Synchronous Rendezvous for Networks of Active Drifters in Gyre Flows

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Abstract. We develop a synchronous rendezvous strategy for a network of minimally actuated mobile sensors or *active drifters*. The drifters are tasked to monitor a set of Lagrangian Coherent Structure (LCS) bounded regions, each exhibiting gyre-like flows. This paper examines the conditions under which a pair of neighboring agents achieves synchronous rendezvous by leveraging the environmental dynamics in their monitoring region. The objective is to enable drifters in adjacent LCS bounded regions to rendezvous in a periodic fashion to exchange and fuse sensor data. We propose an agent-level control strategy to regulate the drifter speed in each monitoring region so as to maximize the time the drifters are connected and able to communicate at every rendezvous. The strategy utilizes minimal actuation to ensure synchronization between neighboring pairs of drifters to ensure periodic rendezvous. The intermittent synchronization policy enables a locally connected network of minimally actuated mobile sensors to converge to a common orbit frequency.

Keywords: synchronous rendezvous, marine robotics, coupled oscillators

1 Introduction

We are interested in the deployment of minimally actuated drifters, *active drifters*, or similarly power-constrained mobile sensors for large scale ocean monitoring applications. Swarms of active drifter can cover more area and obtain simultaneous measurements at many distinct geographic locations which is important when tracking spatio-temporal processes in the ocean. Furthermore, unlike their *passive* counterparts, active drifters can adapt, albeit in a limited fashion, their sampling strategies to maximize information gain. As such, swarms of active drifters can be more cost effective than larger, more capable autonomous surface, underwater, and remotely operated vehicles (ASVs, AUVs, or ROVs). Nevertheless, these mobile sensors have limited storage, communication, and power budgets and must rely on *energy aware* motion control and coordination strategies for data harvesting, exchange, and upload.

The intermittent and short-range interactions between these drifters give rise to a particular type of dynamic and sparse sensor network. This network stays disconnected for most of the time, and has brief periods in which small, isolated cliques are formed. Cliques may share nodes, but not at the same time. Questions of interest here are under which conditions such cliques are formed, how frequently do they appear, how could information propagate if they share some members, and how can the formation of such cliques be made more robust, given that the nodes can only interact with each other when they are in very close proximity.

Motion plans and control strategies for robots that are part of a mobile sensor network needs to capture the interplay between sensing, communication, and mobility. Existing work has mostly focused on enabling robots to efficiently harvest and transport data from stationary sensors deployed across large geographical regions [1,2]. This is typically done by tasking robots to assist in the data exchange between sensor nodes by physically downloading, carrying, and uploading data from one node to another. Such an approach minimizes the transmission power needed at each node as well as the number of relay nodes in the network. Recent work [3] considered the synchronous arrival of pairs of robots at predefined set of stationary rendezvous points. This problem was defined as "synchronous rendezvous," and distributed agreement protocols where used to ensure that pairs of robots arrived at a set of predefined rendezvous points to exchange acquired information. Along similar lines, the analysis here builds upon the observation that synchronous rendezvous between active drifters in the ocean shares elements with the problem of synchronizing networked nonlinear oscillators. However, since drifter motion is affected by the surrounding geophysical flow, the synchronized arrival of oceanic drifters will have to rely on motion plans and control strategies that are *in concert* with the ocean current patterns. In addition, the concept of waiting time around rendezvous location [3] is no longer relevant in this context: it is unrealistic to expect that power-constrained drifters will persistently fight the current to stay at a certain location. Thus, this paper departs from existing approaches [3], and considers a mobile sensor network striving for synchronous rendezvous at rendezvous positions determined by the *geophysical fluid dynamics*.

Collection of Lagrangian coherent structure (LCS) bounded regions are abundant in the oceans (Fig. 2b). These are material lines that organize fluid-flow transport and can be viewed as the extensions of stable and unstable manifolds to general time-dependent systems [4]. In two-dimensional (2D) flows, Lagrangian Coherent Structure (LCS) are one-dimensional separating boundaries analogous to ridges defined by local maximum instability, and can be quantified by local measures of Finite-Time Lyapunov Exponents (FTLEs) [4,5]. Recently, LCS have been shown to correspond to minimum energy and time optimal paths in the ocean [6]. Despite being global features of the flow field, it has been shown that LCS can be tracked in real-time by teams of autonomous vehicles using only local measurements of the flow velocity [7]. Figure 1 shows a simulation of the dispersion of particulates in a time-varying wind-driven double-gyre flow where the LCS boundaries are marked as red curves and the corresponding velocity field is shown in Fig. 2a. Figure 1 suggests that (a) LCS boundaries behave as basin boundaries and thus fluid from opposing sides of the boundary do not mix; (b) in the presence of noise,[1] particulates can cross the LCS boundaries, and thus LCS denote regions in the flow field where more escape events occur [8]; and (c) it makes sense to decompose the oceanic workspace along LCS boundaries and assign sensors to each LCS-bounded region for large scale monitoring operations [9]. While the model shown in Figs. 1 and

