THE ENTROPY OF COOPERATIVE RADIATION SENSING BY DISTRIBUTED SENSORS

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By expressing the expected collective radiation counts registered by the robotic swarm, at any given workspace location as a function of the prior knowledge about the distribution of radiation intensity in the workspace, the sensor geometry, and the sensors' trajectories, we apply Bayes' rule to obtain the probability density function of local radiation intensity conditioned on the collected measurements and the trajectory of the sensor. From this, we derive analytical expressions for the entropy of the distributed sensing system as a function of time. We show that the entropy of the distributed sensing system is stable and converges to steady state values.

I. INTRODUCTION

The development of algorithms for the automated detection of weak radiation activity is motivated by applications in the field of nuclear nonproliferation: nuclear forensics, where one searches for specks of radioactive material as evidence of illicit activity; or detection of shielded special nuclear material, being smuggled through international points of entry.

One method of approaching this detection problem is by bringing the sensor close to the source. The Signal-to-Noise-Ratio follows an inverse square law with respect to the distance as well as with respect to the area of the sensor. This means by bringing the sensor closer to the source, one can either achieve the same detection efficiency with smaller (thus less expensive) sensors, or improve significantly the detection capabilities using the same sensor technology. The key in such an approach is controlled sensor mobility, and having multiple sensors deployed and coordinated simultaneously will allow scanning reasonably large areas in little time. Our ultimate goal is to design gradient-based cooperative deployment control laws for the robotic swarm, and the entropy formulation presented in this paper is a first required step in this direction.

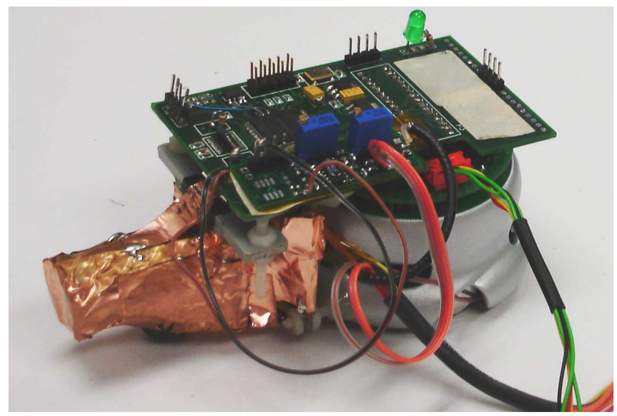

Fig. 1. The experimental platform used in our earlier work [1]. It consists of a Khepera II mobile robot on which a La_2Br scintillator is mounted.

We consider a group of mobile robots equipped with radiation detectors as a distributed, reconfigurable, sensor (Fig. 1). The detector on each one of these mobile robots realizes a communication channel between the environment and the robotic system, and viewed as such, it is characterized by a conditional differential entropy. This quantity is a measure of the uncertainty associated with the corresponding radiation measurement, and can be used to distribute the robots in the workspace in a way that improves the world model constructed based on the measurements.

I.A. Prior Related Work

With relatively inexpensive and capable hardware in communication, computing, and sensing, as well as the many practical applications, research in cooperative control of mobile robots has become a big area in controls research. Applications can be found in coordination of robots for topological mapping. [2,3,4] When coordinating robots for the purpose of mapping, one can set target points for each robot and then control them toward these target points.[2] Such target points are chosen by weighing the cost and utility of each target point. Other approaches[3] define region *frontiers*, based on information-theoretic considerations. Frontiers are regions that are on the boundaries of the area which has been explored and the region which is unexplored. Then the group is coordinated so that these frontier regions are continuously extended. An alternative method is to use an occupancy grid in conjunction with a Bayesian update rule to coordinate the group for topological mapping.[4] In an occupancy grid, the environment is divided into homogeneous cells that contain a probability of being occupied by an obstacle. Another application in cooperative control is found in robotic deployment of sensor networks.[5,6,7,8] Potential fields can be used to disperse robots within an area and coordinate their motion.[5] Potential fields place artificial forces on the agents, and through these forces the desired trajectories are obtained. In a conceptually similar fashion, one can use gradient climbing algorithms to distribute agents in an optimal fashion over the area of interest.[6,7] Agents follow gradients that maximize a static density function that is weighted by a sensor performance function. Gradient climbing can be implemented either in a spatially distributed fashion[6] or without partitioning the area among the team members, which may help reducing computational overhead.[8]

