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MEEG 331 Incompressible Fluids Laboratory

Date Experiment Performed: [redacted]

Submission Date: [redacted]

Objective: To test the theory of hydrostatics for force and pressure center on a submerged plane surface.

Background: This experiment is designed to illustrate hydrostatic pressure distributions and hydrostatic forces on plane surfaces.

- Hydrostatic pressure distributions- in static fluids pressure varies only with vertical distances and is independent of the shape of the container. The pressure is the same at all points on a given horizontal plane in the fluid. The pressure increases with depth in the fluid.

$P = \rho g y + P_{atm}$

- Hydrostatic forces on plane surfaces- the fluid pressure acts normal to the surface of an object and is positive in compression. Integration of the pressure over a submerged surface yields the hydrostatic pressure force acting on the submerged surface. The resultant moment about a specific point can be obtained from the calculated force and the center of pressure.

$F_h = \gamma h_{CG} A$

Equipment:

Torroid device- The torroid device consisted of a level, balance, weight hanger,. The balance was made up of the torroid, which was balanced so that the face of the torroid was vertical with no water present, and an area for mass to counter the moment caused by the hydrostatic force on the vertical face of the torroid.

Precision depth measuring device- A device that enabled us to take measurements of the height of the water, within a tenth of a millimeter.

Symbols Definitions: ρ = density g = gravitational constant y = vertical distance below the free surface.

Fh = horizontal force on the plane. $\gamma = gamma = \rho g$

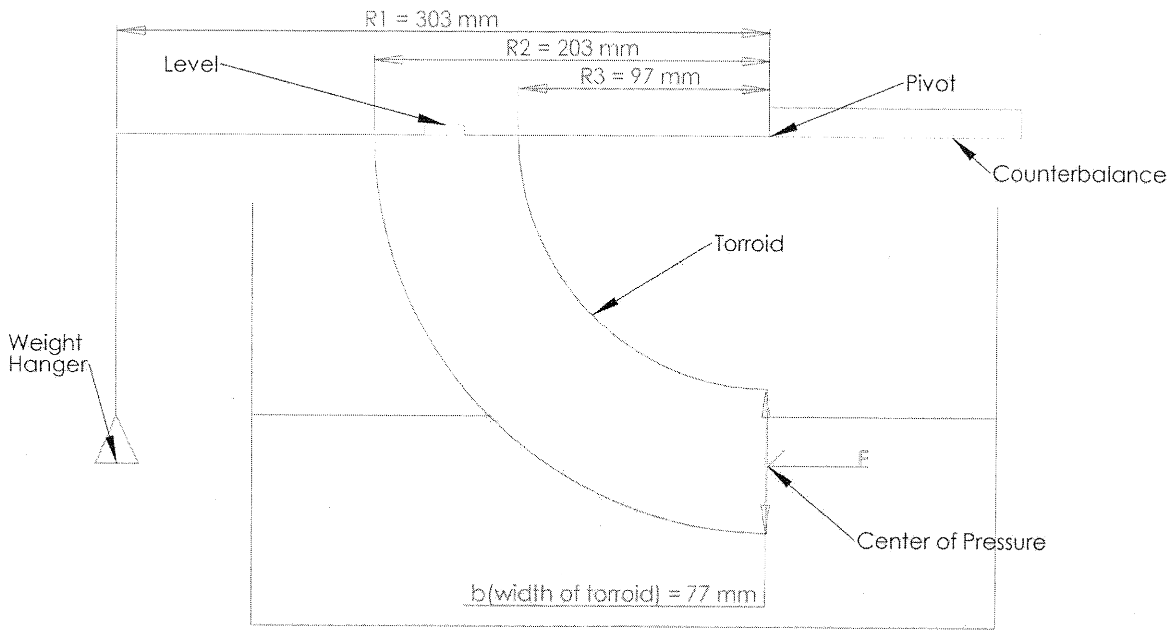
h_{CG} = the distance from the surface to the plate centroid.

*See Figure 1 for schematic diagram. *

Procedure:

1. With the tank empty, we calibrated the balance arm so that the balance arm was horizontal.
2. We filled the tank so that the water level was even with the top edge of the torroid.
3. We then added mass to the balance to level the system again. We also recorded the level of the water. This level we set to a specific height so that all other measurements would relate to this first measurement.
4. We then chose an amount of mass to remove from the balance and removed it. Then we leveled the system by removing water from the system.
5. We repeated this step for the rest of the data points. Each time we tried to choose weights in order to get equal amounts of change in the water height.

