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Nonholonomic Stabilization with Collision Avoidance for Mobile Robots

Herbert G. Tanner, Savvas Loizou and Kostas J. Kyriakopoulos

Control Systems Laboratory,
Mechanical Engineering Department,
National Technical University of Athens,
9 Heroon Polytechniou St, Athens, 157 80, Greece
htanner@ieee.org, {sloizou, kkyria}@central.ntua.gr

Abstract

This paper presents a motion planner and nonholonomic controller for a mobile robot, with global collision avoidance and convergence properties. An appropriately designed (*dipolar*) potential field is combined with discontinuous state feedback. A new class of Lyapunov functions is introduced and used for nonholonomic navigation. The obstacle avoidance and global asymptotic stability properties are verified through simulations.

1 Introduction

Nonholonomic systems have received increased attention in the last years. The survey paper [1] illustrates the intensity of research effort in the area. This interest is motivated by the large class of mechanical systems that fall in this category and the challenging problem of stabilization. The class of nonholonomic systems includes mobile robots, underwater vehicles, aircrafts, surface vessels and underactuated manipulators. Stabilization of nonholonomic systems is quite involved, mainly due to the fact there is no smooth (or even continuous) time invariant static state feedback law that can stabilize such systems [2].

The stabilization problem has been approached from two main directions: discontinuous state feedback [3, 4, 5] and time varying state feedback [6, 7, 8, 9]. A parallel direction is the differential flatness approach [10, 11] that utilizes dynamic feedback linearization. Discontinuous feedback approaches have generally exhibited faster convergence rates compared to time-varying approaches. The use of homogeneous feedback [9] narrowed the gap.

A large class of nonholonomic systems consists of mobile robots and the issue of collision avoidance frequently arises. The robot workspace topology and the obstacles are naturally expressed in the task space coordinates and most of the coordinate transformations used to tackle the stabilization problem distort this topology and render the obstacle representation in the new coordinates impossible.

The problem of stabilization in obstacle environments has been traditionally decomposed into two stages: the first stage includes motion planning and usually results in a collision free path. Motion planning techniques [12, 13], typically produce a holonomic path. These algorithms are not complete, in the sense that they may fail to yield a possible solution, except for potential fields generated by navigation functions [13]. Second stage involves following the planned path. The major difficulty here is the holonomic nature of the path which usually makes it infeasible for the nonholonomic robot. What is more, this approach completely excludes the possibility of replanning the motion in real time to address the issue of navigating in dynamic environments. In the two stage approach followed in [14] collision avoidance is pursued through refining the discretization of the previously computed holonomic path.

This paper addresses the above problem for the case of unicycle-type mobile robots and presents a methodology with provable global convergence properties. Planning of motion is realized in a feedback manner and implemented in real time. Convergence is established by a discontinuous feedback law. This paper is partially based on the earlier results of [15, 16]. The control law is redesigned to yield improved convergence rates and the admissible workspace representation is completed using a systematic procedure to take into account the robot volume. To the author's knowledge, this is the first complete methodology for real time nonholonomic navigation amongst obstacles.

The rest of the paper is organized as follows: Section 2 outlines the construction of the globally converging potential field. The control law is presented in section 3 where the convergence properties of the closed loop system are analyzed. Section 4 presents simulation results for a number of nontrivial nonholonomic navigation tasks. Finally, section 5 summarizes the conclusions and indicates current research directions.

2 Nonholonomic Motion Planning

The approach is based on the fact that globally convergent potential functions (navigation functions) [13] can be constructed on sphere worlds. It departs from the methodology of [13] by introducing a sequence of transformations under which the obstacle regions and the robot volume collapse to single points. On the "point world", the construction of navigation functions is straightforward and tuning of the field is significantly easier. Furthermore, contrary to the approach in [13] which is applicable only to point-robots, the transformations introduced are designed so that the free space represented in the point world accounts for the robot volume for arbitrarily shaped robot and obstacles, at an arbitrary degree of approximation.