Most of the research in navigation for groups of robots for the purpose of distributed exploration and mapping assumes a static, time-invariant environment. Because of this, a good model of the environment is required. In application like the one considered in this paper, this assumption is not realistic. Furthermore, most approaches to group navigation are not reactive with respect to the sensed environment, and therefore unable to cope with a dynamic environment. There are some notable exceptions[9], in which the control of the group is done in a way to reduce state estimate uncertainty. This type of navigation is named *information surfing*, because agents are driven to maximize their information gain. Information surfing[9] has been used for topological mapping and surveillance, which is, however, quite different from radiation mapping due to the underlying statistics.

The goal in this paper is to enable information surfing for radiation mapping. We need to recast the problem of “which locations are interesting to visit first" within the framework of nuclear statistics, and identify the important differences and similarities that will guide cooperative control design at a later stage.

I.B. Overview of the Approach

First we express the expected collective radiation counts registered by the robotic swarm, at any given workspace location as a function of the prior knowledge about the distribution of radiation intensity in the workspace, the sensor geometry, and the robots' trajectories. With Bayes rule we obtain the probability density function of local radiation intensity conditioned on the collected measurements and the robot's trajectory. This allows us to derive analytical expressions for the entropy of the distributed sensing system as a function of time. The robotic swarm should only decrease the measurement entropy, facilitating a more accurate description of the radiation world around it.

Assuming that the total number of radiation counts registered is a strictly increasing function of time, an assumption that is reasonable in the presence of background radiation, we show that the entropy of the distributed sensing system is stable and converges to steady state values. The entropy formulation in this paper will be the basis for the gradients that will guide the robots. The time-varying nature of the entropy function characterizing measurement uncertainty makes the stability analysis of the closed loop system particularly challenging. Thus, the time-asymptotic analysis of the entropy function and its derivatives, out of which desired motion directions for the sensors will be derived, is expected to be central to the stability and convergence analysis of the cooperative sensing system.

I.C. Paper Organization

The rest of the paper is organized as follows. In Section II we describe our model for radiation measurements and the associated statistics used in this work. Then in Section III we show that by viewing the radiation detector as an information channel between the environment and the system, we can associate it with concepts such as differential entropy and mutual information. Section IV presents our main result, which describes the asymptotic properties of the mutual information associated with nuclear measurement in a static environment. In Section V we outline how to generalize our approach from a single sensor to the distributed sensor network setting. Finally, Section VI summarizes the discussion and concludes the paper.

II. THE PROBABILISTIC DYNAMICS OF RADIATION MEASUREMENTS

Low-rate counting of radiation from nuclear decay is described by the Poisson statistics, where the probability to register n counts in t seconds, from a source assumed to emit an average of μ counts per second (cts/s) is

$$P(n,t) = \frac{(\mu \cdot t)^n}{n!} e^{-(\mu \cdot t)}. \tag{1}$$

In [10], the authors describe how to model radiation measurements from a moving source with a stationary sensor. In our approach we take that same idea but turn it around and look at a stationary source with a moving sensor. We can describe the expected number of source counts, μ, to be registered by a moving sensor as

$$\mu = \chi \cdot \alpha \int_0^t \frac{1}{r^2(t)} \, \mathrm{d}t \,, \tag{2}$$

where χ is the cross sectional area of the sensor, α is the activity of the source, and $r(t)$ is the instantaneous distance of the source to the sensor.

We can now describe the probability density function (PDF) associated with the random variable c, which is the total number of counts recorded, for a moving sensor as

$$f(c) = \frac{(\mu)^c}{c!} \cdot e^{-(\mu)}, \tag{3}$$

where μ is expressed in (2).

