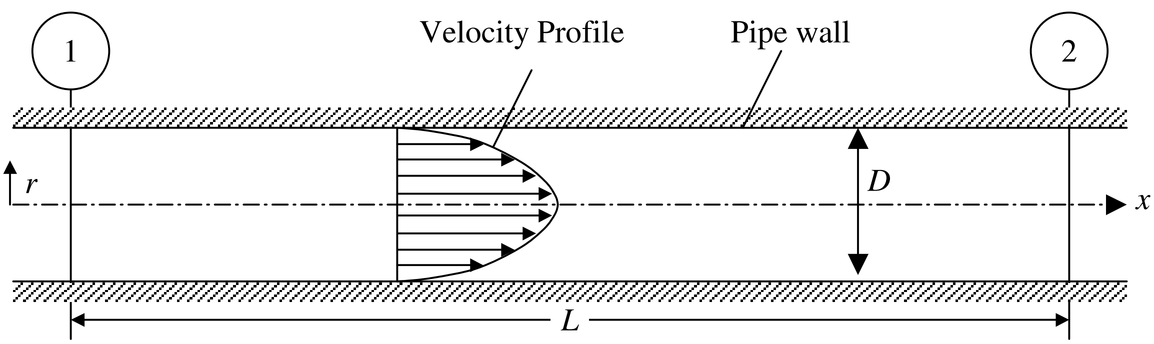
**MEEG102 Introduction to Mechanical Engineering, Spring 2007**
Engineering Simulation and Modeling Laboratory
**Instructors:** Professors Ajay K. Prasad and Lian-Ping Wang

**Problem I. Application of Error Analysis to Pipe Flow**
**Problem II. Computer modeling of Viscous Flow through an Orifice Plate**

**Fluids** (liquids and gases) are typically transported using circular pipes. The fluid could be crude oil (such as an Alaskan oil pipeline), water (such as the water mains supplying water to your house), air (such as the pipes that bring conditioned air into buildings), natural gas (such as the one that supply natural gas to your house), blood (in your arteries and veins), etc.

Although fluid flow is likely to be **unsteady** and **turbulent** in most practical cases, we will consider the simpler case of **steady laminar flow** in which the flow **streamlines** are well ordered and parallel to each other. The fluid immediately next to the wall is stationary due the **no-slip** boundary condition, and the flow at the centerline is the fastest.

**I.** Consider an example of laminar flow in an Alaskan oil pipeline, with the **velocity profile** assuming a shape called the **parabolic** velocity profile.




In the figure above, the pipe diameter is $D$, its length is $L$, and it is transporting a fluid of **viscosity** $\mu$, and **density** $\rho$. The volume **flow rate** is $Q$. The flow experiences frictional drag resulting in a continuous **pressure drop** as one proceeds downstream. The pressure drop from station 1 to 2 is given by $\Delta P = P_1 - P_2$. There is a simple relationship, called the **Hagen-Poiseuille** equation, that governs the pressure drop for laminar pipe flow given $D$, $L$, $Q$ and $\mu$:

$$\Delta P = \frac{128 \mu L Q}{\pi D^4}$$

You are given the following information:

$D = 1.0$ m
$L = 100$ km
$\mu = 1$ kg/(m-s)
$\rho = 800$ kg/m$^3$
$Q = 0.05$ m$^3$/s

1. Compute the **Reynolds number** for this flow. The Reynolds number, denoted by Re, is the single most important parameter for classifying the flow conditions in pipe flow.

$$\mathrm{Re} = \frac{\rho U D}{\mu}$$

   Here, $U$ is the **average flow velocity** given by $Q/A$, where $A$ is the **cross-sectional area** of the pipe.

2. Comment on whether the flow is lamainar or turbulent. (Usually, if Re < 2300, the flow is laminar. The flow becomes turbulent for higher Re.)

