Low-Range Interaction Periodic Rendezvous Along Lagrangian Coherent Structures

Cong Wei, Herbert G. Tanner, Xi Yu, and M. Ani Hsieh.

***Abstract*— This paper presents synchronous rendezvous conditions for minimally actuated and very short-range communicating mobile sensors in open sea environments. The working assumption is that the ocean currents of interest can be approximated by gyres or eddy flows arranged over a grid, in which each gyre is delineated by Lagrangian coherent structures. Sensor interactions can only occur between sensors in neighboring gyres, when they drift in close proximity, and over short time periods where the required distance is maintained. Within these application-dictated constraints, a cooperative synchronization controller is designed to establish and robustify periodic sensor rendezvous. Both the rendezvous conditions, as well as the rendezvous controller are tested and validated in simulation.**

I. INTRODUCTION

There is history of deployments of low cost vehicles in swarms for environmental observation and monitoring [1], [2]. In oceanic applications, such swarms can be used for sampling complex submesoscales dynamics or for tracking ocean features [1]–[4]. Among these features, a wind driven double gyre flow is of interest [5]. This ocean circulation is widespread across the mid-latitude sea surface [6] (Fig. 1). Leveraging these currents, ocean vehicles can cover distances by simply drifting without spending their energy reserves [7].

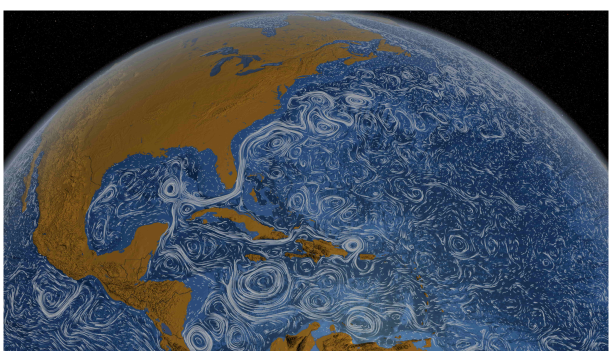

Fig. 1: Visualization snapshot (August 2005) of ocean surface currents. Source: NASA/JPL.

Here we consider a collection of minimally actuated mobile sensors or vehicles that rely predominantly on the dynamics of the surrounding environment for navigation within the workspace. We assume that the vehicles are capable of communicating and interacting with each other over extremely short spatial ranges. Such conditions arise when semi-passive floating sensors, or *active drifters*, are deployed over large areas to track and monitor various biological, chemical, and physical spatio-temporal processes that occur in the ocean. Since these sensors may have limited storage, communication, and power capacity, they must rely on *energy aware* motion control and coordination strategies for data harvesting, exchange, and upload. As such, motion plans and control strategies for these vehicles must capture the interplay between sensing, communication, and mobility.

The intermittent and short-range interactions between these drifters give rise to a particular type of dynamic and sparse sensor network. This network stays disconnected for most of the time, and has brief periods in which small, isolated cliques are formed. Cliques may share nodes, but not at the same time. Questions of interest here are under which conditions such cliques are formed, how frequently do they appear, how could information propagate if they share some members, and how can the formation of such cliques be made more robust, given that the nodes can only interact with each other when they are in very close proximity. This paper presents conditions for the active formation and maintenance of these cliques by formulating this as a *synchronous rendezvous problem* [8]. Specifically, it presents conditions for synchronous rendezvous for a semi-passive mobile sensor network, and a nonlinear synchronization controller to robustly achieve this rendezvous behavior.

The ocean dynamics are abstracted into a grid of gyres or eddies as shown in Fig. 1. Under such conditions, one may want to predict where and when two or more drifters that move along the boundaries of adjacent gyres will come close to each other so as to devise a scheduling strategy to enable a more capable surface (or aerial) vehicles to rendezvous with the drifters to upload data, download new deployment commands, and/or physically recover the sensors. Assuming periodic circulations and in the absence of noise and disturbances, such temporally brief encounters between these drifters should in principle repeat. In this light, the problem of verifying or initializing such intermittent periodic drifter rendezvous behaviors is formulated as an instance of a synchronous rendezvous problem, with the following important distinguishing features: (i) vehicles cannot be reasonably be expected to stop and wait at rendezvous locations, (ii) actuation is expensive and vehicles should leverage the environmental dynamics whenever possible, and (iii) the environmental dynamics are periodic and nonlinear.