The expected number of counts μ is conditioned on the source having activity α, the cross sectional area of the sensor being χ, and the distance between the source and sensor being $r(t)$. Therefore the PDF associated with the random variable c is formally

$$f(c) = f(c|\alpha, \chi, r(t)). \tag{4}$$

In [1] it is seen that Bayes rule allows us to calculate $f(\alpha|c, \chi, r(t))$ using $f(c|\alpha, \chi, r(t))$. As new measurements are taken by the sensor, we update the distribution using the equation

$$f(\alpha|c, \chi, r(t)) = \frac{f(\alpha) \cdot f(c|\alpha, \chi, r(t))}{f_c(c)}. \tag{5}$$

Function $f(\alpha)$ is the PDF associated with having a source with activity α at a distance $r(t)$. In our formulation we take this to be the uniform distribution, from source activity α_1 to a source activity α_2. This allows us to search and map radiation levels from an arbitrary source with activity α, such that $\alpha_1 < \alpha < \alpha_2$. In fact, $f(\alpha)$ is a function of position too, but when assuming a uniform distribution, the position of the sensor does not matter. From this point, the PDF expressing our initial guess about the source activity will be expressed as

$$f(\alpha) = \begin{cases} \frac{1}{\alpha_2 - \alpha_1}, & \text{if } \alpha_1 < \alpha < \alpha_2 \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Function $f_c(c)$ is the marginal density function of $f(c)$

$$f_c(c) = \int_{\alpha_1}^{\alpha_2} \left[\frac{(\chi \cdot \alpha \int_0^t \frac{1}{r^2(t)} \, \mathrm{d}t)^c}{c!} \cdot e^{-(\chi \cdot \alpha \int_0^t \frac{1}{r^2(t)} \, \mathrm{d}t)} \right] \mathrm{d}\alpha. \tag{7}$$

III. DIFFERENTIAL ENTROPY AND MUTUAL INFORMATION

Viewing our radiation sensor as a communication channel between the robot (receiver) and its environment (sender), we can introduce metrics to describe the transmission of information. An entropy-based metric provides an intuitive way of measuring how much information we gain by a given measurement. The entropy is also known as self information, and is related to uncertainty because as the latter decreases the information gain increases. By using such a measure we can formalize the objective of our control law as reducing the uncertainty of our belief regarding the radiation levels over the area of interest.

Considering a single sensor at first, we imagine an information channel between this sensor and every location in the environment where a radiation source is likely to be present. Thus every such channel is “anchored" at a particular location in the workspace, and the information metric we derive is therefore associated to that location. Concequently, one can define the distribution of such a metric over the whole workspace, and use this distribution to guide measurement collection.

Information theory defines the conditional differential entropy of a continuous random variables A associated with the transmitted signal, (in our case the radiation source activity), and C associated with the received signal, (in our case the number of counts registered by our sensors) as follows,

$$h(A|C) = - \int_{\alpha_1}^{\alpha_2} f(\alpha|c) \cdot \log_2 f(\alpha|c) \, \mathrm{d}\alpha. \tag{8}$$

Note that we are using the definition of the differential entropy for continuous distributions even though the Poisson distribution is discrete. This because our ultimate objective is to derive a continuous time/space control laws based on information gradients. Our workspace is continuous even if our measurement events are discrete.

Using (5), denoting the generalized hypergeometric function $\Omega(\cdot)$ and defining

$$d \triangleq \int_0^t \frac{1}{r^2(t)} \, \mathrm{d}t, \tag{9}$$

we get a closed form solution for the differential entropy in (8) as follows

$$h(A|C) = \frac{-d \cdot \chi}{[\Gamma(c+1, \alpha_1 \cdot d \cdot \chi) - \Gamma(c+1, \alpha_2 \cdot d \cdot \chi)] \log 2} \cdot \Bigg[\Bigg(\frac{c \cdot \Omega(c+1, c+1; c+2, c+2; -\alpha_1 \cdot d \cdot \chi)(\alpha_1 \cdot d \cdot \chi)^{c+1}}{(c+1)^2 d \cdot \chi} - \frac{(c+1)^2 \Gamma(c+2, \alpha_1 \cdot d \cdot \chi)}{(c+1)^2 d \cdot \chi} \Bigg)$$

