Closed Loop Motion Planning and Control for Mobile Robots in Uncertain Environments

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***Abstract*— In this paper we present an extension to the navigation function methodology [6], [7] to the case where unmodelled obstacles are introduced in the workspace. A feedback control law is derived, based on the navigation function built on the initial workspace. Global convergence and collision avoidance properties are established. The derived closed form control law is suitable for real time implementation. Collision avoidance and global convergence properties are verified through computer simulations.**

I. Introduction

Navigation functions [1], [3], [6], [8] have become an important tool for potential field based robot navigation because they yield robust and fast navigation schemes that are important for real time implementation. Among other challenges, real time implementation requires that control systems are able to react promptly and safely to environment changes and be able to deal with uncertainty by modelling and incorporating sensory information on line.

This work is motivated by the need to redesign the potential field each time an environment change is detected. Our approach is to construct a potential field that models all known environment features and combine it with an efficient control scheme that handles additional unknown features.

Most of previous research has focused either on the problem of completely known environments [6] or of completely unknown ones [5]. In this paper we bridge this gap and try to bring the best of both worlds. The navigation problem for partially known environments can be stated as follows: "*For a partially known workspace, find a control law that steers a point robot from any initial state to a given final state, and can incorporate on-line information from newly discovered environment features.*". Our basic idea is to construct a discontinuous vector field which acts as the predefined navigation field away from unmodelled obstacles and as a vorticity-like field close to them. We argue that the flows of such a vector field will drive our system asymptotically to its target state, given that the vorticity-like field is appropriately constructed.

The rest of the paper is organized as follows: Section II introduces preliminary definitions, notation and some technical Lemmas, required for further discussion. Section III describes the construction of the vector field that is used for robot navigation while section IV presents the proposed control law. Section V presents the simulation results and the paper concludes with section VI.

II. Preliminaries

Let the admissible configuration space (workspace) for the robot be $\mathcal{W} \subset \mathbb{R}^2$. The obstacle free subset of the workspace is denoted $\mathcal{W}_{free} \subseteq \mathcal{W}$, and $\varphi : \mathcal{W}_{free} \to \mathbb{R}$ is the potential (navigation) function which models the known environment. Let $\mathcal{O}_i \in \overline{\mathcal{W}}_{free}$ be the i'th known obstacle, $i = 1 \dots n_O$, where n_O is the number of known obstacles. We assume that obstacles boundaries are C^2 curves.

Define the class of periodic, vector valued C^2 functions: $g_j(s+S) = g_j(s)$, with $g_j(s) : \mathbb{R} \to \mathcal{R}(g_j)$, with j an index number referring to an obstacle. The range $\mathcal{R}(g_j) \subset \mathcal{W}_{free}$ of each of these functions will serve as the bounding surface[1] that will bound a newly added obstacle $\mathcal{A}_j \subset \mathcal{W}_{free}$. It is trivial to note that such functions always exists in a bounded workspace $\mathcal{W}$ as long as $\mathcal{A}_j \subset \mathcal{W}_{free} \subseteq \mathcal{W}$ since we can always adjust the range of g_j to be arbitrarily close (in the sense of the minimum distance between set members) to $\mathcal{A}_j$ and still have that $\mathcal{R}(g_j) \subset W_{free}$.

Note that newly discovered obstacles are part of the initially thought "free" workspace, $\mathcal{W}_{free}$. If for a newly added obstacle $\mathcal{A}_j \cap \mathcal{O}_i \neq \emptyset$ then $\mathcal{R}(g_j)$ is extended to bound it as shown in Fig. 1. We assume that $\mathcal{R}(g_j)$ has a nonzero distance d from the contained obstacle's boundary.

The internal region (see [1]) introduces a new obstacle in the workspace, that is denoted by the set $\mathcal{O}_{g_j} \subset \mathcal{W}$. Let S represent the length of $\partial\mathcal{O}_{g_j}$. If the domain of $g_j(s)$, where $s \in \mathbb{R}$ is increasing while describing the obstacle's boundary counterclockwise, is restricted to $s \in [a, b)$ with $b - a = S$ then $g_j(s)$ is required to be a bijection. Obviously $\mathcal{R}(g_j) = \partial\mathcal{O}_{g_j}$. Each $\partial\mathcal{O}_{g_j}$ introduces the following topology (Fig. 1):

1) The $\mathcal{O}_{g_j}$ internal, $\mathcal{J}^-$
2) The $\mathcal{O}_{g_j}$ boundary, $\partial\mathcal{O}_{g_j}$
3) The $\mathcal{O}_{g_j}$ external, $\mathcal{J}^+$