3. Determine the pressure drop $\Delta P$ for this pipeline. Express your answer in pascals (1 Pa = 1 N/m$^2$). Convert your answer to psi by noting that 1 psi = 6895 Pa.

4. What size of pump (in horsepower) is required to drive the flow and overcome the pressure drop? Note that Power = $\Delta P * Q$. You can determine power in watts, and convert to HP by noting that 1 HP = 746 W.

5. Assume that the following **uncertainties** apply to the relevant quantities:

   $\Delta D = \pm\, 0.02$ m
   $\Delta L = \pm\, 0.5$ km
   $\Delta \mu = \pm\, 0.03$ kg/(m-s)
   $\Delta \rho = \pm\, 10$ kg/m$^3$
   $\Delta Q = \pm\, 0.002$ m$^3$/s

   Express each of these errors in % terms.
   (For example, the % error in $D$ is ± 0.02/1.0 = ± 2%.)

6. Using the **Propagation of Errors** idea developed in class, determine the overall uncertainty in $\Delta P$ accruing from the uncertainties in each variable. You may start with the following expression:

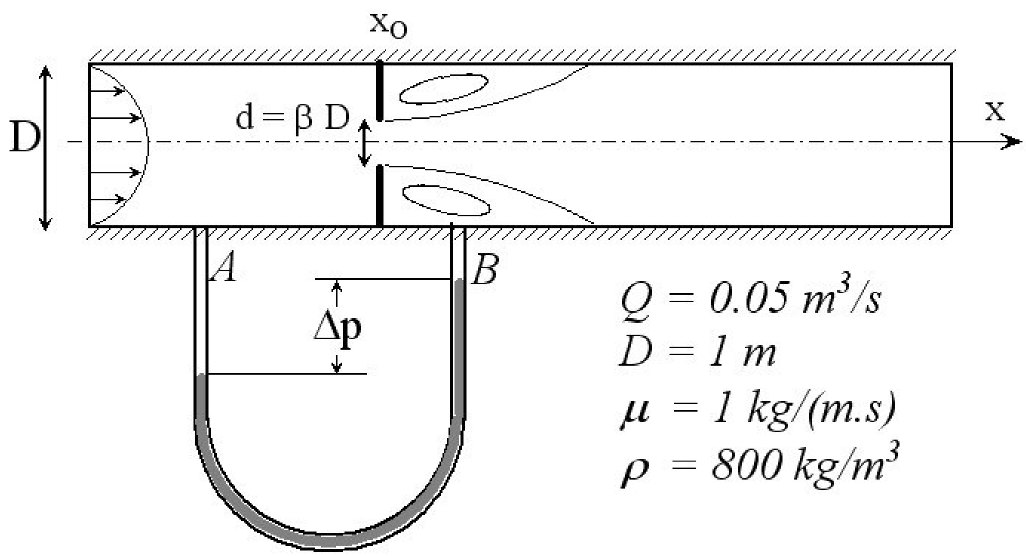
$$\Delta(\Delta P) = \sqrt{\left(\frac{\partial(\Delta P)}{\partial \mu}\Delta\mu\right)^2 + \left(\frac{\partial(\Delta P)}{\partial L}\Delta L\right)^2 + \left(\frac{\partial(\Delta P)}{\partial Q}\Delta Q\right)^2 + \left(\frac{\partial(\Delta P)}{\partial D}\Delta D\right)^2}$$

   Compute each derivative in the above expression, and show that the expression for the **fractional error** can be rewritten in a much simpler form as:

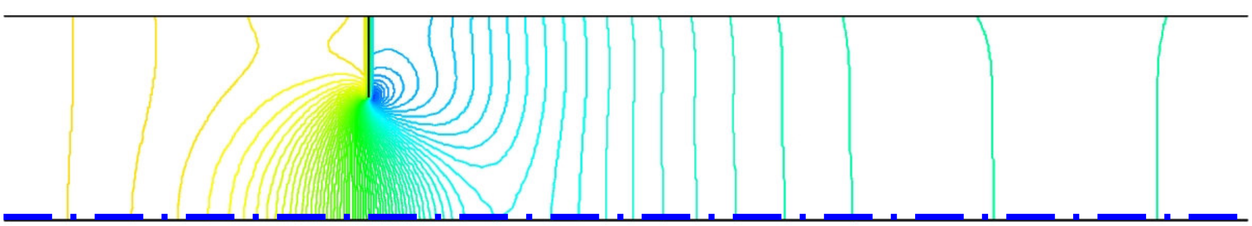
$$\frac{\Delta(\Delta P)}{(\Delta P)} = \sqrt{\left(\frac{\Delta\mu}{\mu}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta Q}{Q}\right)^2 + \left(4\frac{\Delta D}{D}\right)^2}$$

7. Express that error $\Delta\,(\Delta P)/\,\Delta P$ in % terms. If you wanted to reduce the overall error, which variable would you try to measure more accurately *first*?

**II.** An **orifice plate** can be inserted into a pipeline to serve as a flow-rate measurement device. It works through the augmented difference in pressure from the upstream side to the downstream side of the orifice plate. The additional pressure drop due to the orifice depends sensibly on the ratio of the orifice diameter to the pipe diameter, $\beta=d/D$. An example of **pressure distribution** near the orifice is displayed below, showing **rapid pressure drop** through the orifice [Note that only the top half of the pipe is shown due to the **symmetry** of the flow]. You are asked to study how the flow changes due to a thin **concentric** plate of orifice diameter 80%, 60%, and 40% of the pipe diameter, for a given in-coming flow rate. Specifically we are interested in the **pressure difference**, $\Delta P$, between point $A$ and point $B$, where A is one pipe diameter upstream of the orifice and B is $0.5D$ downstream of the orifice.




*Typical pressure distribution with contour interval at 2 Pa for* β=0.6



Solve the flow with Fluent using the four **meshes** provided by your instructor for a straight pipe (the reference case) and pipes with orifice plate at three different $\beta$, and then address the following:

| Model | model0 | model1 | model2 | model3 |
|---|---|---|---|---|
| $\beta$ | 1.0 | 0.8 | 0.6 | 0.4 |

1. First consider the reference model (**model0**) with no orifice present (or $\beta=1$). Determine $\Delta P$ from Fluent model simulation and compare your result with the analytical value based on the equation in part **I**. This value will be referred to as the reference pressure drop $\Delta P_0$.

2. Investigate how ($\Delta P - \Delta P_0$) changes with $\beta$ using the other three Fluent models (**model1**, **model2**, and **model3**). Plot or tabulate your results.

3. Compare fluid pressure, in a same plot, as a function of axial location along **the pipe wall** for different $\beta$. Comment on your results.

4. Compare the **axial velocity profiles** at $x - x_o = -D$, 0, $0.5D$ for the model3 ($\beta=0.4$), in a same plot, and comment on the results. Here $x_o$ is the location of the orifice.

5. If orifice plate is used as a practical flow-rate **measurement device,** what $\beta$ value shall you use? Discuss the **pros and cons** for using too large or too small $\beta$ values.

Submit your finished laboratory report by 5 pm on Friday April 27 to the mailbox of Ms. Stephanie Merkler located in 126 Spencer Lab. Time stamp the front of your report. No late report will be accepted.

Ms. Merkler will be available for office hours between 4 pm - 5 pm on Tuesday and Thursday during the weeks from April 2 to April 27. Her office is 221 Spencer Lab.
E-mail smerkler@gmail.com to meet at additional times.

You may download the error analysis handout from
http://www.me.udel.edu/~prasad/meeg167/error_analysis.pdf

Instructions and notes for Fluent modeling can be found at
http://www.me.udel.edu/~lwang/teaching/MEEG102/orifice.html