Both the shape of the robot and the obstacles is considered as a union of (possibly intersecting) spheres. For clarity purposes, let us consider only one robot sphere, $\mathcal{R}$ and one obstacle sphere $\mathcal{O}$. Let z denote the configuration vector and $b_r(z)$, $b_o(z)$ the real valued functions that represent the robot sphere $\mathcal{R}$ and the obstacle sphere $\mathcal{O}$ boundaries, respectively. Firstly, the robot sphere is transformed to a point through the mapping:

$$\mathcal{H}_1 : z \mapsto \left(\frac{b_r(z)}{b_r(z)+1}\right)^{\frac{1}{2}} z \triangleq h_1$$

where h_1 is the new position of z. This transformation "sucks" the robot sphere volume and stretches the free space to fill the area. The boundary of obstacle $\mathcal{O}$ is deformed under the mapping. In the derived space a new mapping transforms the obstacle boundary to a point:

$$\mathcal{H}_2 : h_1 \mapsto \left(\frac{b_o(\mathcal{H}_1^{-1}(h_1))}{b_o(\mathcal{H}_1^{-1}(h_1))+1}\right)(h_1 - h_{co}) + h_{co} \triangleq h_2$$

where h_{co} a reference point in the obstacle's interior that now represents obstacle $\mathcal{O}$. Due to $\mathcal{H}_1$, b_o exhibits a singularity but there is a limit value that depends on the direction of approach. The limit value can be analytically expressed and therefore allows the definition of both the inverse mappings and the transformation differential at all points. In the space derived after $\mathcal{H}_2$, both the robot sphere and the obstacle sphere are points.

By repeating the procedure for all robot and obstacle spheres one transforms both robot and obstacles to points. The great advantage of having a point world is that its topology allows for the definition of a navigation function, whereas the generally (generally) non convex original space does not. In this point world topology a measure of the robot's proximity to the obstacles is obtained by multiplying the normalized Euclidean distances from the robot points to the obstacle points. The point world the distances do not correspond directly to minimum distances but the procedure ensures that they are class $\mathcal{K}$ functions of the true minimum distances. The proximity measure combines the set of robot points to a single reference point, z_r. In the final space the position of obstacle $\mathcal{O}$, relative to the robot reference point position z_r will be

$$z_o = \prod_{i\in R} \hat{d}_i(h_{2r}^i, h_{2o})[z_r - h_{2o}] + z_r$$

where R is the index set of all robot spheres, $\hat{d}_i$ is the normalized Euclidean distance between the robot point $\mathcal{R}_i$ and the obstacle point $\mathcal{O}_j$, $z_r - h_{2o}$ is the vector from the obstacle point to the robot point and z_r is the reference point that represents the whole robot. The result is space where the robot is represented by a single point and the obstacles are points at a configuration dependent position. A possible choice for a navigation function would be:

$$\hat{\varphi} = \frac{\|z\|^2}{(\|z\|^{2k} + |x|\beta(z))^{\frac{1}{k}}}$$

where $\beta(z)$ can be defined as: $\beta(z) = \prod_{i\in O} \hat{d}_i(z_{oi}, z_r)$ and $k \geq 1$ is a tuning parameter.

Another issue related to potential fields is that the integral curves of the potential fields are holonomic and therefore infeasible for a nonholonomic system. If, however, the potential function is constructed as a *dipolar* potential function [15], then the integral curves are feasible nonholonomic paths which simultaneously stabilize the robot's orientation. Let $z = (x, y, \theta) \in \mathbb{R}^n \times \mathbb{R}^n \times S^1$ and define a norm as $\|z\| \triangleq (x^2 + y^2 + \lambda\theta^2)^{\frac{1}{2}}$, with λ a small positive parameter. Define the dipolar potential function:

$$\varphi(z) = \frac{\|z\|^2}{|x|\beta^{1/k}} \tag{1}$$

The hidden assumption in using potential fields is that the field should be the negated gradient of a potential function. This however restricts the class of potential functions considered and therefore the choice of the control law. In this paper we introduce a class of functions, named *inverse Lyapunov functions*, which tend to infinity at the origin. Their gradient is used to construct a vector field that vanishes at the origin. A dipolar inverse potential function can be constructed as:

$$V(z) \triangleq \frac{|x|\beta^{1/k}}{\|z\|^2} \tag{2}$$

and the potential field that can be constructed from (2) but will not correspond to its gradient:

$$f = \begin{bmatrix} f_x & f_y & f_\theta \end{bmatrix}^T, \text{where} \quad f_r \triangleq \|z\|^4 \frac{\partial V(z)}{\partial r}, r = \{x, y, \theta\}$$

Inverse Lyapunov functions overcome the tuning difficulties encountered in conventional navigation functions. In the present approach, the tuning of the field is decoupled from convergence rate and thus one can have both the benefits of elimination of local minima and fast convergence.

3 Nonholonomic Control

3.1 Inverse Lyapunov Functions

The motivation for extending the definition of the Lyapunov function came from difficulty in adjusting conventional navigation functions in order to achieve improved obstacle avoidance and convergence properties. Using inverse Lyapunov functions we can tune the field without significantly affecting the convergence rate of the system. We proceed by defining the inverse Lyapunov function:

Definition 1. *Let $\mathcal{D} \subset \mathbb{R}^n$ be a domain containing the origin and consider a real smooth valued function $V(x)$: $\mathcal{D} \setminus \{\mathbf{0}\} \rightarrow \mathbb{R}^+$ having the following properties: (i) $V(x) \geq 0, \forall x \in \mathcal{D}$, (ii) $\lim_{x\rightarrow 0} V(x) = +\infty$, (iii) $\dot{V}(x) > 0, \forall x \in \mathcal{D} \setminus \{0\}$ Such a function is called an inverse Lyapunov function.*

Proposition 1. *Consider the continuous system $\dot{x} = f(x)$ and let $x = 0$ be an equilibrium point. Let $\mathcal{D}$ be a neighborhood of 0 and $V : \mathcal{D} \setminus \{0\} \rightarrow \mathbb{R}^+$ be a smooth inverse Lyapunov function.Then, $x(t)$ tends asymptotically to zero.*

Proof. (Schetch) The proof follows the same guidelines as in the classical case [17]. Assume an $\varepsilon > 0$ and choose a ball B_r of radius $r \in (0, \varepsilon]$. Let $\alpha \triangleq \max_{\|x\|=r} V(x)$ and take $\beta \in (\alpha, +\infty)$ Then one can show that $\Omega_\beta \triangleq \{x \in B_r | V(x) \geq \beta\}$ is invariant. Since $V(x)$ is monotonically increasing and unbounded from above, for every sequence $x(t_n), t \rightarrow \infty$ we will have $V(x(t_n)) \rightarrow +\infty$. Thus $V(x(t)) \rightarrow +\infty$ as $t \rightarrow \infty$. Then there will be a T for which $\forall t > T$, $x \in \Omega_\beta \subset B_r \subset B_\varepsilon$ which establishes asymptotic stability. □

It is easily shown that if $V(x)$ is radially unbounded then the result holds globally. The next theorem establishes the equivalence between the existence of a classic Lyapunov function and an inverse Lyapunov function:

Corrolary 1. *Consider the system $\dot{x} = f(x)$. Then a (possibly non smooth) Lyapunov function exists for this system if and only an inverse Lyapunov function exists.*

Proof. Let $V(x)$ be a Lyapunov function for the system $\dot{x} = f(x)$. Then we can define the function: $W(x) \triangleq \frac{1}{V(x)}$. It is clear that W satisfies the first two requirements of Definition 1. For the third a direct calculation yields:

$$\dot{W}(x) = \frac{-1}{V^2(x)} \cdot \dot{V}(x) > 0, \quad \forall x \in \mathcal{D} \setminus \{0\}$$

Therefore, $W(x)$ is an inverse Lyapunov function.