[1] Noise can arise from uncertainty in model parameters and/or measurement noise.

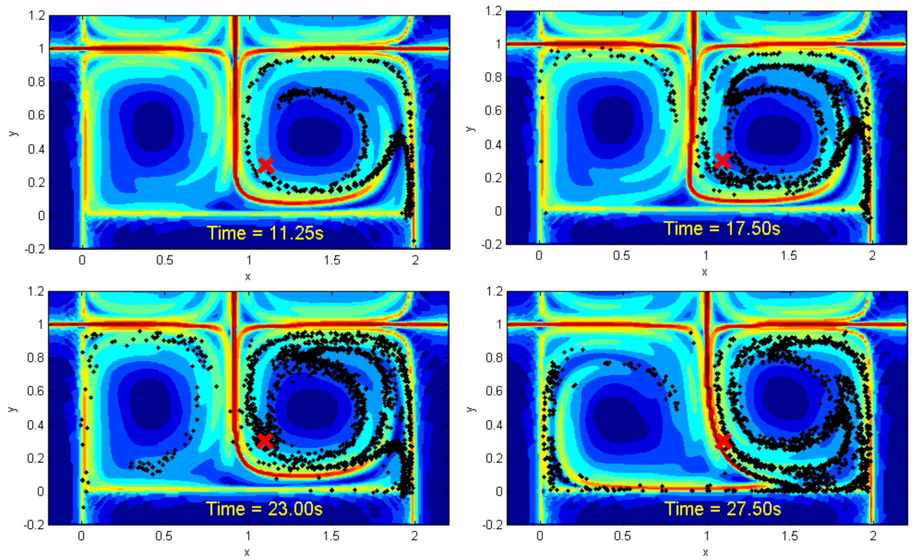


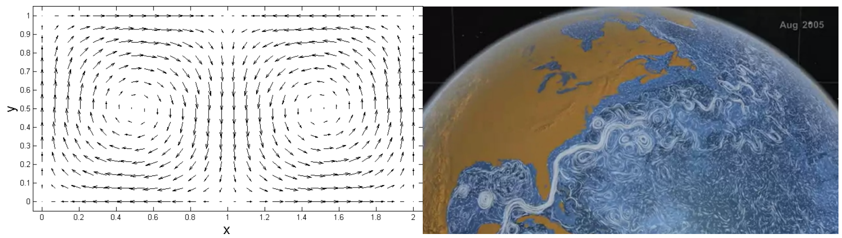
Fig. 1: Simulation of a contaminant spill in a time-varying wind-driven double-gyre flow. The LCS boundaries are shown as red curves and the red x denotes the source position of the spill. Black particles denote particulates emanating from the source. The center vertical LCS boundary oscillates horizontally about $x = 1$.

2a presents an idealized representation of the flow field, a snapshot of the ocean surface currents in August 2005 (Fig. 2b) shows a variety of flow patterns including jets and gyres similar those in Figs. 1 and 2a[2]. In fact, the time-varying wind-driven double-gyre model is often used to model large scale ocean circulation [10].

Leveraging our understanding of LCS, we assume that the workspace can be modeled as a collection of LCS bounded regions exhibiting gyre-like flows. Decomposing the workspace along LCS boundaries allows active drifters to leverage the surrounding fluid dynamics for navigation and thus enabling them to prolong their operational lifespans. Given the tight coupling between the ambient and agent dynamics, the rendezvous problem at hand bears more similarities to the synchronization of networked oscillators often found in physics, biology, neuroscience, and engineering [11–13]. Existing strategies for distributed oscillator synchronization is ill-suited to the problem of persistent sensing and continuous monitoring by networks of active drifters subject to spatiotemporal-dependent intermittent communication [14–16]. Existing strategies does not allow for the *a priori* prediction of the equilibrium consensus state and existing phase or location synchronization. As such, the analysis in this paper focuses on coordination through either *phase* or *location* synchronization.

