GOVERNING EQN FOR DYNAMIC (time-dependent) SOLID-DEFORMATION PROBLEMS

Cauchy's eqn of motion: satisfied by any continuum: solid & fluid.

$$\underbrace{\rho\mathbf{a}}_{acceleration} = \underbrace{\rho\mathbf{g}}_{body\ force} + \underbrace{\nabla\cdot[\tau]}_{surface\ force}$$

FOR LINEAR ELASTIC SOLID UNDER INFINITESIMAL DISPLACEMENT, it is better to use Lagrangian description $(\mathbf{X}, t)$

$$\mathbf{x} = \mathbf{X} + \mathbf{u}$$

If $\frac{\partial u_i}{\partial X_j} \ll 1$, then

$$\frac{\partial[\tau]}{\partial X_j} \approx \frac{\partial[\tau]}{\partial x_j}$$

We can write

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2}|\mathbf{x} = \rho\mathbf{g} + \frac{\partial[\tau_{ij}]}{\partial X_j}$$

with

$$[\tau] = \frac{E}{1+\nu}[\epsilon] + \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{kk}[I]$$

E is elastic modulus, ν is Poisson ratio.
Solved together with proper BC's and initial condition.

THERMAL EFFECTS:

If there is significant temperature change

$$[\tau] = \frac{E}{1+\nu}([\epsilon] - \alpha T[I]) + \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_{kk} - 3\alpha T)[I]$$

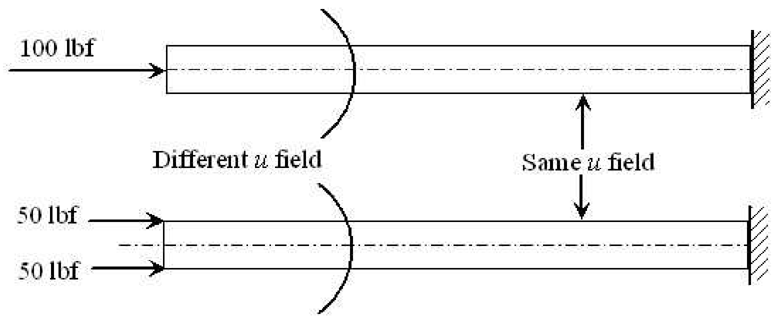
where α: thermal expansion coefficient,
T: temperature relative to initial state temperature.
Then the energy eqn has to be solved also.

$$\rho\frac{d}{dt}(cT) = [\tau] : [\frac{d\epsilon}{dt}] + k\nabla^2 T$$

where c: specific heat,
k: conductivity.
BC's and IC for T have to be specified.

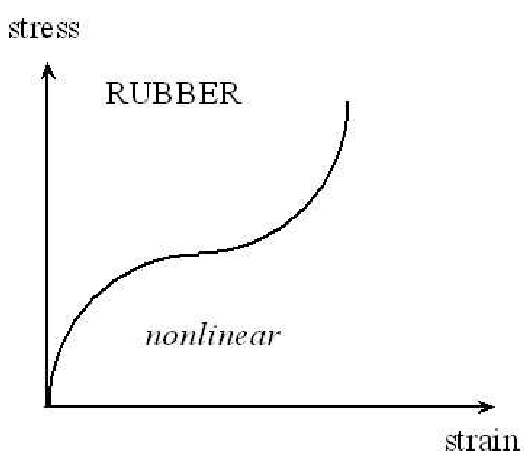
SAINT-VENANT'S PRINCIPLE

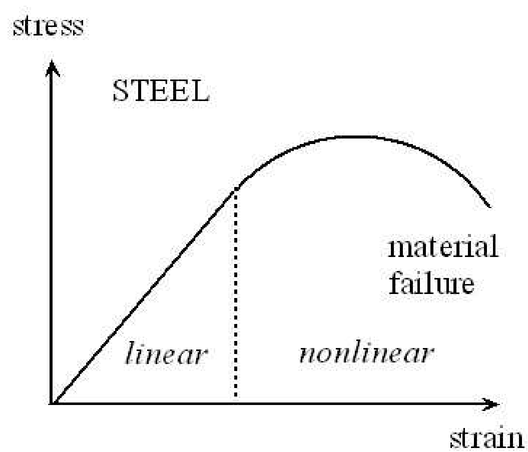
If some distribution of forces acting on a portion of the surface of a body is replaced by a different distribution of forces acting on the same portion of the body, then the effects of the two different distributions on the parts of the body sufficiently far remote from the region of application of the forces are essentially the same, provided that the two distributions of forces have the same resultant force and the same resultant couple.



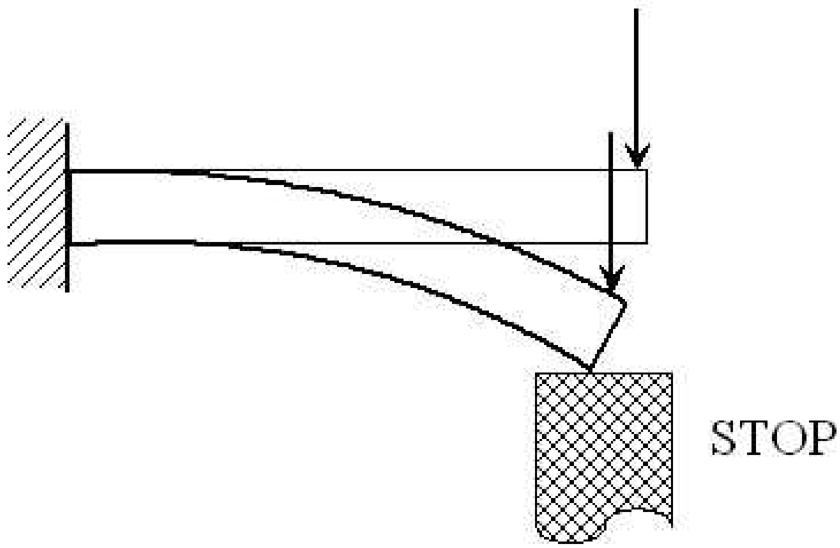
NONLINEARITY IN SOLID MECHANICS

Material Nonlinearity: (nonlinear constitutive relationship / plasticity)





Boundary Nonlinearity: (BC's may change suddenly during analysis)



Geometric Nonlinearity: (large deflections)