The Role of Air Turbulence in Warm Rain Initiation

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Abstract

A significant fraction of the precipitation that falls on Earth is formed by the collision-coalescence of cloud droplets, yet the rate of this process in a turbulent environment during the rain initiation stage is poorly understood. Quantitative parameterization of turbulent collision represents a major unsolved problem in cloud physics, especially for the condensation-coalescence bottleneck with droplets of 15 to 50 \( \mu m \) in radius. Turbulent collisions of small cloud droplets are dynamic events in a complex multiphase flow system affected by a range of scales from those governing the background air turbulence to those characterizing droplet-droplet aerodynamic interactions. Here a hybrid direct simulation tool is used specifically to quantify the turbulent enhancement of the gravitational collision-coalescence. Simulation results and theoretical reasoning show that air turbulence can enhance the collision kernel by an average factor of about 2. Furthermore, larger relative enhancements occur for the two limiting cases of droplet pairs of either very different or similar in size where the gravitational collision alone in the bottleneck range may be ineffective due to either low collision efficiency or low differential settling. An impact study using the most realistic collection kernel suggests that cloud turbulence can significantly reduce the time for warm rain initiation.

Key words: collision-coalescence, cloud microphysics, turbulence, warm rain development.
Atmospheric clouds dominate the visual appearance of Earth when viewed from space. While visible clouds may extend over distances up to hundreds of kilometers, the individual water droplets are typically only 5 \(\mu \text{m}\) to 20 \(\mu \text{m}\) in radius. In warm (i.e., above-freezing) clouds, droplets may further grow by collision-coalescence to form drizzle droplets or raindrops, typically a few hundred microns to several millimeters in diameter. Rainfall produced in such a way (referred to as “warm rain”, in contrast to precipitation formed by ice processes) accounts for about 30% of the total rainfall on the planet and roughly 70% of the total rain area in the tropics [1]. In general, global radiative and hydrological fluxes are strongly linked to microphysical processes in clouds [2] which determine, among other things, the number concentration and size distribution of cloud droplets. Representation of cloud microphysical processes is a source of significant uncertainty in numerical weather prediction and climate models. Critical weather phenomena such as aircraft icing and freezing precipitation often result from warm rain processes, sometimes with deadly consequences [3, 4].

Small cloud droplets (i.e., radii less than roughly 15 \(\mu \text{m}\)) grow efficiently by the diffusion of water vapor; they are unable to grow efficiently by gravitational collisions until their radius reaches about 50 \(\mu \text{m}\) [5]. The latter is due to low terminal velocities and low collision efficiencies of small droplets. In general, it is difficult to explain the rapid growth in cloud droplets in the size range from 15 to 50 \(\mu \text{m}\) in radius for which neither the diffusional mechanism nor the gravitational collision-coalescence mechanism is effective (i.e., the condensation-coalescence bottleneck). An open question is what drives the droplet growth through the bottleneck size range. The onset of drizzle-size drops (\(\sim 100 \mu \text{m}\) in radius) is still poorly understood, and this issue is regarded as one of important unresolved problems of cloud physics. A related issue is the discrepancy between the width of observed and simulated size distributions of cloud droplets [6].

Several mechanisms have been proposed to explain the rapid development of rain in shallow convective clouds. The first mechanism involves entrainment of dry environmental air into the cloud. Although entrainment lowers the cloud water content (and thus has a negative impact on rain development), it can result in broader cloud droplet spectra [7]. The second mechanism involves effects of giant aerosol particles which allow the formation of large droplets by the diffusion of water vapor [8]. It has also been suggested that the droplet spectral width can be broadened by local fluctuations of the water vapor supersaturation [9]. However, numerical results [10] suggest that this effect contributes insignificantly to the
width of the cloud droplet spectrum. Finally, air turbulence can enhance the relative motion of droplets, concentration fluctuations, and collision efficiencies [11–14].

**Turbulent Collision-Coalescence**

This paper examines the effects of air turbulence on the growth of cloud droplets by collision-coalescence. The central issue is the magnitude of the enhancement of the gravitational collection kernel due to the air turbulence, and whether the enhancement can significantly impact rain initiation. The collection kernel is defined as the number of collisions per unit time between droplets of two different sizes, divided by the corresponding number of pairs involved. We will show that, despite the complexity of the problem, recent quantitative studies begin to address these long-standing issues with confidence.