This paper builds on prior work [9], [10] which exam-

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ined synchronous rendezvous conditions and controllers in one and two dimensions, respectively, but under idealized conditions for the ambient flow dynamics that drive the vehicles. Here, a simple nonlinear model of large-scale wind-driven ocean circulation [11] —more realistic compared to the agent orbit models in recent prior synchronous rendezvous work [9], [10]— is incorporated. The additional nonlinearities introduced render the periodic rendezvous and synchronization problem more challenging, to the extend that earlier results [9], [10] do not directly carry over. After presenting sufficient conditions for rendezvous on gyres, the paper introduces a sliding-mode synchronization controller for pair of vehicles to lock them into periodic synchronous rendezvous, under the assumption that the vehicles can only interact with each other when in very close proximity.

The synchronous rendezvous problem arising from the very short-range inter-agent interactions is traced back to almost a decade [8]. The original formulation is not applicable here, mainly because the vehicles considered in this paper cannot realistically stop and wait for other vehicles [12]. Other work on synchronous rendezvous [13], [14] does not translate because rendezvous here does not happen at user-defined points or regions, but is rather determined by the ambient geophysical dynamics; in addition, the constraint that forces vehicles to interact intermittently and only when in very close proximity, precludes earlier analysis based on coupled oscillator models [15], [16].

Leveraging the Khinchine's flatness theorem [17] (Section II), which is the foundation for existence of solutions of integer programming problems, and after the problem addressed here stated (Section III), the paper develops conditions for drifters on idealized ocean circulation models to spontaneously rendezvous (Section IV). Anticipating the importance of robustness to noise and disturbances for control schemes intended for field applications, a sliding mode controller is then developed and its stability established (Section IV). Numerical results are presented to corroborate the conditions derived and the controller design (Section V).

II. Technical Preliminaries

A. The flatness theorem

Consider a d-dimensional lattice $\mathcal{L} \subseteq \mathbb{Z}^d$ embedded in $\mathbb{R}^d$ (Fig. 2). The *feasibility* question for an Integer Linear Programming (ILP) problem [17]–[19] calls one to determine if there exist lattice points in a *convex body* K (a full-dimensional convex compact set), i.e., if $K \cap \mathcal{L} \neq \varnothing$. The answer to this question ultimately relates to a notion of measure of *width* for convex bodies.

Definition 1 ([20]): Let $K \subset \mathbb{R}^d$. The *width* of K along (vector) primitive $v \neq 0$ in $\mathbb{R}^d$ is defined as

$$\mathrm{width}_K(v) = \max\{v^\intercal x \mid x \in K\} - \min\{v^\intercal x \mid x \in K\}$$

The *dual lattice* [21] of $\mathcal{L}$ is defined as $\mathcal{L}^* \triangleq \{y \in \mathbb{R}^d \mid y^\intercal v \in \mathbb{Z},\ \forall v \in \mathcal{L}\}$.

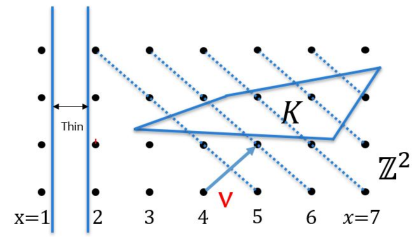

Fig. 2: A convex body on a hyperplane-decomposed $\mathbb{Z}^2$ lattice. Each lattice point is labeled by an integer pair of coordinates.

Definition 2 ([20]): The *lattice width* of K is the minimal width among all directions of $\mathcal{L}^*$:

$$\mathrm{width}(K, \mathcal{L}^*) = \min\Big\{\mathrm{width}_K(y) \mid y \in \mathcal{L}^* \setminus \{0\}\Big\} \quad (1)$$

For a set $K \subset \mathbb{R}^n$, K^* represents its *polar*, i.e., $K^* = \{y \in \mathbb{R}^n \mid y^\intercal x \leq 1\ \forall x \in K\}$. In the same context, the notation $K - K$ is understood in a Minkovski sense, where $K - K = \{x - y \mid x \in K \ni y\}$. The following theorem is key to developing existence conditions for the solutions of ILP problems.