Build upon preliminary work [17], this paper analyzes the conditions for rendezvous for any pair of active drifters undergoing periodic motion in 2D flows. Contrary to [3], the synchronous rendezvous problem for pairs of sensors operating in a geophysical

[2] For the full animation visit http://svs.gsfc.nasa.gov/vis/a000000/a003800/a003827/.



(a) (b)

Fig. 2: (a) Phase portrait of the wind-driven double-gyre model at $t = 0$. (b) Snapshot (August 2005) of visualization of ocean surface currents for June 2005 through December 2007 generated using NASA/JPLs Estimating the Circulation and Climate of the Ocean, Phase II (ECCO2) ocean model.

flow is conceptually closer to the oscillator synchronization problem. Our analysis shows that it is *very rare* for pairs of sensors in gyre-like flows to not ever rendezvous. As such, the proposed control strategy aims to prolong the time drifters can stay within communication range while in the rendezvous zone and improve the robustness of the next rendezvous event. These results are made possible through synthesis of ideas from nonlinear dynamics, transport theory, and distributed control.

The rest of the paper is organized as follows: Section 2 offers a more complete problem statement, while Section 3 presents the analysis of the synchronous rendezvous conditions and the synthesis of the short-range coordination strategies. Section 4 presents simulation results. Conclusions and final thoughts close the paper in Section 5.

2 Problem Statement

Consider a set of N active drifters, each equipped with a low power communication device, operating within their assigned LCS bounded region. Each drifter is assumed to travel along a non-overlapping circular orbit in $\mathbb{R}^2$. The drifters and their corresponding circular orbits are indexed by $i \in \{1, \ldots, N\}$. The position for drifter i is denoted x_i. The drifter is treated as an oscillator whose phase on orbit i at time t is $\theta_i(t) \in (-\pi, \pi]$ with $\phi_i = \theta_i(0)$ being the *initial phase*. If ω_i is the natural frequency of the unactuated drifter, and $u_i(t)$ denotes its control input, its phase will evolve as

$$\dot{\theta}_i(t) = \omega_i + u_i(t) \tag{1}$$

Given such mapping $\dot{\theta}_i(t) = f(\theta_i(t))$, agent i has a fixed period T_i. When $u_i(t) = 0$, $\dot{\theta}_i(t) = \omega_i$, and the unforced drifter's period is denoted τ_i.

Any pair of agents i and j with orbits tangent to each other (sharing an LCS boundary) are *neighbors*. In the neighborhood of the tangent point, drifters can exchange bidirectionally information about their phase, velocity, and frequency. The position of these special tangent points is denoted $\gamma_{i,j}$. A *rendezvous zone* $\Gamma_{(i,j)}$ with radius B_{ij} is defined for drifters i and j around $\gamma_{i,j}$ (Fig. 3)

$$\Gamma_{(i,j)} = \{x \in \mathbb{R}^2 | \|x - \gamma_{i,j}\| < B_{ij}\} \tag{2}$$

The phase of $\gamma_{i,j}$ on the orbit of drifter i is denoted $\psi_{i,j}$. The phase of drifter i when it enters and exits $\Gamma_{(i,j)}$ are denoted $\psi^{-}_{i,j}$ and $\psi^{+}_{i,j} \in (-\pi,\pi]$, respectively. The time instances when i reaches these phases for the m-th time are denoted as $t^{m-}_{i,j,}$ and $t^{m+}_{i,j}$, respectively. The layout of neighboring relations can be abstracted as an undirected graph $\mathscr{G} = (\mathscr{V},\mathscr{E})$, where each agent is a vertex in $\mathscr{V} = \{v_1,\ldots,v_n\}$, and $\{i,j\} \in \mathscr{E}$ if agents i and j have tangential orbits (Fig. 3)

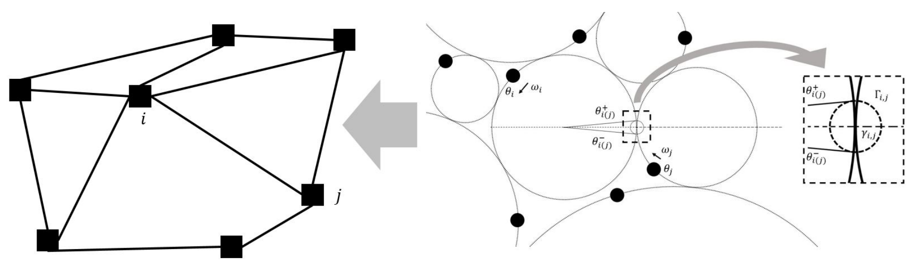


Fig. 3: The layout of 7 agents on their circular orbits and the graph abstraction. Each circle is an approximation of a region bounded by LCS. The right insert gives details on the rendezvous zone, with the entering and the exiting phases.