Conversely, if $W(x)$ is an inverse Lyapunov function satisfying the requirements (i)–(iii) of Definition 1, then we can define the function:

$$V(x) \triangleq \begin{cases} \frac{1}{W(x)}, & x \neq 0, \\ 0, & x = 0 \end{cases}$$

By definition, $V(x)$ is continuous at the origin but it may not be smooth. We can say that $V(x)$ is smooth almost everywhere since the origin is a set of measure zero. Even if $V(x)$ is non differentiable at the origin it is still a valid Lyapunov function since for the proof of stability only continuity at the origin is required. □

3.2 Discontinuous Feedback Control

With the dipolar potential function (2) at hand, one can construct a globally stabilizing feedback law. The inverse dipolar potential function can be represented by two partially defined functions. Partition the configuration space into two connected subsets:

$$M^1 \triangleq \{(x, y, \theta) \,|x \geq 0\} \setminus \{0\} \quad M^2 \triangleq \{(x, y, \theta) \,|x < 0\}$$

These two sets are separated by the surface $\Gamma \triangleq \{(x, y, \theta) \,|x = 0\}$. Then (2) can be expressed as:

$$V(z) = \begin{cases} \frac{x\beta^{1/k}}{\|z\|^2} \triangleq V^1(z) & \text{if } z \in M^1, \\ \frac{-x\beta^{1/k}}{\|z\|^2} \triangleq V^2(z) & \text{if } z \in M^2 \end{cases}$$

Consider the following nonholonomic system:

$$\dot{z} = g_1(z)u_1 + g_2(z)u_2 \tag{3}$$

with $g_1 = \begin{bmatrix}\cos\theta & \sin\theta & 0\end{bmatrix}^T, \quad g_2 = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$ In order for V_1, V_2 to be control Lyapunov functions, we define:

$$u_1 \triangleq k_1 \operatorname{sgn}(f_x \cos\theta + f_y \sin\theta)\,(f_x^2 + f_y^2 + f_\theta^2) \tag{4a}$$

$$u_2 \triangleq \begin{cases} k_2(\theta_d - \theta), & \text{if } w(z) \geq 0, \\ u_1(f_x \cos\theta + f_y \sin\theta) f_\theta^{-1}, & \text{if } w(z) < 0 \end{cases} \tag{4b}$$

where k_1, k_2 positive constants and:

$$\theta_d \triangleq atan2(-\operatorname{sgn}(x) f_y, -\operatorname{sgn}(x) f_x)$$
$$w(z) \triangleq u_1(f_x \cos\theta + f_y \sin\theta) + k_2 f_\theta(\theta_d - \theta)$$

with the functions $\operatorname{sgn}(\cdot)$ and $\arctan2(\cdot)$ defined as:

$$\operatorname{sgn}(x) \triangleq \begin{cases} 1, & \text{if } x \geq 0 \\ -1, & \text{if } x < 0 \end{cases}$$
$$\arctan2(y, x) \triangleq \arg(x, y), \quad (x, y) \in \mathbb{C}$$

We now proceed to show that V^1 and V^2 are inverse Lyapunov functions. To see that take the Lie derivatives with respect to the system vector fields g_i. For $z \in M^j$:

$$\begin{aligned} \dot{V}_j(z) &= u_1 L_{g_1} V^j(z) + u_2 L_{g_2} V^j(z) \\ &= \frac{|f_x \cos\theta + f_y \sin\theta|}{\|z\|^4}(f_x^2 + f_y^2 + f_\theta^2) + \frac{u_2 f_\theta}{\|z\|^4}. \end{aligned}$$

If $w(z) \geq 0$, then $u_2 = k_2(\theta_d - \theta)$ and $\dot{V}_j(z) = \frac{w(z)}{\|z\|^4} \geq 0$ If on the other hand $w(z) < 0$, then $u_2 = -u_1(f_x \cos\theta +$

$f_y \sin\theta) f_\theta^{-1}$ and $\dot{V}_j(z) = 0$. Therefore, $\dot{V}_j(z) = 0$ is positive semidefinite. Let $S \triangleq \{z \in M^j | \dot{V}_j(z) = 0\}$. In an invariant set $\Omega \in S$ there will be $u_1 = u_2 = 0$. However, u_1 does not vanish except for the origin. Using simple arguments one can verify that u_2 makes the set $\{z \in M^j | f_x \cos\theta + f_y \sin\theta = 0\}$ repulsive.

