# Position and Force Control by Reaction Compensation

H. G. Tanner and K. J. Kyriakopoulos
Control Systems Laboratory,
Department of Mechanical Engineering,
National Technical University of Athens,
9 Ir. Politechniou, 157 80 Zografou, Greece
{htanner,kkyria}@central.ntua.gr

### Abstract

The paper presents a new position/force controller, based on the philosophy of the parallel approach. The controller exploits the reaction compensation action of the inverse dynamics position controller and achieves superior transient performance. It incorporates a velocity dependent damping term. Stability is established and conditions for the control parameters are derived. Performance of the proposed controller is verified through computer simulations.

#### 1 Introduction

Pure position control in not sufficient when a manipulator interacts with its environment, in the sence that external forces and torques are not exclusively due to gravity and inertia. In this case, relatively small and probably insignificant position errors may result in large control signals which can have destructive effects either on the environment or the robot or both. For this reason, several force control schemes have been developed aiming at regulating the contact forces exerted by the manipulator to its environment.

The main approaches to robot force control are impedance control, hybrid position/force control and parallel control. Literature is rich in the first two categories, the latter being a relative recent development. All schemes require an environment model, either in controller designing or for stability proving. The dependence of each approach on this model in terms of system performance or even applicability of the method may vary. To compensate for the environment model uncertainty many researches resort to adaptation techniques or learning.

Impedance control was originally proposed by Hogan [1]. The idea is to enforce an adjustable

mechanical impedance relationship between the force and the position error. Proper adjustment of the impedance parameters ensures bounded contact forces. The primary merit of impedance control is that it establishes an adjustable ballanced behaviour of the system between position errors and external force. Force regulation, however cannot be achieved without accurate description of the environment. The value of the contact force at steady state depends heavily on the assumed environment stiffness. For this reason, impedance control has primarily been implemented in the framework of an adaptive scheme [2, 3].

Given a detailed environment description, a widely adopted method is hybrid position/force control. Hybrid position/force control has been introduced in [4]. The 'hybrid' characterization should not be confused with the co-existence of continuous and discrete time subsystems, but rather with the simultaneous control of both position and force in different directions. The task space is partitioned into two orthogonal subspaces [5]. The scheme allows adjustment of position and force dynamics independently. In order, however, for the hybrid position/force control to be implemented, directions in which a desired force is to be applied should be predefined and accurately described. As a result, unexpected collision phenomena can not be handled successfully and although time dependency of the contact surfaces can be taken into account, it must still be known in advance. In order to deal with instability phenomena, full manipulator dynamics were taken into account [6, 7, 8]. Still, the validity of the orthogonal space decomposition has been challenged [9].

The parallel position/force control [10, 11] implements position and force control simultaneously in every direction of the task space, giving priority to force errors over position errors. Due to the integral control action on the force error, the method is not so sensi-

tive to environment model errors. It can also handle unexpected contacts and changes in the task specification. To avoid contradictory control commands and given the unavailability of force time derivatives, the parallel controller does not include any damping for the force, other than the usual velocity damping designed to adjust the position transient. Position and force dynamics however, have different requirements in terms of control signal amplitutes and position damping could be insufficient for the force.

The proposed approach is based on the parallel philosophy and remedies the insufficiency of the position controller damping. It introduces a force damping term in the controller which substitutes the unavailable force derivatives with velocity measurements. A nonlinear gain is used to switch between unconstrained and constrained motion modes. Another novelty is the controller structure which exploits the external force compensation action in the position controller: if external force is to be regulated there is no need to be completely compensated. Partial compensation could result in applying the desired force at the manipulator end effector. The method is compared to the parallel control scheme to reveal its superior performance in terms of transient response.

The rest of the paper is organized as follows: In section 2 the proposed approach is presented and stability conditions are derived. Simulation results are included in section 3. Section 4 summarizes the conclusions drawn in the present work.