We say that drifters i and j are *rendezvous* if $x_i \in \Gamma_{(i,j)} \ni x_j$. Drifters can only be aware of each other's existence and can communicate only while in rendezvous. Drifter i would exercise control action u_i in coordination with a proximal neighbor only during that brief period; the rest of the time, it will merely go with the flow. The objective is to coordinate the motion of the drifters when they are found in rendezvous, so that this rendezvous event can continue to happen periodically, and for the maximal possible duration, without additional actuation.

3 Technical Approach

The problem can be decomposed into two parts: (i) determine the conditions for rendezvous for each pair of neighboring agents given their initial phases and frequencies, and (ii) design the motion controller to establish synchronous periodic rendezvous. Toward this end, the analysis begins with consideration of pairs (i,j) of drifters, and a quest for a decision function $f(\omega_i,\omega_j,\phi_i,\phi_j) \to \{1,0\}$ that determines the possibility of the pair's initial condition to spontaneously enable rendezvous without actuation. Once the conditions for such initial encounter are established, a time optimal controller is designed for the pair to lock them into synchronous periodic rendezvous with monotonically longer rendezvous periods. This controller assumes that the drifters, through a potentially energetically expensive but very brief intervention, can do a *one-time* instantaneously reset of their gyre frequency—practically, by slightly shifting their orbit internally in their gyre. Under the assumption that this one-time reset can be lifted, the analysis proceeds by investigating the case of a connected graph of drifters, and shows how through simple consensus protocols, synchronous rendezvous can be achieved for pairs of agents within the same connected graph, using a uniform common frequency.

3.1 Rendezvous condition for a pair of agents

The case of synchronous rendezvous of one-dimensional oscillators has been already analyzed [17]. In that approach, each oscillator had at most two rendezvous zones on the line (left/right), centered around phases 0 and π, respectively. Conditions to guarantee synchronous rendezvous for any pair are derived through the solution of an integer programming problem [17].

Before the first rendezvous, say between i and j, $u_i = u_j = 0$. If $t^r_{i,j}$ marks the time point when i and j rendezvous for the first time, their phases will evolve as

$$\theta_i(t) = \phi_i + \omega_i t \qquad \theta_j(t) = \phi_j + \omega_j t \qquad 0 \le t \le t^r_{i,j} \tag{3}$$

The first rendezvous, if it happens, is spontaneous and unforced—oscillators i and j are unaware of each other's existence. If it does happen, this means that $\exists k_i, k_j \in \mathbb{Z}$ and $t^r_{i,j} \in \mathbb{R}^+$ such that

$$\begin{aligned} \omega_i t^r_{i,j} + \phi_i &\in \left(\psi^-_{i,j} + 2k_i\pi, \psi^+_{i,j} + 2k_i\pi\right) \\ \omega_j t^r_{i,j} + \phi_j &\in \left(\psi^-_{j,i} + 2k_j\pi, \psi^+_{j,i} + 2k_j\pi\right) \end{aligned}$$

The conditions for these inclusions to hold are probably more easily arrived at if one poses the *inverse* question: under which conditions i and j never meet? Thinking of the time instances when one agent has "just missed" the other, one would need that $\forall (k_i, k_j) \in \mathbb{Z}^2$, it should either be

$$\begin{aligned} \frac{\psi^+_{i,j} - \phi_i + 2k_i\pi}{\omega_i} &\le \frac{\psi^-_{j,i} - \phi_j + 2k_j\pi}{\omega_j} \qquad \text{or} \\ \frac{\psi^-_{i,j} - \phi_i + 2k_i\pi}{\omega_i} &\ge \frac{\psi^+_{j,i} - \phi_j + 2k_j\pi}{\omega_j} \end{aligned}$$

