Flocking with Obstacle Avoidance in Switching Networks of Interconnected Vehicles

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***Abstract*—The paper introduces a set of nonsmooth control laws that enable a group of vehicles to synchronize their velocity vectors and move as a flock while avoiding collisions with each other and with static obstacles in their environment. In addition, all vehicles converge to a common destination point, accomplishing a group mission. The proposed control law can steer each vehicle based on local information that can be obtained from within a spherical neighborhood around it. While only the nearest neighbors and obstacles affect a vehicle's motion, as the vehicles move the neighborhoods change discontinuously. The induced discontinuities in the control law of each vehicle do not affect the stability properties of the group collision free motion and connectivity requirements on the interconnection network can be relaxed due to the common objective.**

I. Introduction

Recent advances in computation and communication have opened the way for the development of a new class of multi-agent systems. In these systems agents are autonomous enough to operate independently, yet they can function collectively as a group and synchronise through communication. The challenge here is for the group to cooperate using minimal computing and communication resourses, without the need for centralized coordination. Such cooperative control schemes are the most promising in terms of their ability to scale with the size of the group.

Nature provides several examples of marvelous decentralized, coordinated behaviors in groups of living organisms. Such cases have been studied in in ecology and theoretical biology, in the context of animal aggregation and social cohesion in animal groups, but also in statistical physics and nonlinear science [30, 29, 14, 12, 24, 4, 10]. Although such phenomena have not been thouroughly investigated and understood, researchers in control theory computer animation and robotics have been trying to develop models that describe and explain several aspects of the cooperative behaviors observed in nature. This is especially true in robotics, in the context of formation control and cooperative control of multi-agent systems, where so much effort has been directed towards the development of autononous, distributed multi-robot systems [11, 2, 16, 19, 5, 13, 7, 25, 8, 15, 3, 9, 31, 18]. In a flocking model proposed by Vicsek *et al.* [30], mobile agents regulate their heading to the average of the headings of themselves and their nearest neighbors plus some additive noise. Numerical simulations in [30] indicate the spontaneous development of coherent collective motion, resulting in the headings of all agents to converge to a common value. Convergence of the Vicsek model was theoretically established in [8]. In a similar model, developed approximately ten years earlier, Reynolds [20] described the coordinated motion in bird flocks as the combined result of three simple steering rules, which each agent independently follows: (i) *Separation:* steer to avoid crowding local flockmates; (ii) *Alignment:* steer towards the average heading of local flockmates; (iii) *Cohesion:* steer to move toward the average position of local flockmates. The superposition of these three rules results in all agents moving in a formation, while avoiding collision.

Motivated by Reynolds model, a set of decentralized control laws were developed [27], giving rise to coherent and coordinated motion for a group of interconnected dynamic mobile agents. The stability analysis that established convergence for the group to the synchronized state where all agents have the same velocity vectors, was based on LaSalle's invariant principle and the algebraic graph theoretic properties of the interconnection network among the agents. It was also shown that stability is still guaranteed if the agent interconnections appear and disappear dynamically [28], so long as the connectivity of the network is preserved.

In this paper, we extend these previous results to the case where there are several static obstacles in the environment, which the agents need to avoid. Similar recent results for obstacle avoidance in vehicle formations treat the obstacles as ficticious vehicles [22]. The proposed treatment can not only ensure obstacle avoidance and flocking in the group, but also allows for the agents to pursue additional objectives which are guaranteed to be met. For the purposes of this paper, these objectives are formulated as convergence to a common destination or rendez-vous point. The dynamic nature of the interconnection network induces control discontinuities. The resulting closed loop dynamics is studied within the framework of nonsmooth systems. Algebraic graph theory properties are exploited to show that arbitrary changes in agent interconnections do not affect the stability of the system. The common objective "binds" the group together so that interconnection connectivity is no longer required. The group can become disconnected and velocity synchronization takes place within each connected component. Eventually, as each component will converge to the rendez-vous point, connectivity will be regained.

This paper is organized as follows: in Section II we define the problem addressed in this paper and sketch the solution approach. The stability analysis follows in Section IV. The results of Section IV are verified in Section V via numerical simulations. Section VI summarizes the results and highlights our key points.

