. This is a predictable result, since stiffer springs lead to greater axial leg forces, something that leads to less ground contact duration. Fig. 6c, also shows that the phase does not change considerably.

Control Parameters. We reveal the effects of desired apex height and desired forward speed to the robot walking behavior, using Fig. 7. We treat each parameter separately:

As illustrated by Fig. 7a, as the desired apex height increases, the mean factor decreases from 40%–45% to 15%–25% for both models. This is a reasonable result: The higher the robot jumps, the less time it touches the ground. The same figure shows that the phase does not change much.

Using Fig. 7b, it is quite interesting to observe that the increase in desired forward speed changes the resulting gait differently — depending on the existence of leg mass. For model A, the increase of forward speed decreases the mean factor from 46% to 32%, i.e., the leg is less time on the ground applying a torque to the body, whereas for model B, the increase of speed increases the phase from 2% to 22%, i.e., the torques are applied for longer time but not successively, and not almost simultaneously — as previously.

Environmental Parameters. Using Fig. 8, we examine the gravity and ground inclination effects to the walking behavior of the robot. We again treat each parameter separately:

By examining Fig. 8a, we note that the increase of ground inclination, increases the phase from 4% to 14%–16% for both models. This is reasonable, since during uphill motion, there is an horizontal component of the gravitational force that pulls the robot back. Thus, it delays the touch-down of the other leg. By Fig. 8a, also, we see that the mean factor does not change much.

As shown in Fig. 8b, the increase of acceleration of gravity increases the mean factor from 22%–32% to 62%–66% for both models. This is also reasonable, since greater gravity acceleration leads to greater weight forces and, thus, to longer ground contacts. As shown in Fig. 8b, the phase does not change significantly. Note that, in all of the Figs. 6–8, the model B curves are richer in pattern distribution — and thrifter in the size of the valid parameter-value ranges. This is due to the non-negligible leg mass: Whenever the robot is on the air, the rotation of its legs leads to the rotation of its body — as follows by the conservation of the total angular momentum. Thus, there occur more changes in the leading pair of legs at each landing of the robot. This also leads to failed simulations that regard many parameter values.

VI. CONCLUSION

We witnessed the exploration of the effects of the most significant robot parameters in the way a quadruped robot locomotes, and we investigated the distribution of the members of a specific pattern class. Some of our results were unknown, or not obvious. These include: (a) the connection between the leg rest length and the robot walking behavior, (b) the observation that, in general, quadrupeds modeled the way we studied, land on their hind legs after a flight phase [The pattern “HF” is the most frequent] and (c) the fact that leg mass conveys a central role to whether a quadruped
robot will adopt a more complex locomoting pattern in its steady state. Thus, (d) we attained a qualitative view of some unexplored properties of quadruped robots, for now we can describe very concisely any quadruped robot motion. We also revealed that more complex patterns exist in legged locomotion: We discovered them, categorized them and studied the parameters that affect them. Future work includes both the derivation of algorithms that compute Hildebrand diagrams for 3D locomotion, and the expansion of the results regarding parameter effects and pattern distributions, in the 3D space, in the hope that this work will ultimately find its way to the industrial design of robots.

REFERENCES