## 2 Position/Force Control

#### 2.1 The position controller

Consider the following set of dynamic equations of motion for a nonredundant manipulator:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau} - \mathbf{J}^T(\mathbf{q})\mathbf{f}$$
 (1)

where  $\mathbf{q} \in \mathbb{R}^n$  is the vector of manipulator joint angles,  $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n \times n}$  is the configuration dependent inertia matrix,  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n \times n}$  is the matrix of Coriolis and centrifugal terms,  $\mathbf{G}(\mathbf{q}) \in \mathbb{R}^n$  is the vector of gravity related terms,  $\boldsymbol{\tau} \in \mathbb{R}^n$  are the joint torques,  $\mathbf{J}(\mathbf{q}) \in \mathbb{R}^{n \times n}$  is the manipulator jacobian matrix and  $\mathbf{f} \in \mathbb{R}^n$  is the vector of forces and torques applied to the manipulator end effector by its environment.

In order to control the motion of the manipulator in the task space, the manipulator dynamics is usually linearized using the inverse dynamics technique:

$$\tau = \mathbf{M}\alpha + \mathbf{C}\dot{\mathbf{q}} + \mathbf{G} + \mathbf{J}^T\mathbf{f} \tag{2}$$

where  $\alpha$  is now the new control input. Note that the environment reactions applied at the end effector are fully compensated. Substituting (2) into (1) yields:

$$\ddot{q} = \alpha$$

The task space dynamics are derived using

$$\alpha = \mathbf{J}^{-1}(\mathbf{a} - \dot{\mathbf{J}}\dot{\mathbf{q}})$$

to obtain

$$\ddot{\mathbf{x}} = \mathbf{a} \tag{3}$$

where  $\mathbf{x}$  is the vector of task space coordinates. Given a desired trajectory,  $\mathbf{x}_d$ ,  $\dot{\mathbf{x}}_d$ ,  $\ddot{\mathbf{q}}$ , the new input,  $\mathbf{a}$  takes the form

$$\mathbf{a} = \mathbf{K}_a^{-1} (\mathbf{K}_a \ddot{\mathbf{x}}_d + \mathbf{K}_v (\dot{\mathbf{x}}_d - \dot{\mathbf{x}}) + \mathbf{K}_p (\mathbf{x}_d - \mathbf{x}))$$
(4)

where  $\mathbf{K}_a$ ,  $\mathbf{K}_v$ ,  $\mathbf{K}_p$  are positive definite matrices. Setting  $\mathbf{e}_p = \mathbf{x}_d - \mathbf{x}$ , and substituting (4) into (3) results in linear and stable error dynamics:

$$\mathbf{K}_a \ddot{\mathbf{e}}_p + \mathbf{K}_v \dot{\mathbf{e}}_p + \mathbf{K}_p \mathbf{e}_p = 0$$

### 2.2 Partial reaction compensation

The above scheme gives an infinitely stiff manipulator motion. If the end effector forces are not compensated completely in (2), but instead  $\tau$  is given by

$$\tau = \mathbf{M}\alpha + \mathbf{C}\dot{\mathbf{q}} + \mathbf{G} + \mathbf{J}^T\mathbf{h} \tag{5}$$

then the linearized configuration dynamics will be

$$\ddot{\mathbf{q}} = \boldsymbol{\alpha} + \mathbf{M}^{-1} \mathbf{J}^T (\mathbf{h} - \mathbf{f})$$

and (1) takes the form

$$\ddot{\mathbf{x}} = \mathbf{a} + \mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T(\mathbf{h} - \mathbf{f})$$

Substituting a from (4) gives

$$\mathbf{K}_a \ddot{\mathbf{e}}_p + \mathbf{K}_v \dot{\mathbf{e}}_p + \mathbf{K}_p \mathbf{e}_p = -\mathbf{K}_a \mathbf{J} \mathbf{M}^{-1} \mathbf{J}^T (\mathbf{h} - \mathbf{f}) \quad (6)$$