$$
\begin{aligned}
&\cdot\Bigg(\frac{c\cdot\Omega(c+1,c+1;c+2,c+2;-\alpha_2\cdot d\cdot\chi)(\alpha_2\cdot d\cdot\chi)^{c+1}}{(c+1)^2 d\cdot\chi} \\
&\qquad -\frac{(c+1)^2\Gamma(c+2,\alpha_2\cdot d\cdot\chi)}{(c+1)^2 d\cdot\chi}\Bigg) \\
&\cdot\Bigg(\frac{(c+1)\Gamma(c+1,\alpha_1 d\chi)(\alpha_1 d\chi+\log(e^{-\alpha_1 d\chi})(\alpha_1 d\chi)^c)}{(c+1)^2 d\cdot\chi} \\
&\qquad -\frac{c\cdot\Gamma(c+2)\log(\alpha_1)}{(c+1)^2 d\cdot\chi}\Bigg) \\
&\cdot\Bigg(\frac{(c+1)\Gamma(c+1,\alpha_2 d\chi)(\alpha_2 d\chi+\log(e^{-\alpha_2 d\chi})(\alpha_2 d\chi)^c)}{(c+1)^2 d\cdot\chi} \\
&\qquad -\frac{c\cdot\Gamma(c+2)\log(\alpha_2)}{(c+1)^2 d\cdot\chi}\Bigg)\Bigg] \\
&\qquad -\frac{\log\left(\frac{d\cdot\chi}{\Gamma(c+1,\alpha_1\cdot d\cdot\chi)-\Gamma(c+1,\alpha_2 d\cdot\chi)}\right)}{c!\log 2}, \qquad (10)
\end{aligned}
$$

Note that (10) is a time and position dependent quantity.

It is known that continuous differential entropy can not be directly associated with self-information – contrary to the discrete scheme[11]. One concept that *does* carry over from the discrete setting is the *mutual information*

$$I(X;Y)\triangleq h(X)-h(X|Y).$$

Mutual information quantifies the mutual dependence of the two random variables, X and Y. It tells us how knowing one variable, Y, reduces our uncertainty about the other variable, X. We exploit the property of mutual information expressed in the following Lemma.

Lemma III.1. *([12]) $I(X;Y)\geq 0$ with equality iff X and Y are independent.*

For our problem, the mutual information is defined as

$$I(A;C)=h(A)-h(A|C),$$

where

$$
\begin{aligned}
h(A)&=-\int_{\alpha_1}^{\alpha_2} f(\alpha)\,\mathrm{d}\alpha \\
&=-\int_{\alpha_1}^{\alpha_2}\frac{1}{\alpha_2-\alpha_1}\cdot\log\left(\frac{1}{\alpha_2-\alpha_1}\right)\mathrm{d}\alpha \\
&=(\alpha_2-\alpha_1)\left[\frac{1}{\alpha_2-\alpha_1}\cdot\log\left(\frac{1}{\alpha_2-\alpha_1}\right)\right]\triangleq K,
\end{aligned}
$$

which is a constant. With $h(A|C)$ as in (10), mutual information is described as

$$I(A;C)=K-h(A|C). \qquad (11)$$

Equation (11) expresses how knowing the number of radiation counts, reduces our uncertainty regarding the presence of the source A.

IV. ASYMPTOTIC PROPERTIES OF MUTUAL INFORMATION

The following Lemma summarizes the main result of our paper.

Lemma IV.1. *Let $I(q,p_i,t)$ be the mutual information of the radiation sensor information channel. Then,*

$$\lim_{t\to\infty}\frac{\partial I(q,p_i,t)}{\partial t}=0.$$

Proof: Taking the mutual information defined in equation (11), we find its time derivative to be

$$\frac{\partial I(q,p_i,t)}{\partial t}=\frac{\partial h}{\partial d}\frac{\partial d}{\partial t}.$$