Figure 1


R1 = 303 mm
R2 = 203 mm
R3 = 97 mm
Level
Pivot
Counterbalance
Torroid
Weight
Hanger
Center of Pressure
b(width of torroid) = 77 mm

Fluid Dynamics Lab 1 : Incompressible Fluids Lab

	Measured Data			Calculated Data		
	Control Variable	Measured Variable	Force (N)	Center of Pressure (mm)	Mf (Nm)	Mw (Nm)
Measurment Number:	Water Height mm	Mass on Scale g				
1	140.0	212	4.234	70.67	.70988	.62996
2	136.4	200	3.951	68.27	.66723	.5943
3	132.6	185	3.663	65.73	.62326	.54973
4	124.0	155	3.052	60.00	.52803	.46059
5	112.8	120	2.340	52.53	.41352	.35656
6	102.6	90	1.773	45.73	.31943	.26744
7	94.4	70	1.375	40.27	.25138	.20801
8	83.1	45	.908	32.73	.16954	.13572
9	71.4	25	.527	24.93	.10042	.07429
10	52.8	5	.133	12.53	.0262	.01486

Other Measured Data:

Water Temperature	R1	R2	R3	b
22° C	30.3cm	20.3cm	9.7cm	20-12.3 =7.7cm

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Fluid Dynamics Lab 1: Incompressible Fluids Lab

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Measured Data

Measurment Number	Height of Water (mm)	Mass on Scale (g)
1	140.0	212
2	136.4	200
3	132.6	185
4	124.0	155
5	112.8	120
6	102.6	90
7	94.4	70
8	83.1	45
9	71.4	25
10	52.8	5

Calculated Data

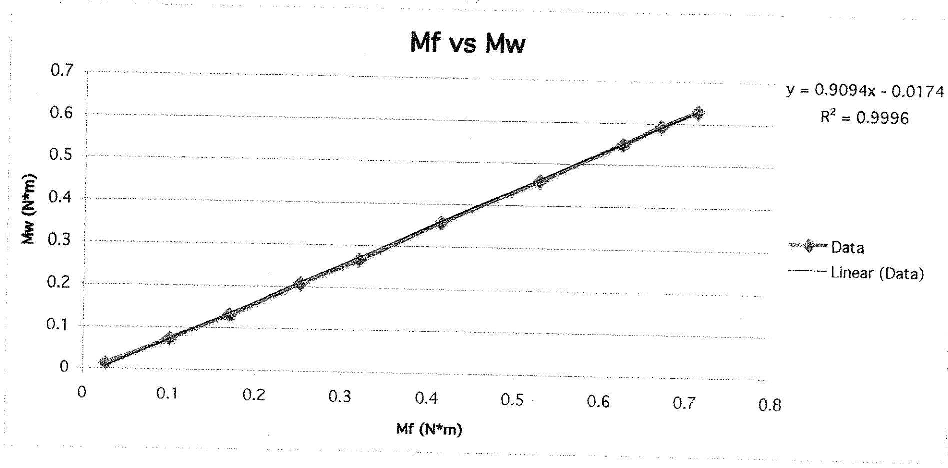
Calculated height of water (mm)	Area (mm^2)	Force (N)	Center of pressure From Free Surface (mm)	Mf (N*m)	Mw (N*m)	Difference between Mf and Mw	Relative Error Mf	Relative Error Mw
106.0	8162.0	4.234	70.67	0.70988	0.62996	0.0799191	0.059648083	0.059648083
102.4	7884.8	3.951	68.27	0.66722	0.5943	0.0729192	0.057802327	0.057802327
98.6	7592.2	3.663	65.73	0.62326	0.54973	0.0735307	0.062686343	0.062686343
90.0	6930.0	3.052	60.00	0.52803	0.46059	0.0674442	0.068220854	0.068220854
78.8	6067.6	2.340	52.53	0.41352	0.35658	0.0569392	0.073937031	0.073937031
68.6	5282.2	1.773	45.73	0.31943	0.26744	0.0519887	0.0885876	0.0885876
60.4	4650.8	1.375	40.27	0.25138	0.20801	0.0433765	0.094422088	0.094422088
49.1	3780.7	0.908	32.73	0.16954	0.13372	0.0358245	0.118130675	0.118130675
37.4	2879.8	0.527	24.93	0.10042	0.07429	0.0261369	0.149599212	0.149599212
18.8	1447.6	0.133	12.53	0.0262	0.01486	0.0113436	0.276276701	0.276276701

Other Measured Data

Water Temperature	22
R1	303.0
R2	203.0
R3	97.0
b	77.0

Other Data

Water Density	998	kg/m^3
Gravity	9.807	m/s^2



Data

height of water

$R_2 - R_3 - h_0 + h = h_w$

$h_0 = 140\,mm$

Area

$wh \Rightarrow b \cdot h_w$

Force

(height = 140 - measure. + h)