II. Problem Description

Consider N agents, moving on the plane with the following dynamics:

$$\dot{r}_i = v_i \tag{1a}$$
$$\dot{v}_i = u_i \quad i = 1, \ldots, N\,, \tag{1b}$$

where $r_i = (x_i, y_i)^T$ is the position of agent i, $v_i = (\dot{x}_i, \dot{y}_i)^T$ is its velocity and $u_i = (u_{x_i}, u_{y_i})^T$ its control inputs. Relative position vectors are denoted $r_{ij} = r_i - r_j$. The control input consists of three components (Figure 1):

$$u_i = a_i + \alpha_i \; + f_i. \tag{2}$$

The first component, a_i, is derived from an artificial potential field that is responsible for cohesion and separation of the agents within the group. It is generated by a function V_i which encodes relative position information between agent i and its neighbors. The second component, α_i regulates the velocity vector of agent i to the average of its own and that of its neighbors. The last component, f_i is a second potential field term that is used for navigation of the agent in its environment. This term ensures that the agents avoids stationary obstacles and moves towards a predefined position.

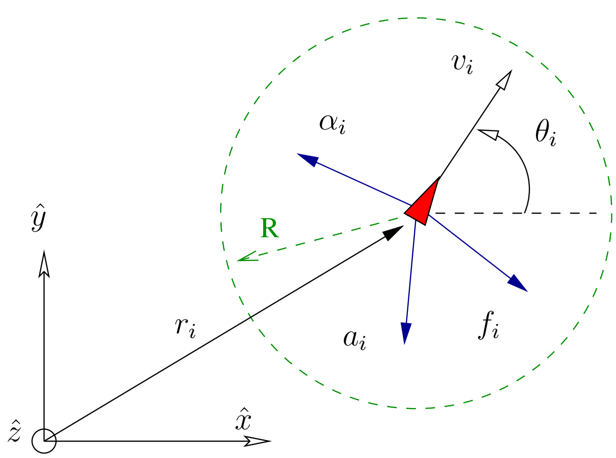

Fig. 1. *Control forces acting on agent i.*

The problem is to determine the input components so that the group exhibits a stable, collision free flocking motion and accomplishes a common mission. Flocking motion is being understood technically as a convergence property on the agent velocity vectors and their relative distances, while a mission will be convergence of the flock to a specified point in the workspace.

III. Control Law with Dynamic Topology

In this section we present a realization of the control law (2) that achieves the control objective. The steering policy of each agent is based only on local state information from its nearest neighbors. The graph $\mathcal{G}$, represents the nearest neighboring relations:

Definition III.1 (Neighboring graph) *The neighboring graph, $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, is an undirected graph consisting of:*

- *a set of vertices (nodes), $\mathcal{V} = \{n_1, \ldots, n_N\}$, indexed by the agents in the group, and*
- *a set of edges, $\mathcal{E} = \{(n_i, n_j) \in \mathcal{V} \times \mathcal{V} \mid n_i \sim n_j\}$, containing unordered pairs of nodes that represent neighboring relations.*

Let $\mathcal{N}_i$ denote the index set of neighbors of i,

$$\mathcal{N}_i \triangleq \{j \mid \|r_{ij}\| \leq R\} \subseteq \{1, \ldots, N\}.$$

Since the agents are in motion, their relative distances can change with time, affecting their neighboring sets. The time dependence of the neighboring relations gives rise to a switching graph. For each edge incident to agent i, we define an inter-agent potential function, U_{ij} which should satisfy:

Definition III.2 (Interaction potential) *The interaction potential function U_{ij} is a nonnegative function of the distance $\|r_{ij}\|$ between agents i and j, such that*

1. *$U_{ij}(\|r_{ij}\|) \to \infty$ as $\|r_{ij}\| \to 0$,*
2. *U_{ij} attains its unique minimum when agents i and j are located at a desired distance.*
3. *U_{ij} is increasing near $\|r_{ij}\| = R$.*