Let **h** be given as:

$$\mathbf{h} = \mathbf{f} + (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T)^{-1}\mathbf{v}$$

where  $\mathbf{v}$  is a new input. Substitution in (6) yields:

$$\mathbf{K}_a \ddot{\mathbf{e}}_p + \mathbf{K}_v \dot{\mathbf{e}}_p + \mathbf{K}_p \mathbf{e}_p = -\mathbf{v} \tag{7}$$

Assuming a linear environment model of the form

$$\mathbf{f} = \mathbf{K}_e(\mathbf{x} - \mathbf{x}_0)$$

v can be given as

$$\mathbf{v} = \mathbf{K}_{fp} \mathbf{e}_f + \mathbf{K}_{vf} (\dot{\mathbf{f}}_d - \mathbf{K}_e \dot{\mathbf{x}}) + \mathbf{K}_{fi} \int \mathbf{e}_f ds \qquad (8)$$

where  $\mathbf{e}_f \triangleq \mathbf{f}_d - \mathbf{f}$  is the force error,  $\mathbf{K}_e$  is the environment stiffness matrix and  $\mathbf{K}_{fp}, \mathbf{K}_{vf}, \mathbf{K}_{fi}$  appropriate gain matrices. The above controller has a full PID action (contrary to the PI action of the parallel scheme), using velocity measurements to substitute the respective force term. The difference here is that instead of letting the position controller do the damping for the whole system, we introduce a new term which allows to control the dynamic behavior of the force controller independently. This improves the transient performance of the system considerably. The position/force control scheme combines the versatility of the parallel approach with the superior perfomance of hybrid position/force control methodologies.

In order for the controller to perform satisfactorily in both cases of position controlled and position/force controlled motion,  $\mathbf{K}_{fv}$  must incorporate some sort of *switching*. In the absence of reliable force derivative measurements, velocities have to be used instead, rendering the force damping term entirely dependent of velocities. If  $\mathbf{K}_{fv}$  is constant, then force damping would interfere with position damping. To deal with this effect,  $\mathbf{K}_{fv}$  could be designed such that it is (almost) zero in the case where there is no force error and activate itself as soon as force error is detected. This transition can be made in a smooth way as fast as it is desired. The switching function can be tuned in such a way that  $\mathbf{K}_{fv}$  to be considered piecewise constant for the purposes of analysis.

The closed loop system is obtained by substition of (8) into (7)

$$\mathbf{K}_{a}\ddot{\mathbf{e}}_{p} + \mathbf{K}_{v}\dot{\mathbf{e}}_{p} + \mathbf{K}_{p}\mathbf{e}_{p}$$

$$= -\mathbf{K}_{fp}\mathbf{e}_{f} - \mathbf{K}_{fv}(\dot{\mathbf{f}}_{d} - \mathbf{K}_{e}\dot{\mathbf{x}}) - \mathbf{K}_{fi}\int\mathbf{e}_{f}ds \quad (9)$$

### 2.3 Stability Conditions

Using the linear environment model we obtain

$$\mathbf{e}_n = \mathbf{x}_d - \mathbf{x}_0 - \mathbf{K}_e^{-1} \mathbf{f}$$

Equation (9) can be written in compact form as:

$$\mathbf{K}_{a}\mathbf{K}_{e}^{-1}\ddot{\mathbf{e}}_{f} + (\mathbf{K}_{v}\mathbf{K}_{e}^{-1} + \mathbf{K}_{fv})\dot{\mathbf{e}}_{f} + (\mathbf{K}_{p}\mathbf{K}_{e}^{-1} + \mathbf{K}_{fp})\mathbf{e}_{f} + \mathbf{K}_{fi}\int\mathbf{e}_{f}ds = \phi_{d} \quad (10)$$

where

$$\phi_d \triangleq \mathbf{K}_a(\ddot{\mathbf{x}}_0 - \ddot{\mathbf{x}}_d) + \mathbf{K}_v(\dot{\mathbf{x}}_0 - \dot{\mathbf{x}}_d) + \mathbf{K}_p(\mathbf{x}_0 - \mathbf{x}_d) + \mathbf{K}_a\mathbf{K}_e^{-1}\ddot{\mathbf{f}}_d + \mathbf{K}_v\mathbf{K}_e^{-1}\dot{\mathbf{f}}_d + \mathbf{K}_p\mathbf{K}_e^{-1}\mathbf{f}_d$$

