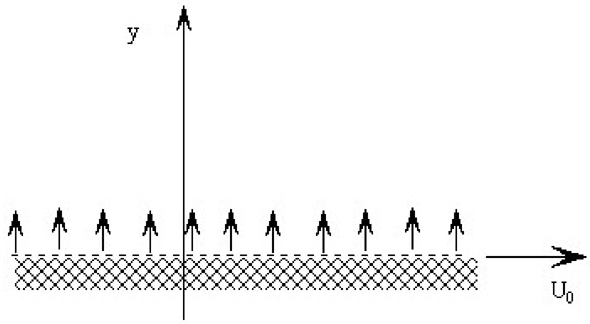
**MEEG 630, Intermediate Fluid Mechanics**

Homework Set #9: Boundary layer solutions

1. A semi-infinite body of a viscous incompressible fluid is bounded by a flat porous plate. At time t=0, the plate is suddenly set in motion at a constant velocity $U_0$ in the x-direction. At the same time injection of fluid through the plate starts. The injection velocity is in the y-direction (normal to the plate). The flow is laminar at all times.




   (a) What is the most general form of any solution for the velocity component v (in the y-direction)? Assume the injection velocity, $V_0(t)$, depends on time only.
   (b) Write the differential equation from which the velocity component $u$ may be obtained.
   (c) Suppose that the partial differential equation in (b) is to be converted into an ordinary differential equation for the single independent variable $h = y/\sqrt{2nt}$ . How must the injection velocity be specified to make this possible?

2. Near the stagnation point on a cylinder in a uniform flow $U_0$, the outer free-stream velocity can be approximately represented by the potential flow solution

$$U(x) = 2U_0 \sin q \simeq \frac{2U_0\, x}{R}$$

Thus, for laminar flow the approximate boundary layer equation near the stagnation point may be written as

$$u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} = 4\frac{U_0^2}{R^2}x + n\frac{\partial^2 u}{\partial y^2}$$

Find a transformation of the form (i.e., determine m, n, A and B)

$$y = B\,x^m F(h), \quad \text{with} \quad h = \frac{Ay}{x^n}$$

that reduces the boundary layer equations to

$$\frac{d^3F}{dh^3} + F\frac{d^2F}{dh^2} - \left(\frac{dF}{dh}\right)^2 + 1 = 0\,.$$

3. A narrow two-dimensional jet issue into an infinite region of an undisturbed fluid of the same kind as the jet, as shown. The flow is laminar everywhere. Some of the undisturbed fluid will now be set in motion because of the shear forces produced by the jet. The region that is significantly affected by the jet is thin compared to the distance along the jet. The equation of motion for the x-direction, therefore, takes the same form as that for a boundary layer on a flat plate. The boundary conditions, however, now are

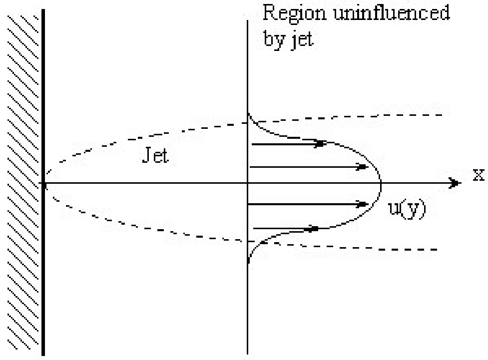
$$at \quad y = 0, \quad \frac{\partial u}{\partial y} = 0,$$

$$at \quad y \to \infty, \quad u = 0$$

It is desired to convert the partial differential equation into an ordinary one by introducing the similarity variable

$$h = \frac{y}{x^m}$$

and letting $y = x^n f(h)$.




(a) Set up an integral for the local x-momentum flux across a vertical plane, and let $h$ be the integration variable. The momentum flux should be the same through all vertical cross sections, as there are no lateral forces acting on the fluid. From this fact, determine the relation between m and n.

(b) Rewrite the differential equation in terms of $f$ and its derivatives and determine the values of m and/or n, so that the resulting differential equation is an ordinary one. Then state the resulting equation.

4. Find the thickness of the boundary layer $d$ and the total drag on one side of a flat plate of length $l$, assuming the boundary layer to be laminar over the whole length. Compare the results obtained by assuming (a) a sinusoidal [ $u/U = \sin\left(\frac{p}{2}\frac{y}{d}\right)$ for $y \le d$ ] and (b) a parabolic [ $u/U = 2\frac{y}{d} - \left(\frac{y}{d}\right)^2$ for $y \le d$ ] profile.