Arenberg [15] in 1939 argued that air turbulence could be a major factor in determining the growth of cloud droplets. Over the years, several studies were devoted to the characterization of the effects of air turbulence (see [16] for an overview of the historical development in this area). During the last 15 years, an increasing number of studies have emerged in both engineering and atmospheric literature concerning the collision rate of inertial particles in turbulent flow (see [16] for relevant references). These studies suggest that the collection kernel of cloud droplets could be enhanced by several effects of air turbulence: (1) the enhanced relative motion due to differential acceleration and shear effects; (2) the enhanced average pair density due to local clustering (“preferential concentration”) of droplets; (3) the enhanced settling rate, and (4) the enhanced collision efficiency. The enhancement depends, in a complex manner, on the size of droplets (which in turn determines the response time and terminal velocity) and the strength of air turbulence (i.e., the dissipation rate, flow Reynolds number, etc.).

While almost all of the past studies point to the collision enhancement by air turbulence, one has to be careful in applying the results to cloud droplets. This is because most studies in the engineering literature fail to address the problem within the relevant parameter space. For cloud droplets, the two key physical parameters are the droplet inertial response time \( \tau_p \) and the still-fluid droplet terminal velocity \( v_T \). Due to the relatively low flow dissipation rate in clouds, the collision-coalescence of cloud droplets tends to be governed by dissipation-range scales of the air turbulence. For the background air turbulence, the dissipation-range motions are characterized by the Kolmogorov time \( \tau_k \) and the Kolmogorov velocity \( v_k \). The
Stokes number $St$, the ratio $\tau_p/\tau_k$, emphasized in early studies of particle-laden flows, is not the only parameter governing the interaction of droplets with air turbulence. The nondimensional settling velocity, $S_v \equiv v_T/v_k$, is the second key parameter, typically one order of magnitude larger than $St$ [17]. This implies that the gravitational sedimentation determines the interaction time between the cloud droplet and the small-scale flow structures. Most of the published results on droplet clustering and collision rate from numerical simulations and theoretical studies assume no sedimentation and, as such, are not directly applicable to cloud droplets.

Recent systematic studies of the collection kernel for cloud droplets have been undertaken through either direct numerical simulation [13, 14, 18, 19] or a kinematic/stochastic representation of turbulence [11, 12]. These studies provide not only quantitative data on the turbulent collision kernel, but also reveal the physical complexity of the collisional interactions in a turbulent flow.

In parallel, several attempts have been made to address the impact of selected aspects of air turbulence on the time evolution of the droplet size spectrum. It has been demonstrated that collection kernels taking into account the effect of air turbulence on relative motion of droplets can lead to the acceleration of large droplet and raindrop formation [20]. The analysis in [21] implies that preferential concentration of droplets and local fluid acceleration due to cloud turbulence can substantially accelerate the formation of large droplets that trigger rain. Another analysis [22] illustrates that the selectively enhanced settling velocity due to air turbulence could make droplets grow rapidly from 20 to 80 $\mu$m and that this mechanism does not depend on the level of cloud turbulence. These studies, however, are based on turbulent collection kernels derived from either approximate or empirical formulations of the air turbulence and/or the motion of cloud droplets and, consequently, should be treated as primarily being qualitative.

**Hybrid Simulation Approach**

Motivated by the issues explained above, we have developed a consistent and rigorous simulation approach to the problem of turbulent collisions of cloud droplets [14, 23]. The basic idea of the approach is to combine direct numerical simulation (DNS) of the background air turbulence with an analytical representation of the disturbance flow introduced by droplets (Fig. 1). The approach takes advantage of the fact that the disturbance flow due to droplets
is localized in space and there is a sufficient length-scale separation between the droplet size and the Kolmogorov scale of the background turbulent flow. This hybrid approach provides, for the first time, a quantitative tool for studying the combined effects of air turbulence and aerodynamic interactions on the motion and collisional interactions of cloud droplets. The disturbance flow is coupled with the background air turbulence through the approximate implementation of the no-slip boundary conditions on each droplet. Dynamical features in three dimensions and on spatial scales ranging from a few tens of centimeters down to 10 µm are captured. Both the near-field and the far-field droplet-droplet aerodynamic interactions could be incorporated [24], with possible systematic improvements of their accuracy.