Equation (10) represents a linear trird order system. If the gain matrices are given the following diagonal structure:

$$egin{aligned} \mathbf{K}_a &= \mathbf{k}_a \mathbf{I} & \mathbf{K}_{fv} &= \mathbf{k}_{fv} \mathbf{I} \ \mathbf{K}_v &= \mathbf{k}_v \mathbf{I} & \mathbf{K}_{fp} &= \mathbf{k}_{fp} \mathbf{I} \ \mathbf{K}_p &= \mathbf{k}_p \mathbf{I} & \mathbf{K}_{fi} &= \mathbf{k}_{fi} \mathbf{I} \end{aligned}$$

then by regarding as  $k_a, k_{fv}, k_v, k_{fp}, k_p, k_{fi} > 0$  the jth element of  $\mathbf{k}_a, \mathbf{k}_{fv}, \mathbf{k}_v, \mathbf{k}_{fp}, \mathbf{k}_p, \mathbf{k}_{fi}$ , the stability of the third order system is ensured as long as

$$k_{fi} < \frac{(k_v + k_e k_{fv})(k_p + k_e k_{fp})}{k_a}$$

Whenever the above condition is satisfied, the force is regulated. The controller gives priority to the force error at the expence of the position error. The latter is treated as a bounded disturbance on the dynamics of the former. In case of zero force error, the scheme provides a common inverse dynamics position controller.

# 3 Computer Simulations

# 3.1 Simulation setup

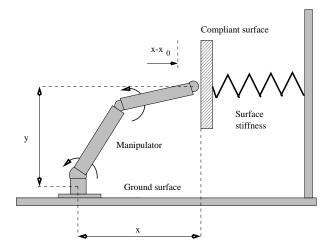


Figure 1: Setup of the simulation example

The system considered for the simulation example is a two link rotational joint manipulator that interacts with a planar compliant surface, located at a distance  $x_0 = 1.2$ m from the base of the manipulator in the horizontal direction (Fig. 1). The environment compliant behavior is depicted in the figure as a spring with constant stiffness  $k_e = 150$ N/m. The position of the end effector is given by the cartesian coordinates (x,y). The force exerted by the compliant surface to the manipulator end effector depends on the difference  $x - x_0$ .

The desired force  $f_d$  during unconstrained motion is set to zero. Once a nonzero force measurement is detected,  $f_d$  can switch to another desired value. The objective is to control both the position of the end effector along any unconstrainted direction and the contact force along the constrained directions, without specifying a priori these directions. In this process the proposed controller is compared to a parallel controller. The gains of the parallel controller have been selected over a wide range of values such that near-optimum performance is obtained, in terms of force overshoot and force error after a period of one time unit. This performance is compared to the best performance of the proposed controller over a range of values for its control parameters.

In each case, the force control scheme has been built around the same position controller which was tuned to yield satisfactory trajectories during unconstrained motion. The gains of the position controller remain unaltered and the gains of the force controller are selected independently.

#### 3.2 Force and position regulation

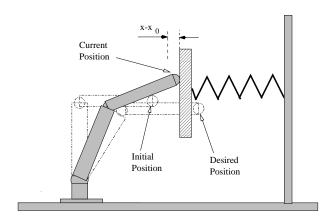


Figure 2: Regulation of position and force

In the case of force regulation the robot end effector moves from the initial position  $(x_i, y_i) = (1, 1)$ ,

to the desired position  $(x_d, y_d) = (1.5, 0.886)$ m (Fig. 2). During this motion collides with the compliant surface. The contact force is then measured and regulated to a constant value of 20N.