Lemma 1: If φ is a navigation function and $h \in \mathcal{O}_{g_j}$ then $\varphi(h)$ attains a minimum value for some $h \in \partial\mathcal{O}_{g_j}$

[1]we use the term bounding surface here to denote that since $\mathcal{R}(g_j)$ slices $\mathcal{W}_{free}$ in two regions, the region where the new obstacle is contained is termed internal, the external region is the other slice and their boundary is the bounding surface

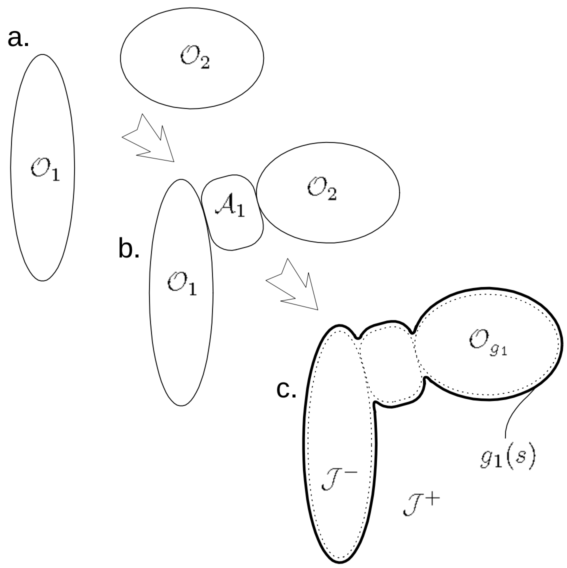

Fig. 1. a. Initial workspace b. Workspace with an additional obstacle $\mathcal{A}_1$ c. The resulting bounding obstacle $\mathcal{O}_{g_1}$

Proof: Since φ is a navigation function for the system $\dot{x} = -\nabla\varphi$, any dense invariant set should contain the origin. Let $h_{b_{\min}} = \underset{h_b \in \partial\mathcal{O}_{g_j}}{arg\min} \left(\varphi\left(h_b\right)\right)$. If $\varphi\left(h_i\right) \leq \varphi\left(h_{b_{\min}}\right)$ for some $h_i \in \left(\mathcal{O}_{g_j} \backslash \partial\mathcal{O}_{g_j}\right)$, that would have meant that $\mathcal{O}_{g_j}$ is an invariant set. But this is not true since $\mathcal{O}_{g_j}$ is not of measure zero and it does not contain the origin (otherwise the problem would be unsolvable). Hence $\varphi\left(h_i\right) > \varphi\left(h_{b_{\min}}\right)$. ■

Lemma 2: The following relations hold:

1) $\nabla\varphi\left(h_{b_{min}}\right) \cdot \nabla g_j\left({g_j}^{-1}\left(h_{b_{min}}\right)\right) = 0$
2) $\nabla\varphi\left(h_{b_{min}}\right) \cdot \nabla^{\perp} g_j\left({g_j}^{-1}\left(h_{b_{min}}\right)\right) < 0$

where $\nabla g_j(s) = \left[\frac{\partial g_j^x(s)}{\partial s} \;\; \frac{\partial g_j^y(s)}{\partial s}\right]$, $\nabla^{\perp} g_j$ is perpendicular to ∇g_j and is directed towards $\mathcal{J}^+$ (Fig. 2) and $h_{b_{\min}}$ as defined in the proof of lemma 1.

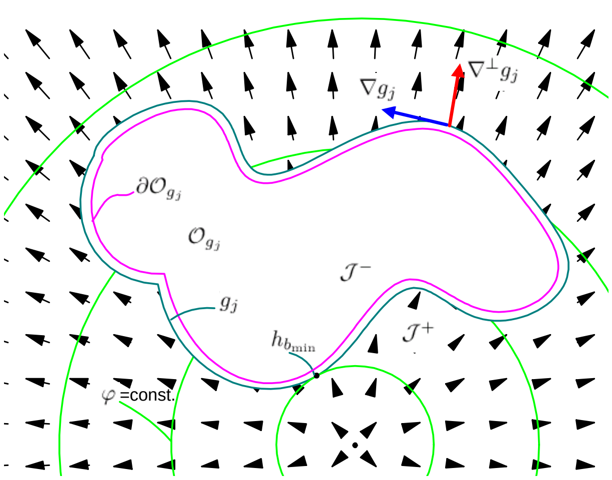

Fig. 2. Unmodelled obstacle in a quadratic (obstacle free) potential field

Proof: The derivative of φ along s at the point of minimum should be zero. Hence

$$\frac{\partial\varphi\left(g_j^x\left(s\right), g_j^y\left(s\right)\right)}{\partial s} = 0$$