Theorem 1 (Khinchine's flatness theorem [17]): For any convex body $K \in \mathbb{R}^n$, either

$$\mu(K, \mathbb{Z}^n) \triangleq \inf\{s \geq 0 : \mathbb{Z}^n + s\,K = \mathbb{R}^n\} \leq 1 \qquad \text{or}$$
$$\lambda_1\Big((K-K)^*, \mathbb{Z}^n\Big) \triangleq \inf_{v \in \mathcal{L}^* \setminus \{0\}} \mathrm{width}_K(v) \leq f(n)$$

In the above, μ and $f(n)$ are referred to as the *covering radius* and the *width* function of K, respectively. The main idea of the theorem is that a convex body will contain lattice points if its covering radius is small enough; otherwise, if it is "fairly flat," it can fit inside the lattice without intersecting with any of its points.

III. Problem Statement

A. Basic formulation

Let us denote (x, y) the position of a single vehicle, with position parameterized in a Cartesian way relative to origin of coordinates. With the amplitude and scale parameters of the gyre flow dynamics (see Fig. 3) denoted A and s, respectively [22], a vehicle that simply drifts in the gyre current will move as

$$\dot{x} = -\pi A \sin\frac{\pi x}{s}\ \cos\frac{\pi y}{s} \quad (2a)$$
$$\dot{y} = \pi A \cos\frac{\pi x}{s}\ \sin\frac{\pi y}{s} \quad (2b)$$

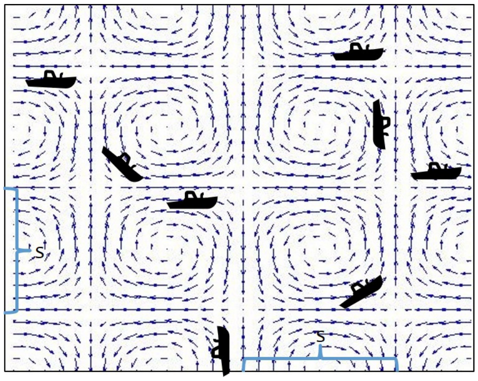

Fig. 3: Sketch of wind driven gyre flow when $A = 0.02$, and $s = 1$. Surface vehicles drifting along such gyres have the opportunity to rendezvous along Lagrangian Coherent Structures.

Consider a planar array of gyres (Fig. 3) and index each with an integer pair (i, j) that determine their location on the array. Inside each gyre and for $C \in [-1, 1]$, (2) admits a family $\Phi_{(i,j)}$ of invariant orbits $\Phi_{(i,j)} = \bigcup_C \Phi^C_{(i,j)}$ with

$$\Phi^C_{(i,j)} = \left\{(x, y) \in \mathbb{R}^2 \mid \sin \tfrac{\pi x}{s} \ \sin \tfrac{\pi y}{s} = C\right\} \tag{3}$$

Now consider two vehicles drifting along neighboring gyres centered at (i, j) and $(i + s, j)$, very close to the associated Lagrangian Coherent Structure (LCS), each having dynamics of the form

$$\dot{x} = -\,\pi A \sin \tfrac{\pi x_1}{s} \ \cos \tfrac{\pi y_1}{s} + u_x \tag{4a}$$
$$\dot{y} = \pi A \cos \tfrac{\pi x_1}{s} \ \sin \tfrac{\pi y_1}{s} + u_y \tag{4b}$$

which is essentially an actuated version of (2). Assume that the vehicles can only exchange information if in very close proximity; when they are in rendezvous. Rendezvous between vehicles happens when they are at a distance of at most δ.

Definition 3 (Rendezvous): Two vehicles, each drifting along $\Phi^C_{(i,j)}$, $\Phi^{-C}_{(i+s,j)}$, respectively, are at time t within Euclidean distance $d(t)$. For $\delta > 0$, the vehicles rendezvous at time $\tau \in \mathbb{R}_+$ if $d(\tau) \leq \delta$.

Problem 1: Design a bounded cooperative control law ensuring that at steady state two vehicles driven by (4) will spend spend the maximum possible time in rendezvous without applying control effort.

