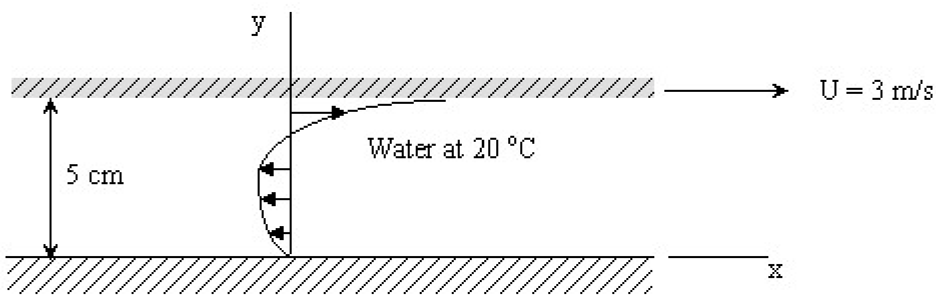
MEEG 630, Intermediate Fluid Mechanics

Homework Set #7: Unidirectional Flows

1. Consider a steady, laminar, unidirectional flow driven by an externally imposed pressure gradient $(\mathrm{d}p/\mathrm{d}x > 0)$ and a flat belt moving at 3 m/s. The bottom wall is fixed. The fluid viscosity is μ = 0.001 kg/(m.s).



a) Solve the velocity distribution for this flow.
b) Calculate the pressure gradient required to cause a zero net flow rate.
c) At what pressure gradient will the shear stress be zero on the bottom wall?

2. Consider the annulus formed between a fixed rod of radius r_0 and a fixed tube of radius r_1.

(a) Find the velocity profile $u_z(r)$ if a pressure gradient $\Delta p / L$ is applied in the direction of the rod axis. Neglect gravity.
(b) Find the average velocity U in the annulus.
(c) Plot $u_z(r)/U$ as a function of $\tilde{r} = \dfrac{r-(r_1+r_0)/2}{(r_1-r_0)/2}$ for $\beta \equiv \dfrac{r_1+r_0}{r_1-r_0}$ =1.2, 2, 4, 8, and 100 (5 curves in one figure). Comment on your plot.

3. Consider the laminar flow of a fluid layer falling down a plane inclined at an angle θ with the horizontal. If h is the thickness of the layer in the fully developed stage, show that the velocity distribution is

$$u = \frac{g \sin\theta}{2\nu}(h^2 - y^2),$$

where the x-axis points along the free surface, and y-axis toward the plane. Also find the volume flow rate and the wall shear stress.

4. Suppose a line vortex of circulation Γ is suddenly introduced into a viscous fluid at rest. Show that the solution is

$$u_\theta = \frac{\Gamma}{2\pi r} e^{-r^2/4\nu t}$$

Sketch the velocity distribution at different times. Calculate and plot the vorticity, and observe how it diffuses outward.

5. Consider the development from rest of a plane Couette flow. The flow is bounded by two rigid boundaries at $y = 0$ and $y = h$, and the motion is started from rest by suddenly accelerating the lower plate to a steady velocity U. The upper plate is held stationary. Notice that similarity solutions cannot exist because of the appearance of the parameter h. Show that the velocity distribution is given by

$$u(y,t) = U\left(1 - \frac{y}{h}\right) - \frac{2\,U}{\pi}\sum_{n=1}^{\infty} \frac{1}{n} \exp(-n^2\pi^2 \frac{\nu t}{h^2}) \sin\frac{n\pi y}{h}$$

Sketch the flow pattern at various times, and observe how the velocity reaches the linear distribution for large times. Hint: You may introduce a new variable

$$q(y,t) \equiv u - U\left(1 - \frac{y}{h}\right)$$

so that $q(y,t)$ satisfies homogeneous BC's. You can then solve $q(y,t)$ by the method of separation of variables.