The first partial derivative takes the form

$$\frac{\partial h}{\partial d}=\sum_{i=1}^{12}A_i, \qquad (12)$$

where A_1 through A_{12} are fractional terms that make up the whole expression. For the first term

$$A_1\triangleq\frac{-(1+c)\Gamma(1+c,\alpha_1\cdot d\cdot\chi)^2}{(1+c)dc!(\Gamma(1+c,\alpha_1 d\chi)-\Gamma(1+c,\alpha_1 d\chi)^2)\log 2},$$

we find that

$$\lim_{d\to\infty}A_1=0,$$

due to having $c!$ in the denominator, which is the total number of radiation counts collected during the mission. Notice that the total number of counts c, grows much faster than d does, because of the added effect of background radiation and source. Other terms that show this same behavior are:

$$A_2=\frac{(-1-c)\Gamma(1+c,\alpha_2 d\chi)^2}{(1+c)d\cdot c!(\Gamma(1+c,\alpha_2 d\chi)-\Gamma(1+c,\alpha_1 d\chi)^2))\log 2},$$

$$
\begin{aligned}
A_3=&\frac{(-1-c)e^{(-\alpha_1\alpha_2)d\chi}\Gamma(1+c,\alpha_1 d\chi)}{(1+c)dc!(\Gamma(1+c,\alpha_2 d\chi)-\Gamma(1+c,\alpha_1 d\chi)^2)\log 2} \\
&\cdot\Bigg(\frac{\alpha_1 d e^{\alpha_2 d\chi}\chi(\alpha_1 d\chi)^c(1+\alpha_1 d\chi\Gamma(1+c))}{(1+c)dc!(\Gamma(1+c,\alpha_2 d\chi)-\Gamma(1+c,\alpha_1 d\chi)^2)\log 2} \\
&-\frac{(\alpha_1 d e^{\alpha_2 d\chi}\chi(\alpha_1 d\chi)^c)2e^{(-\alpha_1-\alpha_2)d\chi}\Gamma(1+c,\alpha_2 d\chi)}{(1+c)dc!(\Gamma(1+c,\alpha_2 d\chi)-\Gamma(1+c,\alpha_1 d\chi)^2)\log 2}\Bigg),
\end{aligned}
$$

$$
\begin{aligned}
A_4=&-\alpha_1(1+c)e^{-(\alpha_1+\alpha_2)d\chi+\alpha_2 d\chi}\chi(\alpha_1 d\chi)^c\Gamma(2+c) \\
&\cdot\frac{\Gamma(2+c,\alpha_1 d\chi)-\Gamma(2+c,\alpha_2 d\chi)+c\Gamma(1+c)\log(\frac{\alpha_1}{\alpha_2})}{(1+c)dc!(\Gamma(1+c,\alpha_2 d\chi)-\Gamma(1+c,\alpha_1 d\chi)^2)\log 2},
\end{aligned}
$$

$$A_5 = -(\alpha_2 d\chi)\alpha_2(1+c)e^{-(\alpha_1+\alpha_2)d\chi+\alpha_1 d\chi}\chi$$
$$\cdot \frac{(1+c+\alpha_2 d\chi\Gamma(2+c))\Gamma(1+c,\alpha_2 d,\chi)}{(1+c)dc!(\Gamma(1+c,\alpha_2 d\chi)-\Gamma(1+c,\alpha_1 d\chi)^2)\log 2},$$

$$A_6 = -(\alpha_2 d\chi)\alpha_2(1+c)e^{-(\alpha_1+\alpha_2)d\chi+\alpha_1 d\chi}\chi$$
$$\cdot \frac{\Gamma(2+c)\Gamma(2+c,\alpha_1 d,\chi)}{(1+c)d\cdot c!(\Gamma(1+c,\alpha_2 d\chi)-\Gamma(1+c,\alpha_1 d\chi)^2)\log 2},$$

$$A_7 = -(\alpha_2 d\chi)\alpha_2(1+c)e^{-(\alpha_1+\alpha_2)d\chi+\alpha_1 d\chi}\chi\Gamma(1+c,\alpha_1 d\chi)$$
$$\cdot \frac{(1+c+\Gamma(2+c))\left(\alpha_1 d\chi+\log(\frac{e^{-\alpha_1 d\chi}(\alpha_1 d\chi)^c}{e^{-\alpha_2 d\chi}(\alpha_2 d\chi)^c}\right)}{(1+c)dc!(\Gamma(1+c,\alpha_2 d\chi)-\Gamma(1+c,\alpha_1 d\chi)^2)\log 2},$$