Note that since the vehicles can only interact with each other and coordinate when they are already in rendezvous, this problem only has a solution if the vehicles can come into a first rendezvous spontaneously. Then, control action can be applied to ensure subsequent periodic rendezvous; presumably, with each iteration their coordinating control action brings them closer to their longest possible rendezvous, which will occur if both vehicles converge to the common saddle point of their neighboring gyres (Fig. 3) simultaneously.

IV. Technical Approach

It should come as no surprise that the approach to solving Problem 1 proceeds in two stages: (i) determine if and when two vehicles drifting on neighboring invariant orbits will rendezvous, and (ii) design the controller that is activated only when the vehicles are in rendezvous and synchronizes them so that they establish periodic rendezvous and prolong their close encounter for as long as the flow dynamics allow.

A. Spatial conditions for rendezvous

Let us focus on a single gyre in Fig. 3 and define a number of symmetry lines that will facilitate the analysis. Placing a coordinate system at the center of the gyre, the symmetry lines of interest are defined in pairs

$$l_1(x, y) = \{(x, y) : y = x\} \cup \{(x, y) : y = -x + s\} \tag{5a}$$
$$l_2(x, y) = \{(x, y) : x = \tfrac{s}{2}\} \tag{5b}$$
$$l_3(x, y) = \{(x, y) : y = \tfrac{s}{2}\} \tag{5c}$$

Note that for the array of gyres shown in Fig. 3 there will be a family of such lines, and that each of the loci defined above for one gyre are shared by others.

The distribution of vehicle speed in (2) is not uniform. It can be verified that the speed of a vehicle drifting along $\Phi^{-C}_{(i+s,j)}$ is expressed as

$$\|V\| = \pi A \sqrt{1 - C^2 - \cos^2 \tfrac{\pi x}{s} \ \cos^2 \tfrac{\pi y}{s}} \tag{6}$$

and represented as a polar plot in Fig. 4, where speed $\|V\|$ is parameterized in terms of magnitude and phase relative to a gyre-centered frame. Speed peaks along the low-curvature

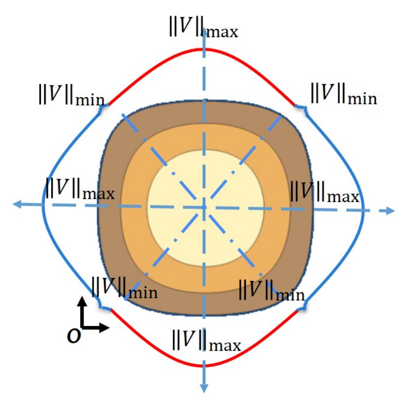

Fig. 4: Symmetry axes and velocity norm distribution along the orbit with $C = 0.1$, $s = 1$, and $A = 0.02$.

areas of $\Phi^{-C}_{(i+s,j)}$, specifically on l_2 and l_3, and is minimal at the high-curvature areas that appear on l_1. The associated extremal speed values are $\|V\|_{\max} = \pi A\sqrt{1 - C^2}$ and $\|V\|_{\min} = \pi A\sqrt{C^2 - C^4}$, respectively.

Take $0 < C \ll 1$ and consider two neighboring orbits $\Phi^C_{(i,j)}$, $\Phi^{-C}_{(i+s,j)}$. (Neigborhing gyre flows have opposite directions, so both sides in (3) flips sign.) Picking $|C|$ small ensures that the two orbits are spatially close, in a Euclidean set distance sense, to each other. Consider two vehicles, one at coordinates $A_1 : (x_1, y_1)$ drifting along $\Phi^C_{(i,j)}$, and another along $\Phi^{-C}_{(i+s,j)}$ at coordinates $A_2 : (x_2, y_2)$.

Let A'_1 denote the symmetric to A_1 relative to LCS and onto $\Phi^{-C}_{(i+s,j)}$ (Fig. 5). Note that $|A'_1A_2| \leq |A_1A_2|$ due to symmetry, which implies that the minima of $|A'_1A_2|$ provides a very close lower bound of $|A_1A_2|$, and motivates casting the problem of achieving rendezvous between the two vehicles as a problem of minimizing the distance between A'_1 and A_2 which *move according to the same unforced dynamics* (2).