$$= \frac{\partial\varphi}{\partial g_j^x}\frac{\partial g_j^x}{\partial s} + \frac{\partial\varphi}{\partial g_j^y}\frac{\partial g_j^y}{\partial s} = \nabla\varphi\left(g_j(s)\right) \cdot \nabla g_j\left(s\right)$$

$$\Rightarrow \nabla\varphi\left(h_{b_{\min}}\right) \cdot \nabla g_j\left(g_j^{-1}\left(h_{b_{\min}}\right)\right) = 0$$

The second part of the Lemma can be proven in a similar way as Lemma 1. If $-\nabla\varphi\left(h_{b_{\min}}\right)$ pointed towards the interior of $\mathcal{J}^-$, that would mean that there exists a point $h_i \in \mathcal{J}^-$ such that $\varphi\left(h_i\right) \leq \varphi\left(h_{b_{\min}}\right)$ which is not true as shown in the proof of Lemma 1. Hence

$$\nabla\varphi\left(h_{b_{min}}\right) \cdot \nabla^{\perp} g_j\left({g_j}^{-1}\left(h_{b_{min}}\right)\right) < 0$$

■

We now define the vector valued function

$$a(s) = g(s) + k \cdot \rho \cdot \widehat{\nabla g}^{\perp}(s)$$

with $\delta > 0$ and $k \in \{-1, +1\}$. The hat over a vector denotes that it is normalized, i.e. $\widehat{v} = \frac{v}{\|v\|}$. We need this function to be bijective. This is guaranteed if we require that

$$a\left(s_m\right) \neq a\left(s_n\right) \;\forall\; s_m \neq s_n \tag{1}$$

with $s_m, s_n \in [0, S)$. Locally condition (1) is equivalent to a restriction on curvature. To show this we examine the limiting behavior of $a(s_m) - a(s_n)$ when $s_m \rightarrow s_n$. So setting:

$$\frac{\partial a(s)}{\partial s} = 0$$
$$\Rightarrow \frac{\partial g(s)}{\partial s} = -\rho \cdot k \cdot \frac{\partial \widehat{\nabla g}^{\perp}(s)}{\partial s}$$

And solving for ρ we get:

$$\rho = \frac{1}{k} \cdot \frac{\left({g_x}'(s)^2 + {g_y}'(s)^2\right)^{\frac{3}{2}}}{{g_y}'(s)\,{g_x}''(s) - {g_x}'(s)\,{g_y}''(s)} \tag{2}$$

where $(g_x, g_y) = g$. Examining eq. (2) we can see that it corresponds to the inverse of the curvature of $g(s)$ or equivalently to the radius of $g(s)$ at the point s . This implies that locally, the requirement (1) is equivalent to choosing a $\rho > 0$ such that:

$$\rho < \rho_m = \min_{s \in [0, S)} \left| \frac{\left({g_x}'(s)^2 + {g_y}'(s)^2\right)^{\frac{3}{2}}}{{g_y}'(s)\,{g_x}''(s) - {g_x}'(s)\,{g_y}''(s)} \right| \tag{3}$$

III. Navigation Vector Field

A. Belt Zones

The vector field that is used to generate desired motion directions for the robot is primarily based on a predefined navigation function. However, the system needs to be able to avoid initially unmodelled obstacles, discovered through sensing during execution, and still be able to reach the goal configuration.

To construct such a vector field we are going to use the concept of sliding motion along surfaces of discontinuity. This type of motion is generated by two additional vector fields, which are attached across the boundary $\partial\mathcal{O}_{g_j}$ of each $\mathcal{O}_{g_j}$. We call the region where those vector fields are defined the "belt zone", and these vector fields, "belt zone" vector fields. The "belt zone" is the region close to the $\partial\mathcal{O}_{g_j}$ and is thought to be composed of an "internal belt" and an "external belt" region. The "internal belt" region width is fixed for every $\partial\mathcal{O}_{g_j}$. The geometry of the "external belt" depends on the initial conditions. The "belt zone" concept is represented in Fig. (3).