The simulation is first conducted for the parallel scheme. The gains for the parallel scheme were chosen so as to give the best performance over a wide set of values. The selection was made after extensive simulations. The optimum values were obtained for a neighborhood of the point  $(k_a, k_v) = (0.001, 0.7)$  with  $k_a$  ranging in  $[10^{-4}, 0.03]$  and  $k_v$  in [0.1, 3]. The simulation was repeated under the same conditions using the proposed controller. The controller gains where selected as follows:  $k_{fp} = 1000$ ,  $k_{fv} = 5$ ,  $k_{fi} = 6000$ . The resulting contact force trajectories for both controllers are presented in Figure 3.

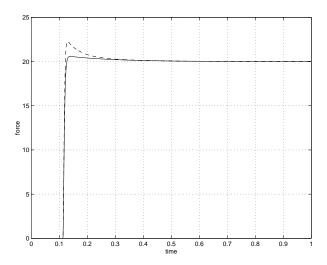


Figure 3: Contact force regulation: parallel control (dashed); reaction compensation (solid)

As it can easily seen, overshoot has been reduced by 47%. The force error, has also been reduced by 6%.

### 3.3 Force and position tracking

In the case of force and position tracking the endeffector is required to track a sinusoidal trajectory in the y direction while remaining at a constant  $x_d$  position. The  $x_d$  position will not be achieved however, because before reaching it collides with the compliant surface. The contact force is then required to track a sinusoidal trajectory (Fig. 4).

The control parameters were the same as in the regulation simulations. The results are given in Figure 5 and in Figure 6 for the proposed controller.

It must be noted that in both tracking controllers, no adaptive scheme has been implemented. As ex-

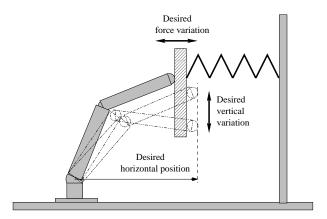


Figure 4: Tracking of position and force

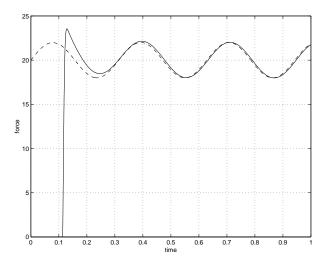


Figure 5: Force tracking with the parallel controller

plained in [10], a common desirable feature of parallel control schemes is that they do not use explicit information on the environment stiffness. Environment stiffness uncertainty was the primary limitation preventing the use of typical parallel controllers for force tracking [12], since it influences the transient behavior during interaction. In the simulations no adaptive scheme for the environment stiffness in any of the two methods compared was implemented since exact values could be used.

### 3.4 Parameter Sensitivity Analysis

To investigate the influence of the environment stiffness estimate on the transient responce of the system, the scheme was tested in a number of cases where erroneous stiffness estimates were used.

The proposed controller uses an estimate of the en-

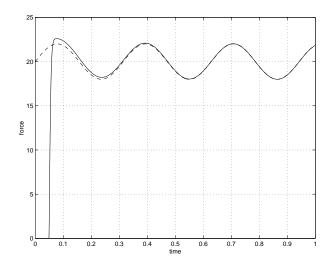


Figure 6: Force tracking with the proposed controller

vironment stiffness within the force damping term in order to obtain an estimate of the force variation. This is necessary due to the unavailability of direct force derivative measurements which requires the derivation of the needed quantities from velocity measurements. On the other hand, the parallel controller makes no use of force derivatives and therefore an estimate for the environment stiffness is not required.

Although the desired behavior can always be achieved by appropriately tuning the control parameters of the proposed controller, it would still be interesting to see the influence of an erroreous environment stiffness estimate to the controller's performance. For this reason we have conducted a series of simulations introducing error in the estimate used by the controller.