Direct derivation yields

$$\frac{\mathrm{d}|A'_1A_2|}{\mathrm{d}t} = \frac{(x'_1 - x_2)(\dot{x}'_1 - \dot{x}_2) + (y'_1 - y_2)(\dot{y}'_1 - \dot{y}_2)}{|A'_1A_2|}$$

With some algebraic manipulation using (2), and (3) for $\Phi^{-C}_{(i,j+1)}$, it follows that

$$\frac{\mathrm{d}|A'_1A_2|}{\mathrm{d}t} = -\pi AC\Big\{(x'_1 - x_2)\Big[\cot\tfrac{\pi y'_1}{s} - \cot\tfrac{\pi y_2}{s}\Big] - (y'_1 - y_2)\Big[\cot\tfrac{\pi x'_1}{s} - \cot\tfrac{\pi x_2}{s}\Big]\Big\}$$

which when one sets to zero for $|A'_1A_2| \ll s$, multiple solutions emerge:

$$\begin{aligned}
S^1_{\mathsf{min}} &= \{(x'_1, x_2, y'_1, y_2) \mid y_2 - y'_1 = -(x_2 - x'_1)\}\\
S^2_{\mathsf{min}} &= \{(x'_1, x_2, y'_1, y_2) \mid y_2 - y'_1 = x_2 - x'_1\}\\
S^1_{\mathsf{max}} &= \{(x'_1, x_2, y'_1, y_2) \mid x'_1 = x_2 \,\wedge\, y'_1 + y_2 = s\}\\
S^2_{\mathsf{max}} &= \{(x'_1, x_2, y'_1, y_2) \mid y'_1 = y_2 \,\wedge\, x'_1 + x_2 = s\}
\end{aligned}$$

The minimum distance between A'_1 and A_2 is obtained when the coordinates of A'_1 and A_2 belong to either S^1_{min} or S^2_{min}, i.e., when A'_1 and A_2 are symmetric relative to l_1 (see (5)). The following definition is illustrated in Fig. 5.

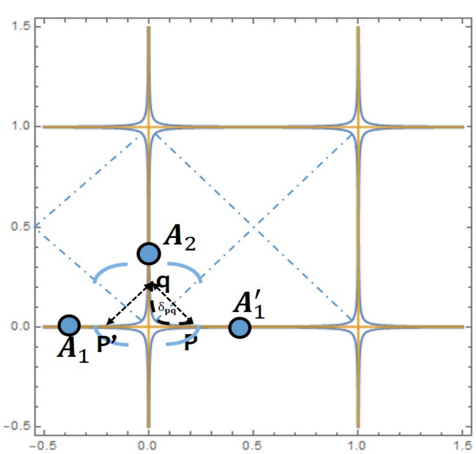

Fig. 5: Neighboring invariant orbits for $C = 2 \times 10^{-4}$.

Definition 4 (Proximity zone): Let $p \in \Phi^{-C}_{(i+s,j)} \ni q$, $p' \in \Phi^C_{(i+s,j)}$, and p' be symmetric to p relative to the boundary shared by $\Phi^{-C}_{(i+s,j)}$ and $\Phi^C_{(i+s,j)}$, such that $p-q$ having slope ± 1, and $\|p' - q\| = \delta \ll s$. The δ-proximity zone $\delta_{pq} \subset \mathbb{R}^2$ is inscribed by the line defined by vector $p-q$ and the segment of orbit $\Phi^{-C}_{(i+s,j)}$ between points p and q, denoted $\widehat{pq}$.

With the terminology of Definition 4 rendezvous between agents 1 and 2 occurs if $A'_1 \in \delta_{qp} \ni A_2$ for some $t \in \mathbb{R}_+$.

Proposition 1: Consider two vehicles $A_1(t)$ and $A_2(t)$, evolving according to (2) along orbits $\Phi^C_{(i,j)}$ and $\Phi^{-C}_{(i,j+s)}$, respectively, with period T_C. Let A'_1 be the projection of A_1 on $\Phi^{-C}_{(i,j+s)}$, and denote p and q the end points of the curve $\Phi^{-C}_{(i+s,j)} \cap \delta_{pq}$. With s an arbitrary point on $\Phi^{-C}_{(i+s,j)}$, define

$$T^p_i = \int_{A_i(0)}^{p} \frac{\mathrm{d}s}{\|V(s)\|} \qquad T^q_i = \int_{A_i(0)}^{q} \frac{\mathrm{d}s}{\|V(s)\|}$$

for $i \in \{1, 1', 2\}$, and without loss of generality assume that $T^p_i < T^q_i$. Denote η_i the number of periods elapsed before $A_i \in \delta_{pq}$. Then the two points will rendezvous if the region in the η'_1-η_2 plane

$$\left\{(\eta_{1'}, \eta_2) \in \mathbb{R}^2 \,\middle|\, \frac{T^p_2 - T^q_{1'}}{T_C} \leq \eta_{1'} - \eta_2 \leq \frac{T^q_2 - T^p_{1'}}{T_C}\right\}$$

has a nonempty intersection with the lattice $\mathbb{Z}^2$.