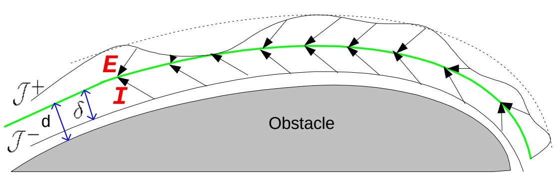

Fig. 3. Vector fields present in the "belt zone"

Let

$$\beta(s) = g_j(s) - \delta \cdot \widehat{\nabla g_j}^{\perp}(s)$$

with $0 < \delta < \rho_m$ .The "internal region" is the set:

$$\mathcal{I}_j = \{q : q = \lambda g_j(s) + (1-\lambda)\beta(s), \\ \lambda \in [0,1], s \in [0,S)\}$$

The "external region" is the set:

$$\mathcal{E}_j = \{q : q = \lambda g_j(s) + (1-\lambda)\sigma(s), \\ \lambda \in [0,1], s \in [0,S)\} \setminus \partial\mathcal{O}_{g_j}$$

where

$$\sigma(s) = g_j(s) + \Delta(\varphi(g_j(s)), \eta(s), s) \cdot \widehat{\nabla g_j}^{\perp}(s)$$

$$\eta(s) = \frac{-\nabla\varphi(g_j(s)) \cdot \widehat{\nabla g_j}^{\perp}(s)}{\|\nabla\varphi(g_j(s))| \cdot \left\|\widehat{\nabla g_j}^{\perp}(s)\right\|}$$

and

$$\Delta(x,y,s) = \\ = \begin{cases} \frac{\delta}{2}\left(\frac{x-\varphi_k}{1-\varphi_k} + \frac{s^2}{\epsilon+s^2} m(y)\right) & x \geq \varphi_k \\ \frac{\delta}{2}\frac{s^2}{\epsilon+s^2} m(y) & x < \varphi_k \end{cases}$$

with

$$m(y) = \max\left(0, \frac{\sin(\theta_{esc}) - y}{\sin(\theta_{esc}) + 1}\right)$$

where s is the distance travelled in the belt zone, $\epsilon << 1$ a positive number, $\frac{\pi}{2} > \theta_{esc} > 0$ is the minimum angle between the local tangent at $\partial\mathcal{O}_{g_j}$ and $-\nabla\varphi$ for which the system is allowed to leave $\partial\mathcal{O}_{g_j}$.

Let $h_{k_j} \in \partial\mathcal{O}_{g_j}$ be the point where the system intersects $\partial\mathcal{O}_{g_j}$ for the k_j'th time (for notational brevity, index "$_j$" might be dropped where it can be assumed by the context), since it is possible for the system to hit the same boundary for more than one times. Then $\varphi_{k_j} = \varphi(h_{k_j})$ is the value of φ at that point and is maintained while the system is in the belt zone. The choice of the width function $\Delta(q)$ is such that it only vanishes at the point of entry or when $\varphi(q) < \varphi(h_k)$ and the escape angle criterion is met.

To be able to create the belt zones in an environment, as described above, we must require that the minimum curvature ρ_{min} of any unmodelled obstacle is at least $\rho_{min} > d + \delta > 0$. The parameters $d > \delta > 0$ can be chosen to be arbitrarily small to satisfy the ρ_{min} requirement. In practice, since ρ_{min} is not known in advance, it is determined by the capabilities of the robot sensory equipment. In this case the sensor perceived obstacle perimeter is a smoothed version of the actual one, with $\rho_{min-perceived} \geq \rho_s$, where ρ_s is the minimum curvature the sensory equipment can detect. Parameters d and δ can then be chosen to satisfy $\rho_s > d+\delta > 0$.

B. Belt zone vector fields

We wish to choose the vector fields in such a way that the Filippov [2] solutions of the differential inclusion (6) will be a sliding motion along the boundary $\partial\mathcal{O}_{g_j}$. Moreover, the system must be able to leave this boundary when it reaches a configuration in which $h \in \partial\mathcal{O}_{g_j}$, where $\varphi(h) \leq \varphi_k$ and the neighboring vector fields point in the appropriate direction.