Using the function $W_j(z) = \frac{1}{V_j(z)}$ as a Lyapunov function candidate, La Salle invariant principle establishes that the system (3) under (4) is asymptotically stable in M^j.

Lemma 1 ([18]). *Let M^1, M^2 be two open connected subsets of $\mathbb{R}^n$ such that $M^1 \cup M^2 = \mathbb{R}^n \setminus \{0\}$. Let $f^i : M^i \to \mathbb{R}^n$, $i = 1, 2$ be two vector fields. Assume also, that there exists a separating surface Γ with $0 \in \Gamma$ and $\Gamma \setminus \{0\} \subset M^1 \cap M^2$. Let C^i, C^2 be two connected components of $\mathbb{R}^n \setminus \Gamma$ and assume that $C^i \subset M^i$ and that f^i points towards C^i on Γ for $i = 1, 2$. Finally assume that f^1, f^2 are asymptotically stable on M^1, M^2. Then, the vector field $f : \mathbb{R}^n \to \mathbb{R}^n$ defined by*

$$f(x) = \begin{cases} f^1(x) & \text{if } x \in (\Gamma \setminus \{0\}) \cup C^1 \\ f^2(x) & \text{if } x \in C^2 \\ 0 & \text{if } x = 0 \end{cases}$$

is globally asymptotically stable.

Let $C^1 \triangleq M^1 \setminus \Gamma$ and $C^2 \triangleq M^2 \setminus \Gamma$. According to the Lemma above, if the vector fields:

$$\begin{aligned} f^1 &\triangleq g_1(z)u_1(z) + g_2(z)u_2(z), \quad z \in M^1 \\ f^2 &\triangleq g_1(z)u_1(z) + g_2(z)u_2(z), \quad z \in M^2 \end{aligned}$$

point towards C^1 and C^2 respectively, then system (3) is globally asymptotically stable. This requirement is equivalent to showing that both C^1 and C^2 are invariant.

Indeed, for f^1 on Γ we have:

$$f^1 = \begin{bmatrix} \beta^{2/k} |\cos\theta| & sgn(\cos\theta)\beta^{2/k} \sin\theta & \pi - \theta \end{bmatrix}^T$$

For all $\theta \neq \pm\frac{\pi}{2}$, f^1 points to C^1. For $\theta = \pm\frac{\pi}{2}$, f^1 is tangent to the boundary of C^1 and non vanishing and therefore C^1 is invariant. On the other hand,

$$f^2 = \begin{bmatrix} -\beta^{2/k} |\cos\theta| & sgn(\cos\theta)\beta^{2/k} \sin\theta & \pi - \theta \end{bmatrix}^T$$

Following the same reasoning it can be seen that C^2 is also invariant. Therefore, the system (3) under the control law (4) is *globally asymptotically stable*.

4 Simulation Results

The simulation setup consists of a rectangular shaped nonholonomic mobile robot moving on the horizontal plane and a Π-shaped obstacle. The mobile robot is represented kinematically by the unicycle equations (3). The objective is for the mobile robot to park inside the cavity formed by

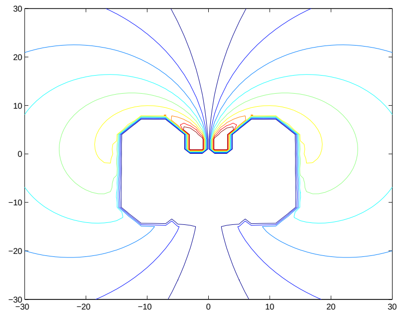

Figure 1: Dipolar field around the obstacle

the obstacle. The inverse dipolar function constructed provides the basis for the development of a globally converging potential field (Figure 1). In the first case (Figure (2)) the robot is positioned at $(x, y, \theta) = (-10, 10, 0)$. The sharp turning close to the initial position and at the neighborhood of the destination is a result of the large control gains k_1, k_2, but is theoretically allowed by the equations of the unicycle. Figure (3) depicts the trajectories of the system.