Proof: Similar to the condition reported in [9], [10]. ■

Corollary 1: Rendezvous will occur if

$$\left[\frac{T^p_2 - T^q_{1'}}{T_C}, \frac{T^q_2 - T^p_{1'}}{T_C}\right] \cap \mathbb{Z} \neq \varnothing$$

B. Controller design for synchronous rendezvous

Now consider a combination of (2) and (4), presenting the scenario of a pair of vehicles where one is actuated and the other is passive:

$$\dot{x}_1 = -\pi A \sin\tfrac{\pi x_1}{s}\cos\tfrac{\pi y_1}{s} + u_x \qquad (7a)$$
$$\dot{y}_1 = \pi A \cos\tfrac{\pi x_1}{s}\sin\tfrac{\pi y_1}{s} + u_y \qquad (7b)$$
$$\dot{x}_2 = -\pi A \sin\tfrac{\pi x_2}{s}\cos\tfrac{\pi y_2}{s} \qquad (7c)$$
$$\dot{y}_2 = \pi A \cos\tfrac{\pi x_2}{s}\sin\tfrac{\pi y_2}{s} \qquad (7d)$$

Assume that the two vehicles rotate about neighboring gyre orbits, $\Phi^C_{(i,j)}$ and $\Phi^{-C}_{(i+s,j)}$ that share a common LCS boundary along the y axis. The control objective would then be coded as follows: for $n \in \mathbb{Z}$,

$$\frac{x_1 + x_2}{2} \to n\,s \qquad\qquad y_1 - y_2 \to 0 \qquad (8a)$$

Without loss of generality, let $n = 0$. (For $n \neq 0$ one can always use new variables $\epsilon_1 = x_1 - 2n\,s$, $\epsilon_2 = x_2$ to recast it as $\epsilon_1 + \epsilon_2 = 0$.) Now consider the state (similarity) transformation

$$R = \begin{bmatrix} 1 & 1 & 1 & -1\\ 1 & -1 & 1 & 1\\ 1 & 1 & -1 & 1\\ 1 & -1 & -1 & -1 \end{bmatrix}$$

and introduce the new variables

$$\begin{bmatrix}\psi_1\\ \theta_1\\ \psi_2\\ \theta_2\end{bmatrix} = R\begin{bmatrix}x_1\\ y_1\\ x_2\\ y_2\end{bmatrix} \qquad \begin{bmatrix}u_1\\ w_1\\ u_2\\ w_2\end{bmatrix} = R\begin{bmatrix}u_x\\ u_y\\ 0\\ 0\end{bmatrix}$$

with which (7) takes the form

$$\dot{\psi}_1 = -2\pi A \sin\tfrac{\pi\theta_1}{2s}\cos\tfrac{\pi\theta_2}{2s} + u_1 \tag{10a}$$
$$\dot{\theta}_1 = -2\pi A \sin\tfrac{\pi\psi_1}{2s}\cos\tfrac{\pi\psi_2}{2s} + w_1 \tag{10b}$$
$$\dot{\psi}_2 = -2\pi A \sin\tfrac{\pi\theta_2}{2s}\cos\tfrac{\pi\theta_1}{2s} + u_1 \tag{10c}$$
$$\dot{\theta}_2 = -2\pi A \sin\tfrac{\pi\psi_2}{2s}\cos\tfrac{\pi\psi_1}{2s} + w_1 \tag{10d}$$

and (8) becomes

$$\theta_1 \to 0 \qquad\qquad \psi_1 \to 0 \tag{11}$$

Finally, define the signum function as

$$\operatorname{sgn}(\tau) = \begin{cases} 1 & \tau > 0 \\ 0 & \tau = 0 \\ -1 & \tau < 0 \end{cases}$$

With these definitions in place, the following statement can be made.