Let $h_{\mathcal{E}} \in \mathcal{E}$, $h_{\mathcal{I}} \in \mathcal{I}$ and s_∂ to be such that $g(s_\partial) = h_\partial$. Then for each $h_{\mathcal{E}}$ and $h_{\mathcal{I}}$, there exists unique h_∂ hence unique s_∂ such that $h_{\mathcal{E}} = \lambda_1 g(s_\partial) + (1-\lambda_1)\sigma(s_\partial)$ and $h_{\mathcal{I}} = \lambda_2 g(s_\partial) + (1-\lambda_2)\beta(s_\partial)$ for some $\lambda_1, \lambda_2 \in [0,1]$ as long as $\delta < \rho_m$. This follows from $g(s)$ being bijective, since we can choose δ to be small enough so we can bring the internal and external belt zone boundaries arbitrarily close to $\partial\mathcal{O}_{g_j}$ which is described by $g(s)$. Locally this is also true because since the distances of the belt zone boundaries are lower than the local radius of $g(s)$ for each s, then the set of points of minimum distance of any $h_{\mathcal{E}} \in \mathcal{E}$ or $h_{\mathcal{I}} \in \mathcal{I}$ from $\partial\mathcal{O}_g$ contains only one point h_∂ of $\partial\mathcal{O}_g$. So we can define the surjective functions $h_\partial^{\mathcal{E}} : q \in \mathcal{E} \to q \in \partial\mathcal{O}_g$ and $h_\partial^{\mathcal{I}} : q \in \mathcal{I} \to q \in \partial\mathcal{O}_g$. Let $s(q) = g^{-1}(q)$ denote the inverse function of g.

We choose for $q \in \mathcal{E}$ (external zone) the vector field:

$$V_{\mathcal{E}}(q) = \theta \cdot k_1 \cdot \nabla g\left(s\left(h_\partial^{\mathcal{E}}(q)\right)\right) - k_2 \nabla g^{\perp}\left(s\left(h_\partial^{\mathcal{E}}(q)\right)\right)$$

where $\nabla g(s) = \frac{\partial g(s)}{\partial s}$ and k_1 and k_2 are positive tuning constants and

$$\theta = \begin{cases} 1 & -\nabla\varphi\left(h_k\right)\cdot\nabla g\left(s\left(h_k\right)\right) \geq 0 \\ -1 & -\nabla\varphi\left(h_k\right)\cdot\nabla g\left(s\left(h_k\right)\right) < 0 \end{cases}$$

For the region $q \in \mathcal{I}$ we choose the vector field:

$$V_{\mathcal{I}}\left(q\right) = \theta\cdot k_3\cdot\nabla g\left(s\left(h_{\partial}^{\mathcal{I}}(q)\right)\right) + k_4\nabla g^{\perp}\left(s\left(h_{\partial}^{\mathcal{I}}(q)\right)\right)$$

with k_3, k_4 positive tuning constants.

We assume for the workspace that the minimum distance between disjoint objects is greater than $\alpha = 2d + 2\delta$ with $d > \delta$. Objects closer than α will be considered as one. The resolution parameter $\alpha < 2\rho_s$, can be chosen to be arbitrarily small.

IV. Control Strategy

We assume that we have a stationary environment and the robot can be described trivially by a fully actuated, first order kinematic model. In the workspace, the robot is represented by a point. The obstacles present in the environment are modelled by the navigation function [6]. The goal is for the robot to be able to navigate using the navigation function constructed on a predefined space, even if several obstacles have been added to the workspace (and are not modelled in the navigation function).

Let us define a vector field through the following multi-function:

$$f\left(q\right) = \begin{cases} -\nabla\varphi\left(q\right) & q \in \mathcal{W}_{free} \\ V_{\mathcal{E}_i}\left(q\right) & q \in \mathcal{E}_i \\ V_{\mathcal{I}_i}\left(q\right) & q \in \mathcal{I}_i \end{cases} \tag{4}$$

Consider the following differential equation:

$$\dot{\mathbf{x}} = f \tag{5}$$

Definition 1 (Filippov [2]): A vector function $\mathbf{x}\left(\cdot\right)$ is called a solution of (5) if $\mathbf{x}\left(\cdot\right)$ is absolutely continuous and

$$\dot{\mathbf{x}} \in K\left[f\right]\left(\mathbf{x}\right) \tag{6}$$

where

$$K\left[f\right]\left(\mathbf{x}\right) = \overline{co}\left\{\lim f\left(\mathbf{x}_i\right)|\mathbf{x}_i \to \mathbf{x}, \mathbf{x}_i \notin N\right\}$$

and N some set of measure zero. Across the surfaces of discontinuity, the set $K\left[f\right]\left(\mathbf{x}\right)$ is a linear segment, joining the endpoints of the vectors f^+ and f^- belonging to the neighboring vector fields.