The interaction potential U_{ij} can be nonsmooth at distance $\|r_{ij}\| = R$, and constant $U_{ij} = V_R$ for $\|r_{ij}\| > R$, to capture the fact that beyond this distance there is no agent interaction. One example of such a nonsmooth potential function is the following, depicted in Figure 2:

$$U_{ij} = \begin{cases} \frac{1}{\|r_{ij}\|^2} + \log \|r_{ij}\|^2\,, & \|r_{ij}\| < R, \\ V_R, & \|r_{ij}\| \geq R \end{cases}$$

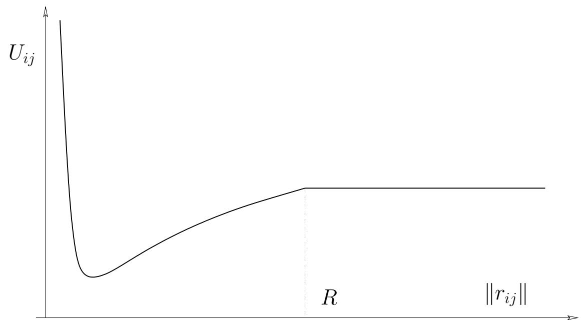

Fig. 2. *A nonsmooth interaction potential function.*

For agent i the (total) interaction potential U_i is formed by summing the potentials due to each of its neighbors:

$$U_i \triangleq (N - |\mathcal{N}_i|)V_R + \sum_{j \in \mathcal{N}_i} U_{ij}(\|r_{ij}\|)$$

where $N_i = |\mathcal{N}_i|$.

We will define an additional potential function that encodes the proximity of the agents to stationary obstacles and the degree to which they have satisfied their portion of the group objective. This potential will be constructed as a navigation function [21]. This formulation will ensure that the group objective will eventually be met, at least when agents are freed from the interaction from their teammates.

Let $V_i(r_i)$ be the mission potential function for agent i. It depends only on the configuration of the particular agent. In the context of this paper, the mission will be the convergence to a particular point in the workspace which will be named r_d. The particular characteristic of this potential function is the way in which the obstacles are implemented: they are designed so that beyond a certain distance d, the function that models each obstacle remains constant and equal to 1. In this way, the only obstacles that are significant for the value of the function and its derivatives are those which are inside a d-neighborhood around the vehicle. For simplicity, the obstacles will be assumed to be points; for a large class of more complicated shapes, diffeomorphic transformations have been proposed that reduce such shapes into points [26]. The obstacle function for obstacle o, as seen from agent i will be:

$$\beta_{io} = \left(1 - \lambda \frac{[\delta(r_i, r_o) - d]^2}{[\delta(r_i, r_o) - d]^2 + 1}\right)^m$$

where $\delta(r_i, r_o)$ denotes the distance between the vehicle and the boundary of the obstacle, which is supposed to be located at position r_o, the exponend m is given as:

$$m \triangleq \frac{1}{2}(\mathrm{sign}(d^2 - \delta(r_i, r_o)^2) + 1)$$

and λ is a constant that can be selected so that the transition of the obstacle function to the value of 1 is smooth:

$$\lambda = \frac{d^2 + 1}{d^2}.$$

The mission potential function for agent i is then defined as

$$V_i(r_i) = \frac{\|r_i - r_d\|^2}{[\|r_i - r_d\|^{2\kappa} + \prod_o \beta_{io}]^{\frac{1}{\kappa}}}$$

where κ is the navigation function tuning parameter.

The control law u_i is defined as:

$$u_i = \begin{cases} -\sum_{j \in \mathcal{N}_i}[(v_i - v_j) + \nabla_{r_i} U_{ij}] & \\ -\nabla_{r_i} V_i - c_d v_i & , V_i > V_{min} \\ -\sum_{j \in \mathcal{N}_i}[(v_i - v_j) + \nabla_{r_i} U_{ij}] & \\ -\nabla_{r_i} V_i & , \text{otherwise} \end{cases} \tag{3}$$

where the last component is a damping term to ensure convergence to the destination point r_d by suppressing oscillatatory motion around it, since the first two terms have no effect on the group kinetic and potential energy. The constant V_{min} defines a sufficiently small level set of the mission potential function.