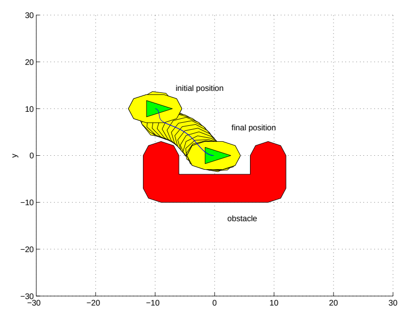

Figure 2: Initial position : $(x, y, \theta) = (-10, 10, 0)$

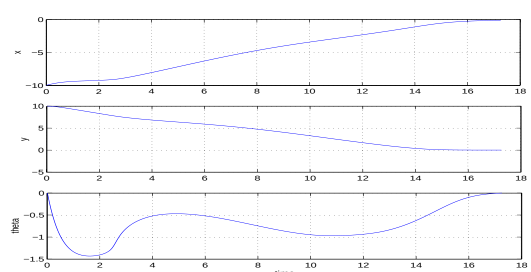

Figure 3: Trajectories with initial conditions: $(-10, 10, 0)$

In the second case the mobile robot is positioned on the x axis with zero orientation angle. The path of the robot is depicted in Figure 4 and the resulting trajectories are given in Figure 5. The initial conditions for the third case are $(x, y, \theta) = (20, 10, \frac{\pi}{2})$. Figure 6 shows that the vehicle successfully reorients itself, avoids the obstacle and reaches

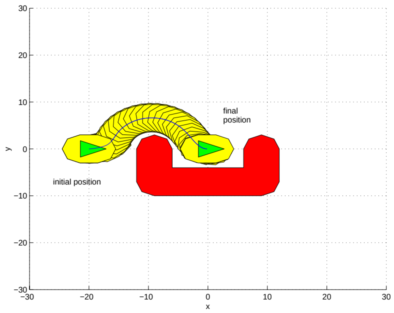

Figure 4: Initial position : $(x, y, \theta) = (-20, 0, 0)$

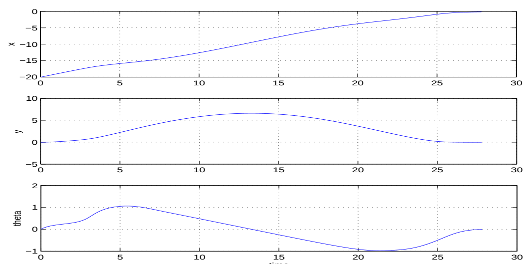

Figure 5: Trajectories with initial conditions: $(-20, 0, 0)$

the desired configuration. The trajectories are depicted in Figure 7. The fourth case the vehicle is initially posi-

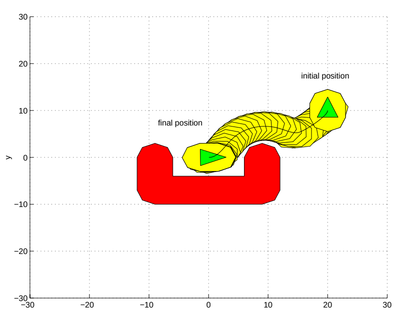

Figure 6: Initial position : $(x, y, \theta) = (20, 10, \frac{\pi}{2})$

tioned behind the obstacle, at a configuration where conventional potential field would exhibit a local minimum. The robot starting from $(x, y, \theta) = (10, -20, \frac{-\pi}{4})$ successfully reaches its destination. The path is given in Figure 8 and the corresponding trajectories in Figure 9. 9. The final case concerns the singular case on the separating surface Γ. In Figure 10 the path of the vehicle with initial conditions $(x, y, \theta) = (0, 20, \frac{\pi}{2})$ is given. It is clear that the separating surface is repulsive. Figure 11 gives system's trajectories.