If one defines

$$\overline{g}_{j,i} \triangleq \frac{\psi^+_{i,j} - \phi_i}{\omega_i} - \frac{\psi^-_{j,i} - \phi_j}{\omega_j} \qquad \underline{g}_{j,i} \triangleq \frac{\psi^-_{i,j} - \phi_i}{\omega_i} - \frac{\psi^+_{j,i} - \phi_j}{\omega_j} \tag{4}$$

then i and j will *have to* rendezvous if $\exists (k_i, k_j) \in \mathbb{Z}^2$ such that

$$\tau_j k_j - \tau_i k_i \in \left(\underline{g}_{j,i}, \overline{g}_{j,i}\right) \tag{5}$$

Building upon the analysis of the case where $\frac{\tau_i}{\tau_j}$ is rational [17], this paper seeks rendezvous conditions for a pair of planar oscillators considering both the case of rational and irrational ratio $\frac{\omega_i}{\omega_j}$. Along a similar line of analysis, the following result is obtained.

Lemma 1. *Assume $\frac{\omega_i}{\omega_j}$ is rational. Let $c \in \mathbb{R}$ be such that both $\frac{2c\pi}{\omega_i}$ and $\frac{2c\pi}{\omega_j} \in \mathbb{Z} \setminus \{0\}$, and denote d the greatest common factor of $\left(\frac{2c\pi}{\omega_i}, \frac{2c\pi}{\omega_j}\right)$. Then agents i and j achieve rendezvous spontaneously iff there exists*

$$\left(\frac{c}{d}\underline{g}_{j,i}, \frac{c}{d}\overline{g}_{j,i}\right) \cap \mathbb{Z} \neq \emptyset \tag{6}$$

Proof. Follow the steps for [17, Corollary 1].

Note that Lemma 1 suggests that for any rational pair of ω_i and ω_j, there exists a time period such that both of the agents have completed integer multiples of rounds and return to their initial phases ϕ_i and ϕ_j. If the rendezvous has not happened until then, then it will not happen ever. Consequently, the verification of whether these agents will rendezvous can be completed within a finite number of tests.

Rewriting ω_i as $\frac{c\tau_i}{d}$, one finds the shortest time period for both agents to complete integer rounds, also the upper bound of the first rendezvous time, to be

$$t_{i,j}^r \leq \tau_{i,j} = \frac{c}{d}\tau_i\tau_j,$$

Time $\tau_{i,j}$ is therefore the shortest common period multiple for i and j. Given that i and j are both in the rendezvous zone at time $t_{i,j}^r$, the next time they both appear in the rendezvous zone (without actuation) will be at $t_{i,j}^r + \tau_{i,j}$. Therefore $\tau_{i,j}$ is also the rendezvous period resulting from just the agents' natural frequencies.

For irrational $\frac{\omega_i}{\omega_j}$, periodic rendezvous for agents i and j cannot be guaranteed. However, it can be shown that i and j always rendezvous in an unforced fashion, based solely on their natural frequencies.

Lemma 2. *Assume $q = \frac{\omega_i}{\omega_j}$ is irrational. If the rendezvous zone for both i and j is not of measure zero (i.e. $\psi_{i,j}^+ \neq \psi_{i,j}^-$), then agents i and j rendezvous infinitely often.*

Proof. Let $z \in \mathbb{Z}$ to be the ceiling of $|\frac{2\pi}{\psi_{i,j}^+ - \psi_{i,j}^-}|$. The orbit of agent i can therefore be divided into z consecutive parts, with one of them to be totally covered by the rendezvous zone for i.

Without loss of generality, take the time point when agent j enters the rendezvous zone as $t = 0$, and the phase of i at this time as $\phi_i = \theta_i(0)$. Before i and j rendezvous for the first time, j has a constant angular velocity ω_j, and enters rendezvous zone at the time points $\frac{2k\pi}{\omega_j}$, for $k \in \mathbb{N}_+$. The corresponding phases of i are $\phi_i + 2k\pi\frac{\omega_i}{\omega_j} = \phi_i + kq\cdot(2\pi)$. If $\exists k$ such that $(kq - \lfloor kq \rfloor)\cdot 2\pi + \phi_i \in [\psi_{i,j}^-, \psi_{i,j}^+]$, then i and j rendezvous.

An irrational q can be approximated arbitrarily close from above and below by a rational, such that there exists a rational number $\frac{a}{b}$ with $a, b \in \mathbb{Z}^+$ such that $\frac{a}{b} < q$, and

$\frac{a+\frac{1}{z}}{b} > q$. If $k = b$, then $(kq - \lfloor kq \rfloor) \cdot 2\pi \in (0, \frac{2\pi}{z})$. If k is an integer multiple of b, then the corresponding phase of i can match every of the z parts on the orbit. Since one of them is a subset of the rendezvous zone, agent i will eventually be in the rendezvous zone when j just enters the zone.