The most important aspect of the approach is that dynamic collision events are detected, along with the direct and consistent calculations of all kinematic pair statistics related to the collision rate [14]. These unique capabilities help establish the following general kinematic formulation of the collection kernel $K_{12}$ [14, 25]

$$K_{12} = 2\pi R^2 \langle |w_r(r = R)| \rangle g_{12}(r = R),$$

(1)

where the geometric collision radius $R$ is defined as $R = a_1 + a_2$, with $a_1$ and $a_2$ being the radii of the two colliding droplets, $w_r$ is the radial relative velocity at contact which combines the differential sedimentation and turbulent transport, and $g_{12}$ is the radial distribution function (RDF) that quantifies the effect of droplet-pair clustering on the collision rate. This formulation is applicable to aerodynamically interacting droplets in a turbulent background flow, showing that the total enhancement factor $\eta_T$ of the collision rate is a product of the enhancement of the geometric collision, $\eta_G$, and the enhancement of the collision efficiency, $\eta_E$ [14].

**Turbulent Enhancement**

This section highlights selected findings obtained using the hybrid DNS approach. Figure 2 shows the net enhancement factor $\eta_T = \eta_E \eta_G$ by air turbulence as a function of $a_2/a_1$ for $a_1 = 30$ µm. To interpret the shape of $\eta_T$, we first note that air turbulence is only effective in altering the local aerodynamic interaction when i) the level of turbulent fluctuations at the scale of $R$ is at least comparable to the differential terminal velocity $(v_{T1} - v_{T2})$, and ii) the aerodynamic interaction time [$\sim R/(v_{p1} - v_{p2})$] is of the order of $\tau_{p2}$, the inertial
response time of the smaller droplet. The first condition may be stated as

$$\frac{R(v_k/\eta)}{v_p_1 - v_p_2} = \frac{9}{2} \frac{\rho}{\rho_w} \frac{\sqrt{\epsilon \nu}}{(a_1 - a_2)|g|} \geq C_1, \quad (2)$$

where $\epsilon$ is the mean viscous dissipation rate of the turbulence, $\nu$ is the air kinematic viscosity, $\rho$ is air density, $\rho_p$ is water density, $g$ is the gravitational acceleration, and $C_1$ is a constant of the order of one. Therefore, this first condition prefers a larger $\epsilon$ and a value of $a_2/a_1$ close to one. The second condition may be stated as

$$\frac{R}{(v_p_1 - v_p_2)\tau_p_2} = \left(\frac{9}{2} \frac{\rho}{\rho_w}\right)^2 \frac{\nu^2/|g|}{a_2^2(a_1 - a_2)} \geq C_2, \quad (3)$$

which favors the two limiting cases of $a_2/a_1 \to 0$ for a given $a_1$, or $a_2/a_1 \to 1$. The above scaling arguments show that a larger $\eta_E$ should occur for the two limits of $a_2/a_1 \to 0$ or $a_2/a_1 \to 1$. It also follows that $\eta_E$ decreases with increasing $a_1$ for a fixed $a_2/a_1$.

A similar qualitative behavior for $\eta_G$ can be inferred. In this case, the first condition, equation (2), must also be satisfied to obtain a significant $\eta_G$ value. Other conditions for enhanced geometric collision through particle clustering would be $\tau_p \sim \tau_k$ [26] and $F_p = \tau_p^3|g|^2/\nu \sim 1$ [18, 22]. For $\nu=0.17 \text{ cm}^2/\text{s}$, $|g|=980 \text{ cm/s}^2$, $\rho_w/\rho \approx 1000$, the condition $F_p \sim 1$ implies a droplet radius at about 21 $\mu$m [18, 22], independent of the flow dissipation rate. The condition of $\tau_p \sim \tau_k$ yields a droplet radius of

$$a(\mu m) \sim \frac{177}{\epsilon (\text{in cm}^2/\text{s}^3)^{0.25}}, \quad (4)$$

which is about 56 $\mu$m and 40 $\mu$m for a dissipation rate of 100 and 400 cm$^3$/s$^2$, respectively. Therefore, for a dissipation rate of 400 cm$^3$/s$^2$, we expect that the preferential concentration is most relevant for droplets in the size range from 21 $\mu$m to 40 $\mu$m, i.e., those in the bottleneck range. We find that $\eta_T$ can typically vary from 1 to 3 when $a_2/a_1 \sim 1$, for the two representative dissipation rates of 100 and 400 cm$^2$/s$^3$. The overall conclusion is that $\eta_T$ is moderate but it occurs at the right place, that is, where the hydrodynamic-gravitational collection kernel tends to be relatively small. Complete compilations and discussions of the simulations results are found in [18, 19].