Since the group may become disconnected as a result of obstacle avoidance maneuvers, we may need to consider $k = 1, \ldots, p \leq N$ different connected components, or subgroups. In order to avoid motion singularities that may arise due to the effect of obstacles, braking of the group may also be intentional. For that reason, we may consider controlling the topology of the group by regulating the neighborhood radius of the agents within each connected component. In every connected component, the agents synchronize their radius to the maximum of their groupmates to enhance connectivity, but when they become immobilized while away from their destination point, they reduce it to break free:

$$R_i = \begin{cases} R_i/2, & v_i = 0, V_i > V_{min}, \\ \max\{R_i, R_j,\ j \in \mathcal{N}_i\} & \text{otherwise} \end{cases} \tag{4}$$

Note that these first two terms depend on the neighboring set of each agent i. Changes in the neighboring set $\mathcal{N}_i$, during motion introduce discontinuities in the control law (3). The stability of the discontinuous dynamics should be analyzed using differential inclusions [6] and nonsmooth analysis [1].

IV. Stability Analysis

In this section we show how the decentralized control laws (3) give rise to a coordinated flocking behavior. Specifically, we prove that all agents of the closed loop system (1)- (3) asymptotically attain a common velocity vector, minimize their artificial potential and avoid collisions with their flockmates.

Our main result is formally stated as follows:

Proposition IV.1 *Consider a group of N vehicles with dynamics* (1)*, each steered by control law* (3) *and interconnection topology rule* (4)*. Then all vehicles will avoid collisions with each other and with static obstacles in their environment, will converge to a common destination point, and away from obstacles each connected component of the group will flock by synchronizing its velocities asymptotically.*

Proof: Consider the following function:

$$Q = \frac{1}{2}\sum_{i=1}^{N}(\sum_{j=1}^{N} U_{ij} + v_i^T v_i + 2V_i). \qquad (5)$$

Function Q is continuous everywhere but nonsmooth whenever $\|r_{ij}\| = R$ for some $(i,j) \in N \times N$. Due to the mission potential terms V_i, and regardless of the connectivity status of the group, the level sets of Q define compact sets in the space of agent velocities v_i and positions r_i. Therefore, relative distances $r_i - r_j$ between agents are also bounded, ensuring the differentiability of $\|r_i - r_j\|$, $\forall i,j \in \{1,\dots,N\}$. Thus

$$\Omega = \{(v_i, r_{ij}) \mid Q \leq c\} \qquad (6)$$

will be compact. Since U_{ij} is continuous at R, it is locally Lipschitz. It is shown that U_{ij} is regular :

Lemma IV.2 ([28]) *The function U_{ij} is regular everywhere in its domain. Moreover the generalized gradient of U_{ij} at R and the (partial) generalized gradient of U_{ij} with respect to r_i at R are empty sets.*

Regularity of each interaction potential function U_{ij} is required to ensure the regularity of U_i, as a linear combination of a finite number of regular functions [1]. Thus, Q is regular as a sum of regular functions. The latter is a necessary condition for all nonsmooth stability theorems. The fact that the generalized gradient of U_{ij} is empty, facilitates the evaluation of the generalized time derivative of Q. When all agents are outside the level set defined by V_{min}, regularity of Q and the property of finite sums of generalized gradients ensures that for the generalized time derivative of Q,

$$\dot{\tilde{Q}} \subset \sum_{i=1}^{N}\left(\bigcap_{\xi_i}\xi_i^T v_i\right) - v^T K\left[(L_t \otimes I_2)v + \begin{pmatrix}\vdots \\ \nabla_{r_i}U_i \vdots\end{pmatrix}\right]$$

where $\xi_i \in \sum_{j=1}^{N}\partial_{r_i}U_{ij}$, L_t is the (time-dependent) Laplacian of the neighboring graph and $\nabla_{r_i}U_i = \sum_{j\in\mathcal{N}_i}\nabla_{r_i}U_{ij}$. Both L_t and $\nabla_{r_i}U_i$ are switching over time, depending on the neighboring set $\mathcal{N}_i$ of each agent i. Recalling that $\partial U_{ij}(R) = \emptyset$ and using some differential inclusion algebra for sums, (finite) Cartesian products and multiplications with continuous matrices [17], we obtain

$$\dot{\tilde{Q}} \subset \sum_{i=1}^{N}(\nabla_{r_i}U_i)^T v_i - v^T K[(L(t)\otimes I_2)v] \\ - \sum_{i=1}^{N} v_i^T \nabla_{r_i}U_i - c_d\|v_i\|^2 = -\overline{co}\{v_x^T L(t)v_x + v_y^T L(t)v_y\} \qquad (7)$$