For errors smaller than 20%, the difference in the force trajectory for the case of force regulation was too small to be recorded. Results for errors of 20%, 30%, 40% are depicted on Figure 7, which is a magnification of the transient region of the force trajectories.

As seen from Figure 7, overshoot increases as the error in the estimate grows, but it does not exceed the value of 15% for a stiffness error of 40%. What is interesting is that convergence rate is faster as the stiffness error increases, a fact that demonstrates an attempt from behalf of the controller to compensate for the relatively large initial force error.

#### 4 Conclusion

This paper presents a position/force controller based on the philosophy of the parallel control scheme

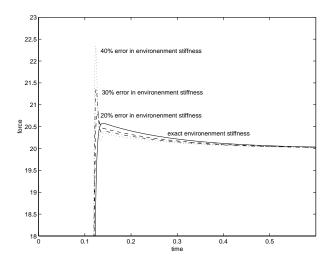


Figure 7: Controller sensitivity on environment stiffness estimate

but with better performance characteristics. The proposed controller exploits the reaction compensation action present in the inverse dynamics position controller. Instead of fully compensating for the reaction force at the end-effector, a partial compenstation according to the proposed controller achieves force regulation and tracking with improved transient behavior.

The proposed controller is compared to the parallel controller in two cases corresponding to force regulation and force tracking respectively. The computer simulations verified the advanced performance characteristics. Due to the fact that the force damping term introduced depends on an estimate of the environment stiffness, the impact of an erroneous estimate of the controller performance is investigated. It is concluded that for rough estimates of less than 20% accuracy the transient behavior is not influenced at all. Larger errors in the environment stiffness introduce some limited overshoot which can be eliminated by appropriate tuning of the controller parameters.

# References

- [1] N. Hogan, "Impedance control: An approach to manipulation; part i-theory; part ii-implementation; part iii-applications," ASME Journal of Dynamic Systems, Measurement and Control, vol. 107, no. 1, pp. 1–24, 1985.
- [2] C. Canudas de Wit and B. Brogliato, "Direct adaptive impedance control including transition

- phases," Automatica, vol. 33, no. 4, pp. 643–649, 1997.
- [3] R. Colbaugh, H. Seraji, and K. Glass, "Direct adaptive impedance control of robot manipulators," *Journal of Robotic Systems*, vol. 10, pp. 217–248, 1993.
- [4] M. H. Raibert and J. Craig, "Hybrid position/force control of manipulators," ASME Journal of Dynamic Systems, Measurement and Control, vol. 103, no. 2, pp. 126–133, 1981.
- [5] T. Yoshikawa, "Force control of robot manipulators," in *Proc. of teh 2000 IEEE International Conference on Robotics and Automation*, (San Francisco), pp. 220–226, April 2000.
- [6] T. Yoshikawa, Foundations of Robotics. MIT Press, 1990.
- [7] O. Khatib, "A unified approach for motion and force control of robot manipulators: The operational space formulation," *IEEE Journal of Robotics and Automation*, vol. RA-3, pp. 43–53, February 1987.
- [8] R. Anderson and M. Spong, "Hybrid impedance control of robotic manipulators," *IEEE Journal* of Robotics and Automation, vol. 4, pp. 549–556, 1988.
- [9] J. Duffy, "The fallacy of modern hybrid control theory that is based on 'orthogonal complements' of twist and wrench spaces," *Journal of Robotic Systems*, vol. 7, no. 2, pp. 139–144, 1990.
- [10] L. Sciavicco and L. Villani, Robot Force Control. Kluwer Academic Publishers, 1999.
- [11] S. Chiaverini and L. Sciavicco, "The parallel approach to force/position control of robotic manipulators," *IEEE Transactions on Robotics and Automation*, vol. 9, no. 4, pp. 361–373, 1993.
- [12] S. Chiaverini, B. Siciliano, and L. Villani, "Force and position tracking: Parallel control with stiffness adaptation," *IEEE Control Systems Maga*zine, vol. 18, no. 1, pp. 27–33, 1998.