Note that Lemma 2 states merely a sufficient condition. Typically, the first rendezvous for i and j occurs much earlier. The Lemma shows that a pair of neighboring agents can achieve spontaneous rendezvous in finite time if $\frac{\omega_i}{\omega_j}$ is irrational. Together with Lemma 1, it narrows considerably the cases that rendezvous may never happen: a rational $\frac{\omega_i}{\omega_j}$ with initial phase lag that does not fit the condition of Lemma 1:

Theorem 1. *Consider a pair of neighboring agents oscillating on their orbits with angular velocity ω_i and ω_j. The agents are* not *able to rendezvous in a pre-defined rendezvous zone* (2) *iff $\frac{\omega_i}{\omega_j}$ is a rational number and $\left(\frac{c}{d}\underline{g}_{j,i}, \frac{c}{d}\overline{g}_{j,i}\right) \cap \mathbb{Z} = \emptyset$, with c and d defined in Lemma 1.*

In practice, it is extremely rare that multiple agents can maintain their frequencies such that the ratio of the frequencies equals an exact rational number. Even for agents that are tasked to maintain said frequencies, uncertainties arising from the agent's motions, measurements, model parameters fluctuations, and minimum actuation can easily result in irrational frequency ratios. From Theorem 1, any pair of agents can almost always spontaneously rendezvous. A detailed sensitivity analysis is beyond the scope of this paper.

3.2 Design of Synchronization Controller

Outside a rendezvous zone $\Gamma_{(i,j)}$ for an (i, j) pair, agent i and j travel at a constant angular velocity, without utilizing actuation. Let us assume that i completes m_i periods, while j does m_j of its own, before they meet for the first time in the rendezvous zone. A coordinating controller applied inside $\Gamma_{(i,j)}$ should first synchronize $\dot{\theta}_i$ and $\dot{\theta}_j$ so that the pair meets again after just one (new) period. It will be assumed that once out of the rendezvous zone, the agents maintain the frequency each has attained upon exit, i.e. $\omega_i(t) = \dot{\theta}_i\big(t^r_{i,j}\big)$ for $t \in \big[t^r_{i,j}, t^{m_i+}_{i,j}\big]$, and $\omega_j = \dot{\theta}_j\big(t^r_{j,i}\big)$ for $t \in \big[t^r_{j,i}, t^{m_j+}_{j,i}\big]$. At the same time, the controller should maximize the time both agents spend in rendezvous. One way to achieve the latter is to regulate their motion so that they are scheduled to hit $\gamma_{i,j}$, which is $\psi_{i,j}$ for agent i and $\psi_{j,i}$ for agent j, simultaneously.

$$\theta_i(t) - \theta_j(t) = \psi_{i,j} - \psi_{j,i} + 2k\pi \qquad k \in \mathbb{Z}$$

In the spirit of the one-dimensional strategy (cf. [17]) the task of synchronization is split between the pair of agents. Agent j is tasked with adjusting its frequency to the average of the pair's angular velocities at the time rendezvous was initiated. For agent j under (1), this means resetting its frequency by applying

$$u_j(t) = \frac{\dot{\theta}_i(t^r_{i,j}) + \dot{\theta}_j(t^r_{j,i})}{2} - \omega_j \tag{7}$$

Agent i now is tasked with tracking this new frequency that j has set for the pair, while simultaneously adjusting its phase to minimize the difference $\left|t_{i,j}^{(m_i+1)-} - t_{j,i}^{(m_j+1)-}\right|$. This is implemented through a minimum-time optimal controller, the choice of which being motivated by the limited period of potential interaction between the agents.

To frame this (linear) minimum-time optimal control problem, define the error states

$$\varepsilon_1 = \theta_i(t) - \theta_j(t) - \left\lfloor \frac{\theta_i(t) - \theta_j(t)}{2\pi} \right\rfloor 2\pi - (\psi_{i,j} - \psi_{j,i})$$
$$\varepsilon_2 = \dot{\theta}_i(t) - \dot{\theta}_j(t)$$

and write the error dynamics in matrix form

$$\begin{bmatrix} \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} + \begin{bmatrix} 0 \\ v_{i,j} \end{bmatrix} \tag{8}$$

with control input $v_{i,j} = u_i - u_j$, noting that u_j is already determined by (7). Then the choice of u_i to bring ε_1 and ε_2 to zero in minimum time is straightforward [18], yielding a bang-bang controller.