For any graph, the right hand of (7) will be an interval of the form $[e,0]$, with $e<0$. Therefore it is always $q \leq 0$, for all $q \in \dot{\tilde{Q}}$. If the graph is connected and the components of the agent velocities are not equal, $v_x, v_y \parallel \mathbf{1}$, then $-\overline{co}\{v_x^T L(t)v_x + v_y^T L(t)v_y\}$ will contain only negative numbers. If the neighboring graph is disconnected, $-\overline{co}\{v_x^T L(t)v_x + v_y^T L(t)v_y\}$ can be expressed in terms of the Laplacians of the connected components:

$$-\overline{co}\{\sum_k v_{xk}^T L_k(t)v_{xk} + \sum_k v_{yk}^T L_p(t)v_{yk}\}$$

where v_{xk}, v_{yk} and $L_k(t)$ correspond to the x, y components of the agent velocities in the p connected component and $L_k(t)$ is the Laplacian of the graph component respectively. Since the Laplacians are positive semidefinite matrices, the set in (7) will contain 0 only when each velocity component of the agents in the connected subgroup are aligned to the vector of ones, since this is the eigenvector corresponding to the singel zero eigenvalue of the connected subgroup's Laplacian.

Applying the nonsmooth version of LaSalle's principle proposed by [23], it follows that for initial conditions in Ω, the Filippov trajectories of the system converge to a subset of the set where $\{v_{xk}, v_{yk} \in \operatorname{span}(\mathbf{1})\}$. This implies that away from the destination point all connected subgroups are going to asymptotically synchronize their velocities and move as a flock.

In the neighborhood of the destination, (7) becomes

$$\dot{\tilde{Q}} \subset -\overline{co}\{v_x^T L(t)v_x + v_y^T L(t)v_y\} - \sum_{i}^{N} c_d\|v_i\|^2$$

and in this case the invariant set has to be in $\{v \mid v_x, v_y = 0\}\}$. For this subset to be invariant, we need $\dot{v} = 0$, in which case,

$$\sum_{j\in\mathcal{N}_i}\nabla_{r_i}U_{ij} = \nabla_{r_i}V_i.$$

However, the term on the left hand side is normal to the vector of ones since for every connected component,

$$(\nabla_{r_i}U_i)_k = -(B_k(t)\otimes I_2)\left[\cdots\ (\nabla_{r_{ij}}U_{ij})_k^T\ \cdots\right]^T,$$

and the range of the incidence matrix of connected component k, $B_k(t)$ is orthogonal to the vector of ones.

Therefore, in steady state, the agents equilibrate in configurations where $\mathbf{1}^T \nabla_{r_i} V_i = 0$ and the mission potential forces balance the interaction potential forces. This can happen either when the agents have surrounded the destination point (to have $\mathbf{1}^T \nabla_{r_i} V_i = 0$) or when they have got stuck in a configuration where due to environment obstacles, the mission potential forces cancel with the interaction forces. In the latter case, reducing the neighborhood radius R, reduces the number of interactions and breaks the symmetry, allowing the group to escape the singularity. ■

V. Simulations

We consider five vehicles that are initiated with random initial positions and velocities in a neighborhood of an initial point in the workspace. The mission for the collection of vehicles is to flock towards the destination point $(0, 0)$ which is located inside a Π shaped obstacle, consisted of several point obstacles placed close to each other. The group has to avoid collisions between vehicles and obstacles and surround the destination point.