$$A_8 = -\alpha_2 d\chi\alpha_1(1+c)e^{-(\alpha_1+\alpha_2)d\chi+\alpha_2 d\chi}\chi\Gamma(1+c,\alpha_2 d\chi)$$
$$\cdot \frac{(1+c+\Gamma(2+c))\left(\alpha_1 d\chi+\log(\frac{e^{-\alpha_2 d\chi}(\alpha_2 d\chi)^c}{e^{-\alpha_1 d\chi}(\alpha_1 d\chi)^c}\right)}{(1+c)dc!(\Gamma(1+c,\alpha_2 d\chi)-\Gamma(1+c,\alpha_1 d\chi)^2)\log 2}.$$

The last four terms that make up (12) are the ones that involve a generalized hypergeometric function, $\Omega(\cdot)$:

$$A_9 = \alpha_1^2 cde^{(-\alpha_1-\alpha_2)d\chi+\alpha_2 d\chi}\chi^2(\alpha_1 d\chi)^{2c}$$
$$\cdot \frac{\Omega(1+c,1+c,2+c,2+c,-\alpha_1 d\chi)}{(1+c)^2(\Gamma(1+c,\alpha_1 d\chi)-\Gamma(1+c,\alpha_2 d\chi))^2\log 2},$$

$$A_{10} = \alpha_1^2 cde^{(-\alpha_1-\alpha_2)d\chi+\alpha_1 d\chi}\chi^2(\alpha_2 d\chi)^{2c}$$
$$\cdot \frac{\Omega(1+c,1+c,2+c,2+c,-\alpha_2 d\chi)}{(1+c)^2(\Gamma(1+c,\alpha_1 d\chi)-\Gamma(1+c,\alpha_2 d\chi))^2\log 2},$$

$$A_{11} = (\alpha_1\alpha_2 cde^{(-\alpha_1-\alpha_2)d\chi+\alpha_1 d\chi}\chi^2(\alpha_1 d\chi)^c$$
$$\cdot \frac{\Omega(1+c,1+c,2+c,2+c,-\alpha_1 d\chi))}{(1+c)^2(\Gamma(1+c,\alpha_1 d\chi)-\Gamma(1+c,\alpha_2 d\chi))^2\log 2},$$

$$A_{12} = (\alpha_1\alpha_2 cde^{(-\alpha_1-\alpha_2)d\chi+\alpha_2 d\chi}\chi^2(\alpha_1 d\chi)^c$$
$$\cdot \frac{\Omega(1+c,1+c,2+c,2+c,-\alpha_2 d\chi))}{(1+c)^2(\Gamma(1+c,\alpha_1 d\chi)-\Gamma(1+c,\alpha_2 d\chi))^2\log 2}.$$

Recall that the generalized hypergeometric function Ω is defined as

$$\Omega(1+c,1+c,2+c,2+c,-\alpha_1 d\chi) = \sum_{n=0}^{\infty}\frac{(1+c)_n(1+c)_n}{(2+c)_n(2+c)_n}\cdot\frac{-\alpha_1^n}{n!}, \quad (13)$$

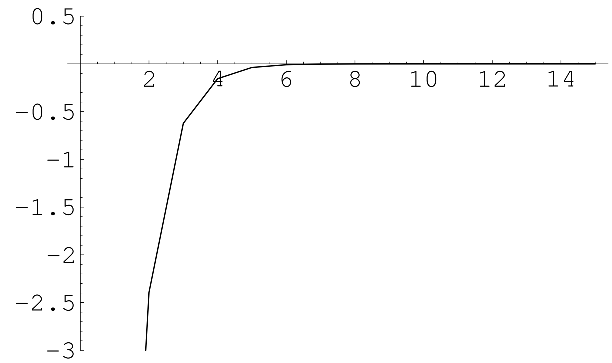

Fig. 2. The behavior of $\frac{\partial h}{\partial d}$ (on the vertical axis) as d (on the horizontal axis) grows large, under the conditions: $\alpha_1 = 1$ cts/s, $\alpha_2 = 10$ cts/s, $\chi = 1$ cm, $c = 2d$.