By construction, switching in eq. (4) happens when the system reaches a distance $d_{min} > \delta$ from some $\mathcal{A}_i$ or $\mathcal{O}_j$. The behavior of the "external zone" at h_k is depicted in Fig. 4. At h_k the system is in the neighborhood of 4 neighboring vector fields: $-\nabla\varphi$, $V_{\mathcal{I}}$, and the two lobes of $V_{\mathcal{E}}$. Let P be the tangential plane at h_k and N its normal vector pointing towards J^+. Computing the differential inclusion at that point we have:

$$K\left[f\right]\left(h_k\right) = \overline{co}\left\{\lim f\left(\mathbf{x}_i\right)|\mathbf{x}_i \to h_k, \mathbf{x}_i \notin N\right\}$$

which for the given neighboring vector fields reduces to the following:

$$K\left[f\right]\left(h_k\right) = -w_1\cdot\nabla\varphi + w_2\cdot V_{\mathcal{I}} + \left(w_3 + w_4\right)V_{\mathcal{E}}$$

with $w_i \in [0,1]$, $i = 1\ldots4$ and $\sum_{i=1}^{4} w_i = 1$. Taking the inner product of $K\left[f\right]\left(h_k\right)\cdot\left(\theta\cdot\nabla g\left(g^{-1}\left(h_k\right)\right)\right)$ with θ as defined in Section III-B one can verify that the resulting set has only nonnegative elements. Then the Filippov solutions of the system will move across the surface of discontinuity in the direction of $\left(\theta\cdot\nabla g\left(g^{-1}\left(h_k\right)\right)\right)$.

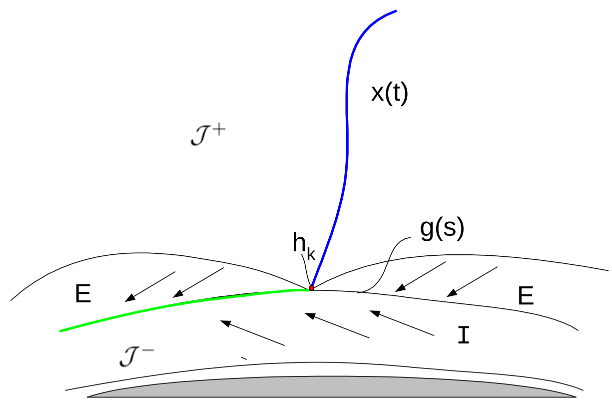

Fig. 4. Neighboring with 4 vector fields

Hence the system will enter the belt zone and start performing sliding motion. When the "external zone" has nonzero width, the Filippov solutions will satisfy:

$$\dot{\mathbf{x}} = f^0$$

where

$$f^0 = a\cdot V_{\mathcal{E}} + (1-a)\cdot V_{\mathcal{I}}$$

and

$$a = \frac{N\cdot V_{\mathcal{I}}}{N\cdot V_{\mathcal{I}} - N\cdot V_{\mathcal{E}}}.$$

The "external zone" has zero width by construction at the point where $s = 0$ (i.e. at h_k), and at the points where $-\frac{N\cdot\nabla\varphi}{\|N\cdot\nabla\varphi\|} \geq \sin\left(\theta_{esc}\right)$, $\varphi \leq \varphi(h_k)$ both hold. At those points the system's trajectories will flow along the solutions of the differential equation:

$$\dot{\mathbf{x}} = -\nabla\varphi$$

Now we can state the following:

Proposition 1: The system

$$\dot{\mathbf{x}} = u$$

under the control law:

$$u = f$$

with f as defined in eq. (4) is globally asymptotically stable, almost everywhere [2]

[2]Almost everywhere i.e. everywhere except a set of initial conditions of measure zero

Proof: We will consider the system as operating in two possible modes: Mode Φ where $q \in \mathcal{W}_{free}$ and mode $\mathcal{B}$ where $q \in \mathcal{E} \bigcup \mathcal{I}$. We will show that while in mode Φ the potential φ decreases, when in mode $\mathcal{B}$ the system stays there for only finite time and the exit potential from mode $\mathcal{B}$ is lower than the entry potential.

Since φ is a navigation function, all initial states in the original environment are brought to the origin, except a set of initial states having measure zero, that lead to unstable saddle points. For $q \in \Phi$ we have that :