**Impact on Warm Rain Initiation**

We next consider the question of how the above turbulent enhancements on the collection kernel alter the size evolution of cloud droplets. An analytical model has been developed for
the geometric collision rate of cloud droplets based on the results from the hybrid simulation approach [27]. The model consists of a parameterization of the radial relative velocity $\langle w_r \rangle$ and a parameterization of the RDF $g_{12}$, both for sedimenting particles. Of significance is the fact that the above parameterizations for both $\langle w_r \rangle$ and $g_{12}$ consider the effects of flow Reynolds number which cannot be fully represented by the hybrid simulations. For example, the parameterization for $\langle w_r \rangle$ makes use of velocity correlations that are valid for both the dissipation subrange and the energy-containing subrange of turbulence [28]. The intermittency of small-scale turbulent fluctuations can be incorporated into the model for RDF [29]. Therefore, our parameterization of the collection kernel, to certain extent, can represent realistic flow Reynolds numbers in clouds. Moreover, we include the effect of $\eta_E$ by interpolating the tabulated simulation results of $\eta_E$ from [19]. The ratio ($\eta_T$) of the resulting turbulent collection kernel to the Hall kernel [30] is shown in Fig. 3 for a typical condition of cloud turbulence. The Hall kernel [30], a hydrodynamical gravitational kernel independent of air turbulence, is used as a base to compare the relative impact of turbulence.

Two important observations can be made from Fig. 3. First, a noticeable enhancement occurs for droplets less than 100 $\mu m$. Second, the overall enhancement is moderate with a value ranging from 1.0 to 4.0 for most regions or an average value of about 2 for droplets in the bottleneck size range. The enhancement factors shown in Fig. 3 are similar to those reported recently in [12], where dramatically different approaches were employed. In general, it is the average enhancement over the bottleneck range that determines the overall impact of turbulence on the warm rain initiation [16].

The above turbulent collection kernel is then used in the kinetic collection equation to solve for droplet size distributions at different times, starting from the initial number density distribution

$$n(x, t = 0) = \frac{L_0}{\bar{x}_0^2} \exp \left( -\frac{x}{\bar{x}_0} \right), \quad (5)$$

where $x$ is the droplet mass, $L_0$ is the liquid water content and is set to 1 $g/m^3$, and $\bar{x}_0$ is the initial average mass of the cloud droplets and is assumed to be $3.3 \times 10^{-9}$ g which corresponds to a mean radius of 9.3 $\mu m$. The above distribution yields an initial relative radius dispersion (the ratio between the standard deviation and the mean droplet radius) of 0.36, which is within the range of 0.1 to 0.4 observed in stratocumulus [31].

The kinetic collection equation is solved by an accurate method [32] which combines
the advantages of flux-based methods and spectral moment-based methods. A small bin mass ratio of $2^{0.25}$ ensures that the numerical solutions are free from numerical diffusion and dispersion errors. The mass distribution $g(\ln r, t) \equiv 3x^2n(x, t)$ is usually plotted at different times in order to examine growth processes [33]. In Fig. 4, we instead plot the local rate of change, $\partial g/\partial t$, as a function of radius for times from 0 to 60 min every 1 min. We compare the results using the turbulent kernel to those using the Hall kernel. The plots naturally reveal the three growth phases first described qualitatively in [33]: (1) the autoconversion phase in which self-collections of small cloud droplets near the peak of the initial size distribution slowly shift the initial peak of the distribution toward larger sizes; (2) the accretion phase in which the accretion mode dominates over the autoconversion mode and serves to quickly transfer mass from the initial peak to the newly formed secondary peak at drizzle sizes; and (3) the large hydrometeor self-collection phase in which the self-collections of drizzle droplets move the second peak toward the raindrop sizes (a few millimeters). By examining the locations corresponding to the maximum and minimum $\partial g/\partial t$, one can unambiguously identify the time intervals of the three phases [16]. In Fig. 4, we indicate in each plot the beginning and end of the accretion phase by two red horizontal lines.

Figure 4 highlights striking differences between the two collection kernels. The intensity of the autoconversion is significantly increased by the turbulent effects as shown by the magnitude of $\partial g/\partial t$ at early times (Fig. 4b) when compared to the base case (Fig. 4a). The time interval for the autoconversion phase is reduced from about 32.5 min (Hall kernel) to only 10.5 min (the turbulent kernel). This demonstrates that turbulence has a strong impact on the autoconversion phase, which is typically the longest phase of warm rain initiation. The time interval for the accretion phase is also significantly reduced and smaller drizzle drops ($\sim 100$ to $300 \mu m$) are produced during this phase.