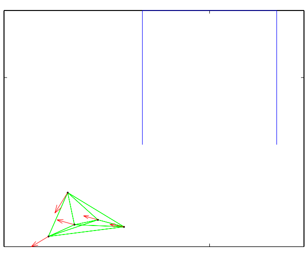

Fig. 3. *Initial state.*

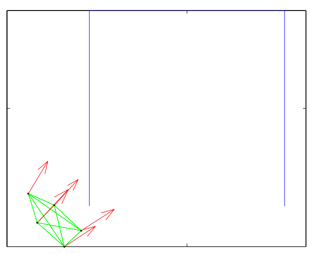

Fig. 4. *The group approaches the obstacle.*

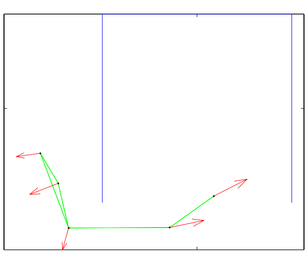

Fig. 5. *Part of the group finds its way to destination.*

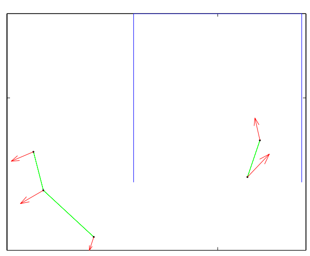

Fig. 6. *The group breaks as one part avoids the obstacle.*

VI. Conclusions

In this paper we showed that a group of autonomous vehicles, in which each agent is steered using local state information from its nearest neighbors and regulates the interconnections with them, can synchronize their velocities if their motion is not inhibited by obstacles, can avoid collisions between the obstacles and each other, and regroup around a predifined point in the workspace

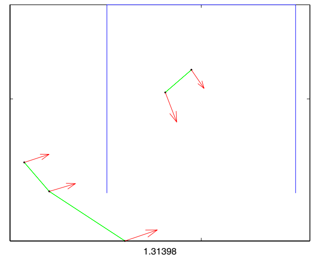

Fig. 7. *The second part approaches again.*

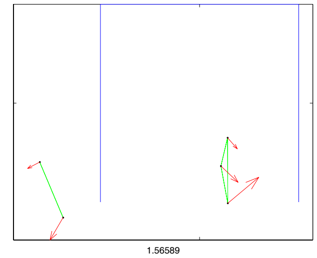

Fig. 8. *One member joins the part that is inside.*

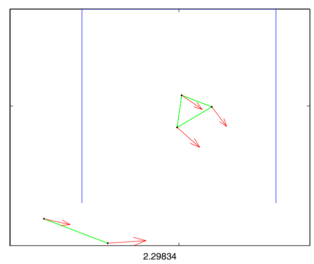

Fig. 9. *The rest maneuver around the obstacle.*

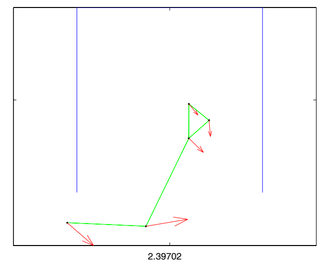

Fig. 10. *The two components are joined.*

accompishing a group objective. The approach demonstrates that besides synchronizing a subset of its state, there is additional control authority for the group to pursue other objectives. When the later are being encoded in a navigation function, the accomplishment of the mission is guaranteed. Stability of motion and convergence to the destination is not affected by the control discontinuities induced by the dynamically changing interconnection topology, and are being established using results from algebraic graph theory and Lyapunov stability analysis for nonsmooth systems.

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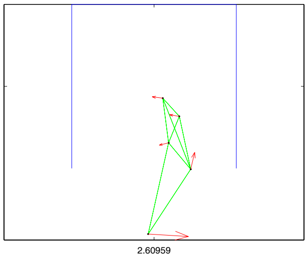

Fig. 11. *A tight formation is created again.*

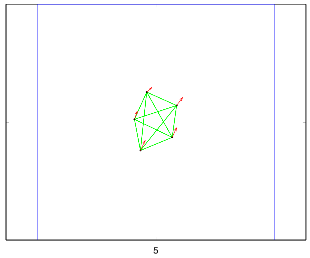

Fig. 12. *The group surrounds the destination.*

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