$$\dot{\varphi}(\mathbf{x}) = -\|\nabla\varphi\|^2 \overset{a.e.}{<} 0$$

with

$$\dot{\varphi}(\mathbf{x}) = 0 \, \forall \mathbf{x} \in \{0\} \bigcup S$$

S being the set of saddle points. A navigation function has exactly that many isolated saddle points as the number of obstacles [3] . Hence under the given control law the potential is strictly decreasing almost everywhere in Φ. Assume now that the system enters the belt zone: $q \in \mathcal{B}$ with entry potential $\varphi(h_k)$. The system will then start tracing the boundary $\partial\mathcal{O}_{g_j}$. A point $q_x \in \mathcal{B}$ is candidate for exit point if $\varphi(q_x) \leq \varphi(h_k)$ and $-\frac{N \cdot \nabla\varphi}{\|N \cdot \nabla\varphi\|} \geq sin(\theta_{esc})$. Proposition 1 guarantees the existence of a set of points of minimum for φ across $\partial\mathcal{O}_{g_j}$, while Proposition 2 guarantees that for the points of minimum, $-\nabla\varphi(q_x)$ points outwards $\mathcal{O}_g$ and moreover that is perpendicular to $\nabla(g)$. Hence we have established the existence of points over $\partial\mathcal{O}_{g_j}$ where the system will exit mode $\mathcal{B}$. An upper bound for the time that the system will remain in $\mathcal{B}$ can always be established, depending on the choice of g. From the assumptions imposed on g, $\nabla g(s)$ is nonzero for all s. Hence $\|f^0\| > 0$ and let $f_0 = \min \|f^0\|$. Then the time the system remains in $\mathcal{B}$ is bounded by $t_{max} = \frac{S}{f_0}$.

Hence we have that for every system switching from Φ to $\mathcal{B}$ and back to Φ, the potential level of φ decreases. If that was not the case, that would have meant that our entry point had minimum potential, which is a contradiction since at the point of minimum potential $-\nabla\varphi$ points outwards as stated in Lemma 2.

It is thus shown that when the system is in Φ the potential decreases, if the system enters $\mathcal{B}$ then there will always be a point where the system will exit $\mathcal{B}$ and for every transition $\Phi \rightarrow \mathcal{B} \rightarrow \Phi$ the potential level of φ decreases. Hence the system will eventually come to rest when φ reaches its minimum, which is, by construction, the desired configuration. Treating φ as a common Lyapunov function, stability is established in the context of switched systems [4]. ■

V. Simulation Results

Computer simulations have been carried out to verify the feasibility and efficacy of the proposed methodology. In the first three case studies, three disks of unit radii were assumed to be the modelled obstacles. In all case studies the unknown obstacle(s) were assumed to be in configurations inhibiting convergence. Modelled obstacles were represented with blue color, non-modelled with red and robot trajectories with black. The robot was assumed to have a radial proximity sensor with limited range capability (only a small fraction of the unmodelled obstacle was visible). A simple sensor data processing algorithm had been applied to interpret sensor readings.

Case Study-1: Two non – modelled (unknown) obstacles were added to the workspace (Fig. 5). The added obstacles were touching the modelled ones, so the robot had to bypass both modelled and un-modelled obstacles to get to the target. The modelled obstacles were placed at $(-4, 6)$, $(4, 6)$, $(0, 3)$. The robot initial configuration was: $\mathbf{x}(0) = (6, 0)$ and the target was set at $\mathbf{x}_{goal} = (0, 0)$. Fig. 5 depicts the first simulation results.

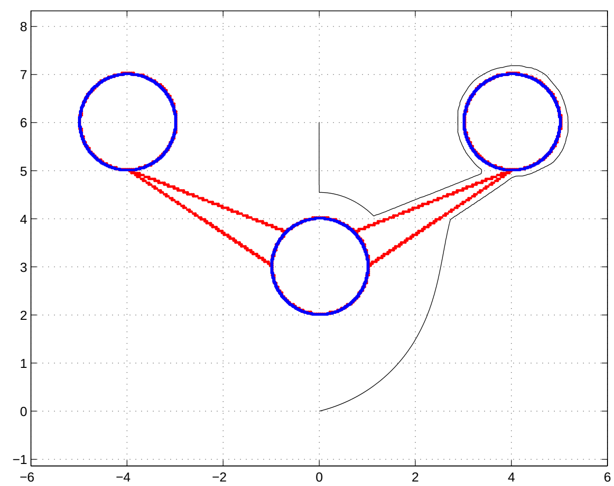

Fig. 5. Robot path in case study-1.

Case Study-2: A different unmodelled obstacle was introduced in the workspace (Fig. 6). The modelled obstacles were placed at $(-2.5, 0)$, $(2.5, 0)$, $(0, 3)$. The robot was placed at $\mathbf{x}(0) = (1.5, 3)$ and the target was the origin. Fig. 6 depicts the second simulation's results.

Case Study-3: the robot was placed at $\mathbf{x}(0) = (0.2, 4.5)$ and the target was the origin (Fig. 7). The workspace was the same as in case study-2. Observe that the robot is heading away from the modelled obstacle and in opposite direction from the target in the beginning. This is due to the use of the navigation function's repulsive potential of the obstacle placed at $(0, 3)$.