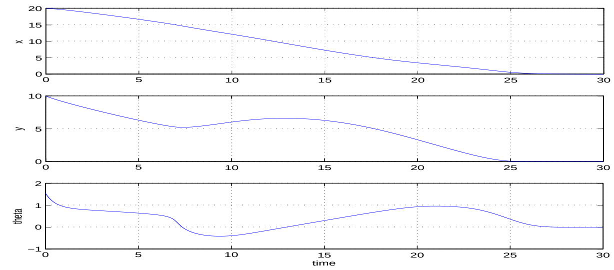

Figure 7: Trajectories with initial conditions: $(20, 10, \frac{\pi}{2})$

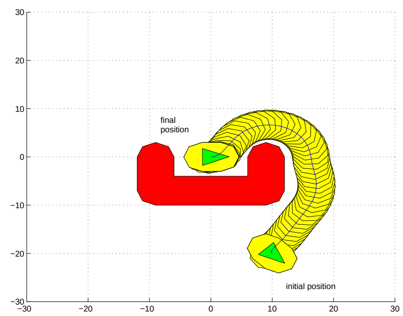

Figure 8: Initial position : $(x, y, \theta) = (10, -20, \frac{-\pi}{4})$

5 Conclusion - Issues for Further Research

In this paper a methodology for nonholonomic mobile robot motion planning and control in obstacle environments is presented. The approach can be readily extended to multi body robots. Appropriately constructed potential functions are combined with discontinuous feedback to provide fast navigation with guaranteed collision avoidance and global asymptotic convergence. The method is applicable to robots of arbitrary shape and is capable to yield a feasible solution wherever such a solution exists. The feedback nature and the simultaneous treatment of motion planning and control makes the method efficient and robust. The nonholonomic controller designed is applicable to unicycle-type mobile robots. The issue of possible curvature constraints is not addressed. Current research directions include extension to multi-robot systems with articulated mechanisms and considering the full dynamics.

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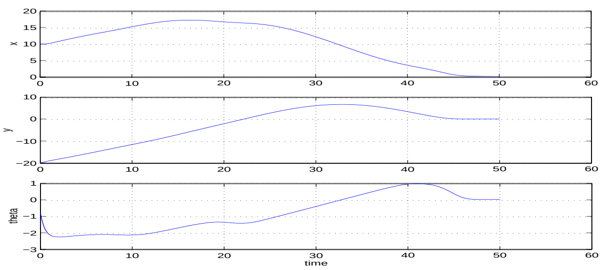

Figure 9: Trajectories with initial conditions: $(10, -20, \frac{-\pi}{4})$

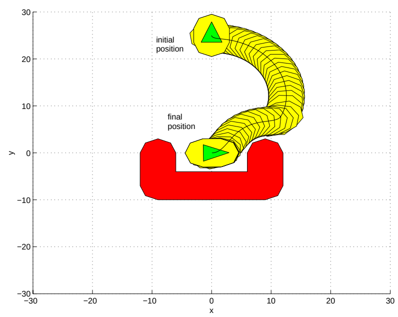

Figure 10: Initial position : $(x, y, \theta) = (0, 20, \frac{\pi}{2})$

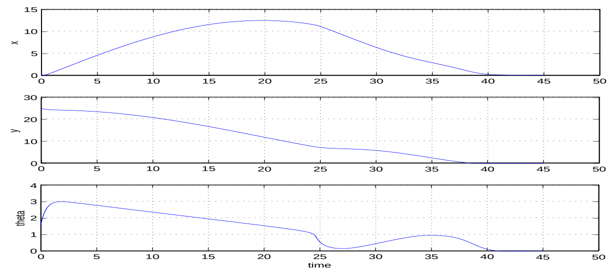

Figure 11: Trajectories with initial conditions: $(0, 20, \frac{\pi}{2})$

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