3.3 Synchronizing multiple agents

The policy of Section 3.1 sets the desired reference for the angular velocity of the pair during their very first rendezvous. After that point, the agents committed to this common period which becomes invariant. One way to extend this synchronization result to chains of oscillators [17] is to have each other agent subsequently meeting an agent with committed frequency, to simply adopt that frequency. It was shown [17] that the implementation of this protocol in one dimension leads to the creation of subgraphs of synchronized agents, with each subgraph having its own committed frequency.

This paper suggests an alternative synchronization policy which can be implemented if all agents are allowed to modify their frequencies on a continuous basis. Interaction between agents is still intermittent and brief, happening only during rendezvous, but now every pair finding itself in rendezvous switches to their average angular velocity

$$\dot{\theta}_i(t)_{new} = \dot{\theta}_j(t)_{new} = \frac{1}{2}\left(\dot{\theta}_i(t_{i,j}^r) + \dot{\theta}_j(t_{i,j}^r)\right) \tag{9}$$

It can be shown that if the whole collection of agents is (intermittently) connected, frequency synchronization propagates to the whole network, and eventually agents converge to a common oscillating frequency:

Theorem 2. *For N oscillators connected over a graph, each having an initial frequency $\dot{\theta}_i(0) = \omega_i$ for $i = 1, \ldots, N$, and assuming that the condition of Theorem 1 is always false for any pair of adjacent agents, then all oscillator frequencies converge to the average of their initial values, i.e. for all $i \in \{1, \ldots, N\}$, $\lim_{t\to\infty} \dot{\theta}_i(t) = \bar{\omega}_N = \frac{1}{N}\sum_{n=1}^{N} \omega_n$.*

Proof. Let $\delta_i(t) = \dot{\theta}_i(t) - \bar{\omega}_N$, and $\Delta(t) = \sum_{i=1}^{N} |\delta_i(t)|$. The theorem's statement is equivalent to having $\lim_{t\to\infty} \Delta(t) = 0$. Pick randomly any pair (i, j) of agents in rendezvous, and consider the time interval between the time of their encounter, $t^r_{i,j}$, and the minimum time $\bar{t}$ between any of them departing the rendezvous region and any other pair of agents coming into rendezvous. For $t \in (t^r_{i,j}, \bar{t})$

$$|\delta_i(t)| + |\delta_j(t)| = |\dot{\theta}_i(t^r_{i,j}) + \dot{\theta}_j(t^r_{j,i}) - 2\bar{\omega}_N|$$

If $\delta_i(t)$ and $\delta_j(t)$ share the same sign

$$|\delta_i(t)| + |\delta_j(t)| = |\delta_i(t^r_{ij})| + |\delta_j(t^r_{ij})| \tag{10}$$

Thus note that $\Delta(t)$ remains constant for $t \in (t^r_{i,j}, \bar{t})$ and is non-increasing over consecutive rendezvous events. It can be shown that $\Delta(t)$ cannot remain constant for ever.

While the network of oscillators has not yet reached consensus on their frequencies, there is bound to be at least one agent with angular velocity greater than $\bar{\omega}_N$, and at least one with angular velocity smaller than $\bar{\omega}_N$. If one pair of those oscillators with frequencies on opposite sides of $\bar{\omega}_N$ are in fact neighbors, then when they meet (and the falsification of the condition of Theorem 1 guarantees they will), $\Delta(t)$ will decrease. Indeed, while $\Delta(t) \neq 0$, rendezvous between neighbors on opposite sides of $\bar{\omega}_N$ is bound to occur because the network is assumed to be connected. Agents on opposite sides of $\bar{\omega}_N$ still interact via shared neighbors: eventually pairs of adjacent agents with frequencies at different sides of $\bar{\omega}_N$ will appear. Otherwise, at least two isolated subgraphs would ex in the network, which would contradict the connectivity assumption.

Remark 1. As mentioned in Section 3.1, a pair of neighboring agents will almost always rendezvous. In fact, the probability that a pair lands on the configuration given by Theorem 1 such that they do not rendezvous is extremely remote. In the even rarer case when these agents cannot interact indirectly via shared neighbors, the network will have to be disconnected and exhibit several isolated components. Each isolated component would achieve its own synchronization.