Proposition 2: Consider the system (10). Let A, s, and σ be positive constants and denote $B_r = \{x \in \mathbb{R}^2 | \|x\| \leq r\}$. Then for the control law

$$u_1 = -\pi(4+\sigma)A\operatorname{sgn}(\tau) \tag{12a}$$
$$w_1 = 2\pi A\Big[1 + \cos\tfrac{\pi\psi_2}{2s}\Big]\sin\tfrac{\pi\psi_1}{2s} \tag{12b}$$

there exists $r > 0$ such that for all $\big(\psi_1(0), \theta_1(0)\big) \in B_r$, $\big(\psi_1(t), \theta_1(t)\big)$ converges to the origin in finite time.

Proof: Plugging (12b) in (10b) yields

$$\dot{\theta}_1 = 2\pi A \sin\tfrac{\pi\psi_1}{2s} \tag{13}$$

Set $\tau = \psi_1 + \theta_1$ and pick $V = \frac{1}{2}\tau^2$ as a Lyapunov function candidate. Then

$$\begin{aligned} \dot{V} &= \tau\,\dot{\tau} \\ &= \tau[2\pi A(\sin\tfrac{\pi\psi_1}{2s} - \sin\tfrac{\pi\theta_1}{2s}\cos\tfrac{\pi\theta_2}{2s}) + u_1] \\ &\leq 4\pi A|\tau| + \tau u_1 \leq -\sigma\pi A|\tau| \leq 0 \end{aligned} \tag{14}$$

and $W = \sqrt{2V} = |\tau|$ satisfies the differential inequality

$$D^+W \leq -\sigma\pi A$$

Invoking the Comparison Lemma [23], it follows that

$$W\big(\tau(t)\big) \leq W\big(\tau(0)\big) - \sigma\pi A$$

Thus the trajectories of (10) reach the manifold $\tau = 0$ in finite time, and because of (14) they cannot escape. Once $\psi_1 + \theta_1 = 0$, (13) becomes

$$\dot{\theta}_1 = -2\pi A \sin\tfrac{\pi\theta_1}{2s} \tag{15}$$

Now when the two vehicles rendezvous $\|\theta_1\| = \|\frac{1}{4}(x_1 - y_1 + x_2 + y_2)\| = o(r)$, and the linearization of (15) at the origin

$$\dot{\theta}_1 = -\frac{\pi^2 A}{s}\theta_1$$

confirms that $\theta_1 \to 0$, which forces (since $\tau = 0$) $\psi_1 \to 0$.

The control objective (11) is thus achieved. ∎

V. Validation

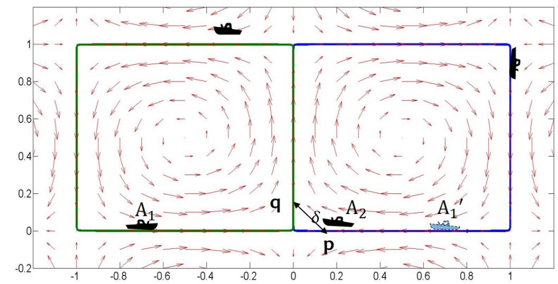

Fig. 6: Invariant orbits $\Phi^{2\times10^{-4}}_{(\frac{1}{2},\frac{1}{2})}$, $\Phi^{2\times10^{-4}}_{(-\frac{1}{2},\frac{1}{2})}$, showing vehicles at locations A_1, and A_2, and the projected vehicle 1 location A_1' on the orbit of 2. A proximity zone is also marked.

Consider vehicle 1 and vehicle 2 drifting on $\Phi^{2\times10^{-4}}_{(1/2,1/2)}$ and $\Phi^{-2\times10^{-4}}_{(-1/2,1/2)}$, respectively. Their initial positions are $(0.85, 1.37\times10^{-4})$ and $(-0.71, 0.8\times10^{-4})$, respectively. The motion of these vehicles on their corresponding orbits can be parameterized by C, and consequently period T_C, as follows:

	C	T_C [sec]	T_i^p [sec]	T_i^q [sec]
A_1'	2×10^{-4}	200.69	16.11	48.22
A_2	2×10^{-4}	200.69	12.61	44.72

Under these conditions, Proposition 1 and Corollary 1 suggest that rendezvous is possible for the two vehicles — Fig. 7 illustrates graphically the inclusion necessary.