If a radar reflectivity factor of 20 dBZ (or the mass-weighted mean droplet radius of 200 $\mu m$) is used as an indicator for the drizzle precipitation, the time needed to reach such a reflectivity (or mean droplet radius) changes from about 2450 s (or 2470 s) for the Hall kernel to 1230 s (or 1250 s) for the turbulent kernel. This two-fold reduction factor increases with either the dissipation rate or rms turbulent fluctuation velocity [16].

Finally, in Fig. 5 we illustrate how the time needed to reach the accretion phase depends on the collection kernel and the width of the initial size distribution. For this purpose, the
following generalized initial number density distribution is considered:

\[ n(x, t = 0) = A \frac{L_0}{\bar{x}_0^2} \exp \left[ - \left( \frac{Bx}{\bar{x}_0} \right)^\alpha \right], \]  

(6)

where \( \alpha \) is varied to produce different initial radius dispersions, and two constants \( A \) and \( B \) are specified such that the physical interpretations of \( \bar{x}_0 \) and \( L_0 \) are unchanged. The relative radius dispersion \( \gamma \) is defined as the ratio between the standard deviation of droplet radius and the mean radius. The case of \( \alpha = A = B = 1 \) is the reference case considered above (Case 1), yielding \( \gamma = 0.3634 \equiv \gamma_0 \). Two other cases with smaller initial radius dispersions are also considered: Case 2 has \( \alpha = 2, A = 0.636618, B = 0.564189 \), which gives \( \gamma = 0.3015 \). Case 3 has \( \alpha = 3, A = 0.566034, B = 0.505479 \), which results in \( \gamma = 0.2825 \). A total of 9 simulations were performed, using different combinations of the initial radius dispersion, the collection kernel type, and the level of flow dissipation rate. The two simulations shown in Fig. 4 are two of the 9 runs shown in Fig. 5. The line fit in Fig. 5 represents

\[ T_A \approx 0.08 \times \left( \frac{\partial g(\ln r, t = 0)}{\partial t} \right)_{\text{max}}^{-2/3} \times \left( \frac{\gamma}{\gamma_0} \right)^{-4}. \]  

(7)

The excellent fit shows that the transition time \( T_A \) from the autoconversion phase to the accretion phase depends primarily on two properties, the initial dispersion \( \gamma \) and the initial autoconversion intensity as measured by the maximum \( \partial g/\partial t \) at \( t = 0 \). Despite the very strong nonlinearity for the autoconversion phase, the above correlation demonstrates a surprisingly simple picture: the rain initiation time is shortened by either increasing the initial radius dispersion \( \gamma \) or increasing the initial autoconversion rate. The latter can be accomplished by turbulent enhancements discussed in this paper. The former may be a result from different characteristics of of the cloud condensation nuclei (CCN; [8]). The two different mechanisms are brought together within the above unified framework for studying the rain initiation processes.

**Summary and Conclusions**

Studies during the last 10 years have significantly advanced our understanding of the effects of cloud turbulence on the collision-coalescence of cloud droplets. There is a sufficient evidence that the air turbulence accelerates the development of drizzle and rain precipitation. Meanwhile, much work is still needed to bring the research in this area to the level of quantitative science. Our systematic efforts to remove uncertainties have led us to conclude that
the enhancement factor by the air turbulence is moderate, which further implies the importance of quantitative measures and the need for rigorous research methodologies. The hybrid direct numerical simulation approach represents the first step in this direction. Current limitations of our hybrid DNS approach and on-going efforts to address these limitations are discussed in [18, 19]. The challenging issues from the perspective of collection kernel parameterization include characterization of radial distribution function, flow Reynolds-number effects, and the modeling of droplet-droplet short-range interactions [18, 19, 27].

Our results advance the current understanding of the growth processes during the warm rain initiation. Since the moderate enhancements by air turbulence occur within the bottleneck range and since the autoconversion is the longest phase of the initiation process, there is now convincing evidence showing that turbulence plays a definite role in promoting rain formation. Recently, a rising adiabatic parcel model has been used to extend the impact study shown here by combining droplet activation, diffusional growth, and turbulent collision-coalescence [34]. Using the same turbulent collection kernel, it has been shown that the warm rain initiation time is reduced by 25% to 40%, demonstrating the important role of air turbulence in a more realistic warm rain model.

