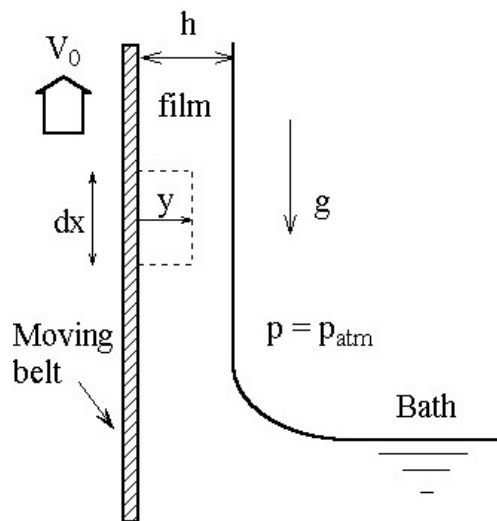


MEEG 630, Intermediate Fluid Mechanics

Homework Set #8: Creeping flows and lubrication approximations

1. A continuous belt passes upward through a chemical bath at velocity V_0 and picks up a liquid film of thickness h , density ρ , and viscosity μ . Gravity tends to make the liquid drain down, but the movement of the belt keeps the fluid from running off completely. Assume that the flow is a well-developed laminar flow with zero pressure gradient, and that the atmosphere produces no shear at the outer surface of the film.

- Set up the equilibrium of forces on a small element of the fluid, one side being bounded by the belt at $y=0$ and the other by the variable y as shown.
- State clearly the boundary conditions at $y=0$ and $y=h$ to be satisfied by the velocity.
- Calculate the velocity profile.
- Determine the rate at which fluid is being dragged up with the belt in terms of μ , ρ , g , h , and V_0 .



2. With the help of the expressions in Appendix B for a spherical coordinate system, show that the stream function for an axisymmetric, incompressible, Stokes flow satisfies equation (64) on page 290 of your text book, namely,

$$\left[\frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right) \right]^2 \psi = 0.$$

(Note that the equation just before (64) is incorrect as I indicated in the class.)

3. Consider a very small, solid spherical particle settling steadily under gravity in an otherwise quiescent viscous fluid. Assume the Stokes solution applies to the flow field around the particle, show that the loss of potential energy of the particle-fluid system is equal to the viscous dissipation in the fluid. What is the percentage of the total dissipation occurring in the region within 10 particle radii from the center of the particle, i.e., $a < r < 10a$?

4. A circular disk of radius a is parallel to and at a small distance h_0 ($\ll a$) from a rigid plane, and the space between them is filled with a very viscous fluid of density ρ and viscosity μ . The pressure at the edge of the disk is equal to atmospheric ambient value. A constant force F (normal to the disk) is now applied to slowly pull the disk away from the plane. How fast will the disk move?