4 Simulation

In this section we show two sets of simulations to illustrate our results. The first one shows how a pair of agents achieve simultaneous rendezvous and synchronize their frequencies as well as their phase lag. The initial phases are $\phi_1 = \frac{14}{16}\pi$, $\phi_2 = -\frac{1}{16}\pi$. Agents' angular velocities are $\omega_1 = \frac{1}{3}$ rad/s and $\omega_2 = \frac{1}{5}$ rad/s. The entering and exiting phases of the first agent is $\frac{15}{16}\pi$ and $-\frac{15}{16}\pi$, while the other agent has $-\frac{1}{16}\pi$ and $\frac{1}{16}\pi$ as its entering and exiting phases.

Analysis following Theorem 1 shows that they are able to achieve rendezvous solely relying on the flow dynamics. Figure 4 shows the agents' frequencies are synchronized immediately during their first rendezvous while their phases are aligned through control action over multiple rendezvous events.

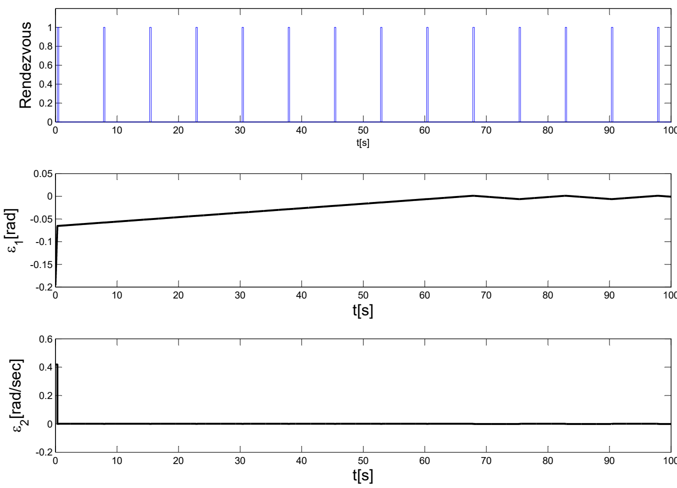
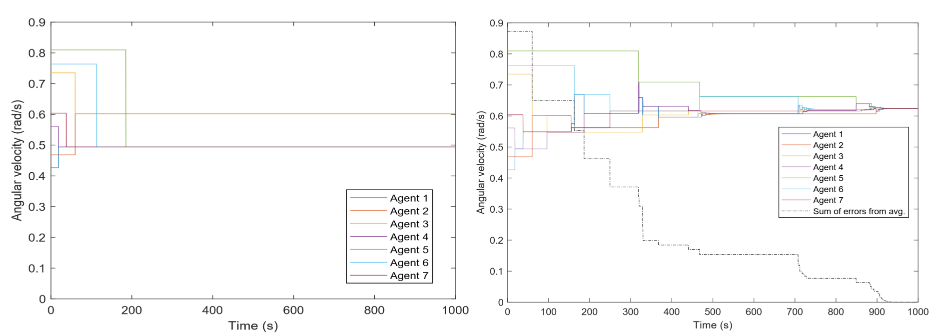


Fig. 4: Two agents synchronized after several consecutive rendezvous.

The second set of simulations show the synchronization of a network of agents. Both pictures show the synchronization of seven agents deployed as shown in Fig. 3. Figure 5a shows that the agents are synchronized with the constraint that only one single switch in the frequency is allowed for one agent. The agents rapidly converge to two different frequencies, and therefore form two sub-groups. Figure 5b shows that when any pair of agents meet each other, they both switch to their average frequencies. The synchronization takes much longer but all agents converged to the same frequency.



(a) (b)

Fig. 5: (a) Agents converge to multiple frequencies and form sub group; (b) Agents converge to the same frequency; the black dash line shows how $\Delta(t)$ decreases.

5 Conclusion and Future Work

This paper addressed a synchronous rendezvous problem for a network of active drifters monitoring large scale ocean regions bounded by LCS. It approximated the coherent structures as a circular orbits tangent to each other, and assuming that the agents flowing along these orbits can only interact while in close proximity, this formulation gave rise to a graph of intermittently interacting 2-D oscillators. Conditions under which a pair of oscillators can rendezvous solely relying on flow dynamics were presented, and controllers were designed to lock them into subsequent periodic rendezvous.

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