Case Study-4: In this case study, the modelled workspace was the one shown with blue color in Fig. 8 and the non-modelled obstacle with red. The robot was placed at $\mathbf{x}(0) = (-11, 6)$ and the target was set at $\mathbf{x}_{goal} = (3, 1)$. And in this scenario, our algorithm successfully converges to the goal configuration avoiding modelled and unmodelled obstacles.

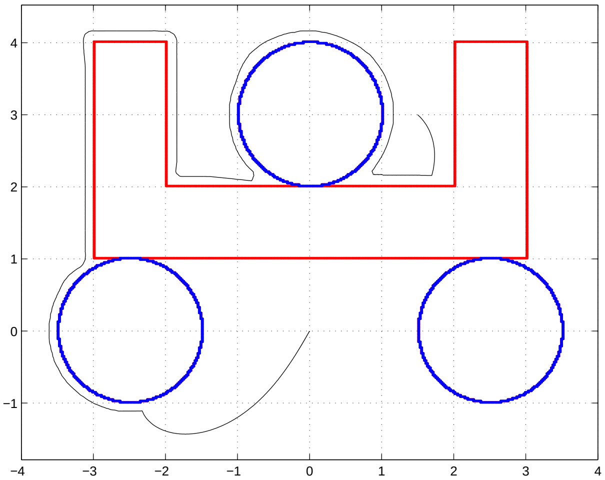

Fig. 6. Obstacles and trajectory for case study - 2

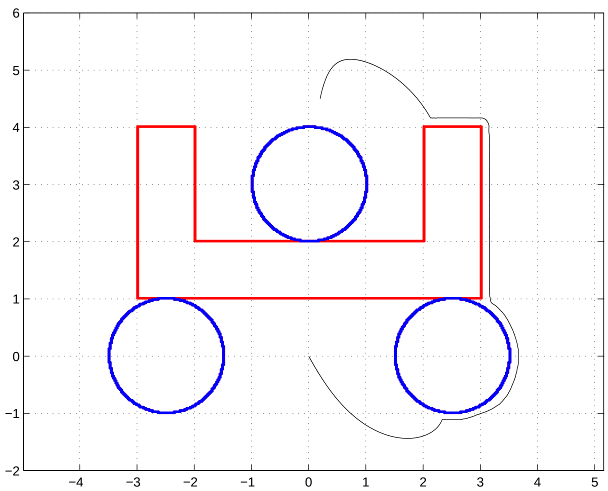

Fig. 7. Obstacles and trajectory for case study - 3

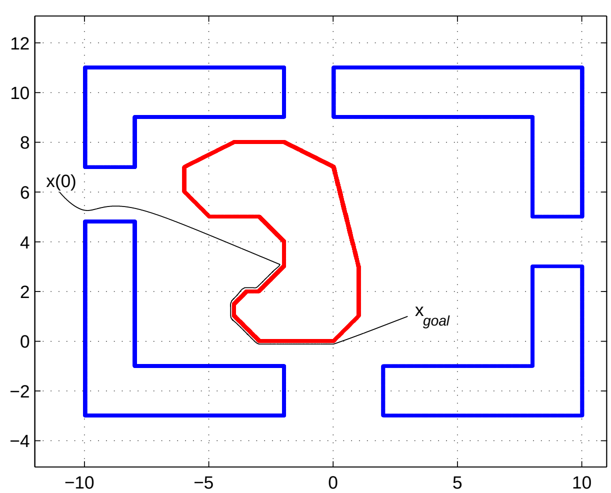

Fig. 8. Obstacles and trajectory for case study - 4

VI. Concluding Remarks

A methodology of navigating a point robot in partially known environments has been derived. Partial knowledge about the robot's environment is modelled using a navigation function. Based on the properties of the navigation functions, an obstacle avoidance scheme is applied to successfully avoid both known and unknown obstacles. The analysis is based on the assumptions that there are no sharp edges in nature, i.e. any sharp edge in nature has a non zero radius of curvature, and that robot sensors can detect a nonzero radius of curvature. The methodology achieves global asymptotic convergence to the target with guaranteed collision avoidance properties. Due to the closed form of the feedback controller, the methodology is particularly suitable for implementation on real time systems with limited computation capability. Current research directions include motion planning in non – static partially known environments, multiple robot scenarios in partially known environments and navigation in partially known three dimensional environments.

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VII. References

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