where $(1+c)_n = (1+c)(1+c+1)(1+c+2)\cdots(c+n)$. Function (13) exhibits oscillatory behavior with n, and because of this the net effect is

$$\Omega(1+c,1+c,2+c,2+c,-\alpha_1 d\chi) = 0.$$

In fact, any reasonably large finite term approximation of Ω will yield outcomes that are numerically insignificant. For this reason, we get no contribution from the terms A_9 through A_{12}, and (12) becomes

$$\frac{\partial h}{\partial d} = \sum_{i=1}^{8} A_i.$$

Note now, that for all terms A_1 through A_8 it holds

$$\lim_{d\to\infty}\frac{\partial h}{\partial d} = 0,$$

and with $c!$ growing much faster than time as captured by the evolution of d (even in the case of only background radiation),

$$\lim_{t\to\infty}\frac{\partial I(q,p_i,t)}{\partial t} = \lim_{t\to\infty}\frac{\partial h}{\partial d}\frac{\partial d}{\partial t} = 0.$$

The asymptotic analysis of the proof of Lemma IV.1 is verified in the plot of the rate of change of the entropy, as a function of variable d, given in Fig. 2.

V. DISTRIBUTED SENSING

When collecting measurements with mutliple detectors attached to mobile robots scattered over a certain area, one can view the system as a single, "shattered" radiation sensor. This distributed, reconfigurable sensor has a cross sectional area equal to the sum of the areas of each individual detector. An individual detector becomes a piece of the single virtual detector and radiation counts collected by any one of the detectors are assumed to be shared. They are treated as counts registered by the single virtual detector.

Recalling equation (10), we note two variables in the expression of differential entropy: the number of counts c, and the function d, which is related to the distance between source and sensor. The question thus becomes, how to define these two parameters, if one needs to evaluate the differential entropy in a distributed (robot) setting.

As we have implied, the number of counts c is taken as the sum of the counts registered collectively by all individual detectors. Function d is dependent on the distance between source and sensor $r(t)$ as shown in (9). Considering a source at a specific location within the workspace, q and a group of mobile sensor platforms positioned at $p_i(t)$, for $i = 1, \ldots, n$, the distance $r(t) = \min_{i\in\{1,\ldots,n\}} \|p_i(t) - q\|$ varies with time, but evolves in a piecewise continuous fashion, so that the integral of (9) can still be computed.

Consequently, one way to carry over the analysis of Sections III–IV to a multi-robot setting is to implement (10) using

$$c = \sum_{i=1}^{n} c_i, \quad d = \int_{t_0}^{t} \frac{1}{\min_{i\in\{1,\ldots,n\}} \|p_i(\tau) - q\|} \mathrm{d}\tau, \qquad (14)$$

where c_i is the total number of counts collected by all n sensor platforms, q is the location of the assumed radiation source, and $p_i(t)$ is the distance between the ith robot and the source at q at time t. Substituting these expressions in (10) makes $h(A|C)$ a function of the number of counts c, the source location q, and time t; thus for every location q in the workspace that could have a radiation source, a function $h_q(c,t)$ is defined. For that particular workspace location again, the corresponding function $I_q(A;C)$ defined using (11) captures the amount of information contained in the collective measurement signal of all n sensors, that have registered a total of $C = c$ counts possibly coming from a source of activity $A = \alpha$ at location q.

VI. CONCLUSIONS

In this paper, we treated a radiation sensor as an information channel between the environment and the automated monitoring system. As such, the measurements that a sensor makes can be associated with entropy and mutual information. The main result described in this paper is a formal analysis of the asymptotic properties of the mutual information associated with a sensor/channel, which concludes that for a static environment, the rate of information that flows from the environment to the system decays with time. Then we carry the analysis over from a single-sensor setting to the distributed sensor network setting, suggesting how the collective measurements can be fused together in the calculation of differential entropy. We expect this analysis to be useful in designing control algorithms for mobile sensor networks tasked with monitoring and mapping radiation distributions.

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