$\rho g h_c A = \rho g h_c (R_3 \ldots)(b)$

$\rho g ((R_2 - R_3 - h_0 + h)/2)(R_2 - R_3 - h_0 + h)(b)$

unit conversion from m⇒mm

$F = \rho g(((R_2 - R_3 - h_0 + h)/2)/1000)((R_2 - R_3 - h_0 + h)(b))/1000)$

Center of Pressure

$$Y_{cp} = h_c + \frac{b h_w^3}{(12)(h_c)(b)(h_w)}$$

$$Y_{cp} = \frac{h_w}{2} + \frac{b h_w^3}{(12)\left(\frac{h_w}{2}\right)(b)(h_w)}$$

$$Y_{cp} = \frac{(R_2 - R_3 - h_0 + h)}{2} + \frac{\cancel{(b)}(R_2 - R_3 - h_0 + h)^{3}}{(12)\frac{(R_2 - R_3 - h_0 + h)}{2}\cancel{(b)}}$$

$$Y_{cp} = \frac{(R_2 - R_3 - h_0 + h)}{2} + \frac{(R_2 - R_3 - h_0 + h)}{6}$$

$$Y_{cp} = \frac{4(R_2 - R_3 - h_0 + h)}{6} \Rightarrow \frac{2(R_2 - R_3 - h_0 + h)}{3}$$

Mf

$Mf = \vec{r} \times \vec{F_f}$

$= (h_0 - h_c)$

$= (R_2 - h_w + Y_{cp})\,F$

unit conversion from m⇒mm $\Rightarrow (R_2 - h_w + Y_{cp})F$

M_w

$M_w = \vec{r} \times \vec{F_w}$

$M_w = R_1 mg$

unit conversion from $m \rightarrow mm$

$M_w = \frac{R_1}{1000}\left(\frac{m}{1000}\right)g$

Differenca between M_f + M_w

$M_f - M_w = \Delta M$

Error

- Errors are unknown $\Rightarrow M = x + y$

$(\Delta M)_{sys} = \Delta x + \Delta y$

$M = M \pm \Delta M$

relative error:

$$\left(\frac{\Delta M}{M}\right)_{sys} = \frac{\Delta x + \Delta y}{x + y} = \frac{x}{x+y}\left(\frac{\Delta x}{x}\right) + \frac{y}{x+y}\left(\frac{\Delta y}{y}\right)$$

$M_f = x_1 + \Delta x$

$M_w = y_1 + \Delta y$

$\Delta R_1 = \Delta R_2 = \Delta R_3 = \Delta b = \pm .5m$

$\Delta h = \pm .05mm \Rightarrow .00005m$

$\Delta M_f = \frac{x}{x+y}\left(\frac{\Delta x}{x}\right)$ $\Delta M_y = \frac{y}{x+y}\left(\frac{\Delta y}{y}\right)$

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Discussion and Error Analysis

- There is a hydrostatic pressure force acting on the circular surface of the torroid. The equivalent force (equivalent to the distributed pressure loading on the surface) will act in a direction perpendicular to the curved surface of the torroid at the location through which it acts. By definition, the center of pressure will lie on this equivalent force's line of action. Geometrically, the curved surface of the torroid is an arc on which all the points are the same, constant radial distance from the hinge on which the torroid is mounted, similar to a circle. If the resultant force line ~~from~~ of the hydrostatic pressure is normal to the surface of the torroid, it is also normal to a tangent line at the edge of the circle. Tangent lines of a circle always intersect radial lines extending to the point of tangency at a right angle. This means that the hydrostatic pressure force acting on the curved surface is coincident with the line extending from the hinge to the point where the equivalent hydrostatic pressure force acts. This means that the moment arm of this hydrostatic pressure force on the curved surface is zero, and therefore no moment is generated by the pressure force on the curved surface.
- One of the major sources of error is the inherent limitation of the measuring devices we used. For instance, the leveling bubble is a qualitative measurement of the angle of elevation relative to the direction of gravity. It is not certain that each trial found the same exact point of balance with only the use of that bubble level. We also used tools like rulers to measure the dimensions of the water level and the geometric properties of the torroid apparatus. Since these errors will cause variations in an unknown "direction" (ie positive or negative) they fall into the category of random error.
  - The device used to measure the height of the water allowed us to accurately measure to one tenth of a millimeter. Therefore $\Delta h = 0.05$ mm.
  - The device used to measure the geometric properties of the system was accurate to the millimeter, therefore $\Delta x = 0.5$ mm.

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