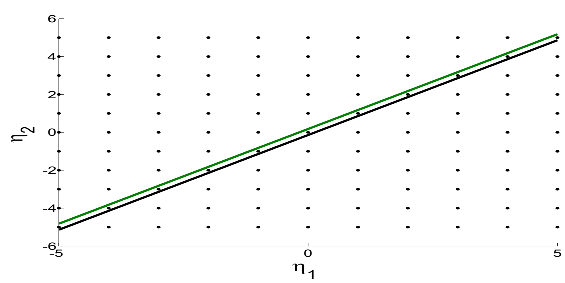

Fig. 7: Convex body of the rendezvous condition for the simulation example within the lattice space.

Once the vehicles do rendezvous, control law (12) is applied, forcing vehicle 1 to regulate its speed relative to vehicle 2, for the brief time window of this encounter. The first rendezvous encounter occurs after 14 seconds and lasts for about a minute (54 seconds, to be exact). The control activity associated with this interaction is seen at the bottom subgraphs of Figs. 8 and 9. The effect of this control action on $x_1 + x_2$ and $y_1 - y_2$ is seen as a transient in the upper subgraphs of these figures, which indicate convergence for both quantities. For the next 147 seconds the vehicles are too far away from each other to interact ($|A_1A_2| > 0.15$). They come into rendezvous again at the 215th simulation second, when another wave of control activity that essentially performs some fine tuning on the controlled quantities. The (smooth) depressions observed in positions around the 200th second in Fig. 8 and around the 150th second in Fig. 9 are due to the velocity distribution along the orbits.

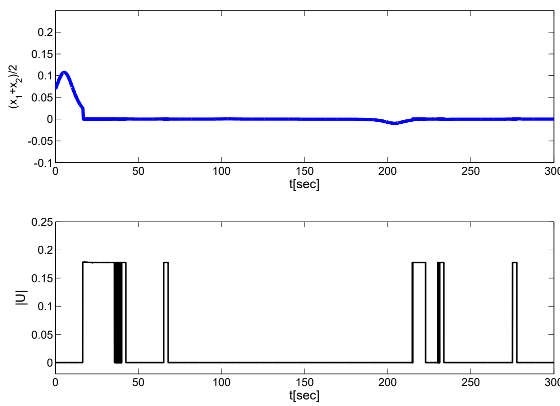

Fig. 8: Time evolution of $\frac{x_1+x_2}{2}$ (up) and the control input $|U| = \sqrt{u_x^2 + u_y^2}$ (bottom).

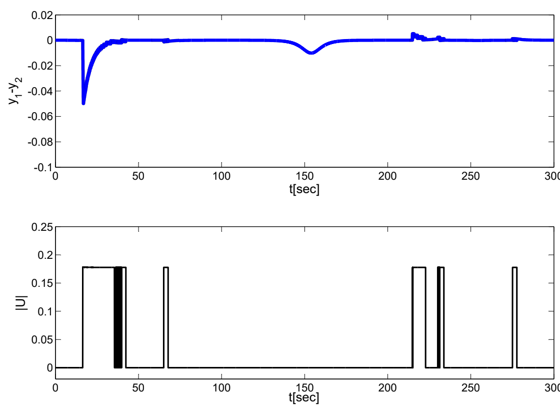

Fig. 9: Time evolution of $y_1 - y_2$ (up); the control input history is repeated from Fig. 8 for easier correlation to position variations.

VI. Conclusion

Mobile sensors drifting along naturally occurring geophysical flows can leverage ambient environmental dynamics to perform monitoring, sensor coverage, or data harvesting tasks, while preserving on board power. In instances where such platforms need to interact with each other or with other vehicles (e.g. for uploading data, updating own parameters, recovery, etc.) but they can only communicate over very short distances (compared to the scale of their motion paths), knowing when and where they will naturally meet can be important. This paper demonstrates that it is possible to predict such encounters under a nonlinear geophysical model with limit cycles, and presents a cooperative control law that can lock the vehicles into periodic and robust rendezvous. Ongoing research involves extending these pairwise synchronization controller to networks of vehicles, over certain communication topologies.

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