Finally, the mechanism discussed here does not exclude other mechanisms that may promote even more rapid rain initiation. In general, these mechanisms may operate simultaneously in the same part of a cloud, or they may act separately in different regions. For instance, both entrainment-mixing and turbulent collisions are expected to promote droplet growth near cloud edges where the cloud water content is reduced but the width of the droplet spectrum and the turbulence intensity are increased. In the core region, where the turbulence intensity is typically lower but the cloud water content is high, the turbulent mechanism can operate jointly with the giant aerosol mechanism, if such aerosol particles are part of the CCN spectrum. These aspects will need to be investigated in the future when the developments discussed in this paper are incorporated into dynamic models of warm precipitating clouds.

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Figure captions

Figure 1 Visualizations from the hybrid simulation approach. The droplets are much smaller than the dissipation-range eddies of the background air turbulence so the disturbance flows due to droplets are represented analytically by Stokes flow solutions. The background air turbulence is solved numerically by integrating the Navier-Stokes equations. The left panel shows vortical structures of the background air turbulence; the center panel shows trajectories of two colliding droplets at the scale of droplets in the turbulent flow; and the right panel is a snapshot of flow vorticity surfaces and the locations of droplets in a subdomain of the computational region.

Figure 2 The net enhancement factor, the ratio of the turbulent collection kernel and the hydrodynamic-gravitational collection kernel, is plotted as a function of the radius ratio $a_2/a_1$, with the larger droplet at 30 $\mu$m in radius. In the legend, $\epsilon$ is the flow viscous dissipation rate and $R_\lambda$ is the Taylor microscale Reynolds number of the simulated background turbulent air flow.

Figure 3 The ratio of a typical turbulent collection kernel to the Hall kernel. The Hall kernel [30] is a hydrodynamical gravitational kernel in still air. The ratio on the 45° degree line is undefined due to the zero value of the Hall kernel. The ratio is essentially one when droplets are above 100 $\mu$m. The flow dissipation rate is 400 $cm^2/s^3$ and rms velocity is 202 $cm/s$.

Figure 4 The rate of change ($\partial g/\partial t$, $g m^{-3} s^{-1}$) of droplet mass density in each numerical bin as a function of droplet radius: (a) solutions using the Hall kernel; (b) solutions using a turbulent kernel at flow dissipation rate of 400 $cm^2/s^3$ and rms fluctuation velocity of 2.0 m/s. There are 61 curves in each plot, representing $t = 0$ to $t = 60$ min with a time increment of 1 min. The curves for $t > 0$ is shifted upwards by a constant in order to distinguish them. The value of $\partial g/\partial t$ can be either positive or negative, with the total integral over the whole size range equal to zero due to the mass conservation. At any given time, a positive $\partial g/\partial t$ for a given size bin implies that the mass density for that size bin is increasing. The two red lines mark the beginning and the end of the accretion phase.

Figure 5 The transition time ($T_A$) from the autoconversion phase to the accretion phase, after empirically adjusted for the initial relative radius dispersion, plotted as a function of the
maximum magnitude of $\partial g / \partial t$ at $t = 0$. The flow rms fluctuation velocity is set to 2.0 m/s. Three different symbols represent three different initial radius dispersions. For each $\gamma$, the three data points correspond to, from left to right, the Hall kernel, the turbulent collection kernel with $\epsilon = 100 \text{ cm}^2/\text{s}^3$, and the turbulent collection kernel with $\epsilon = 400 \text{ cm}^2/\text{s}^3$. 

\[ \eta_T \]

\[ a_2 / a_1 \]

- Pinsky et al. (1999), \( \varepsilon = 100 \text{ cm}^2/\text{s}^3 \)
- \( R_\lambda = 72.4, \varepsilon = 400 \text{ cm}^2/\text{s}^3 \)
- \( R_\lambda = 72.4, \varepsilon = 100 \text{ cm}^2/\text{s}^3 \)
- \( R_\lambda = 43.0, \varepsilon = 400 \text{ cm}^2/\text{s}^3 \)
- \( R_\lambda = 43.0, \varepsilon = 100 \text{ cm}^2/\text{s}^3 \)
$T_A(\gamma/\gamma_0)^4$ (min)

$\frac{\partial g(t = 0)}{\partial t}_{\text{max}}$