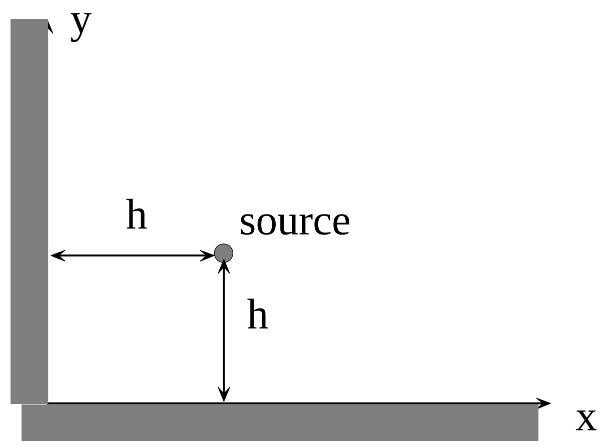
MEEG 630, Intermediate Fluid Mechanics

Homework Set #5: Irrotational Flows

1. A two-dimensional flow is described by a velocity potential $f = x^3 - 3xy^2$. Find the stream function for this flow. Plot streamlines and equal-potential lines for this flow. [Hint: Using polar coordinates, show that $f = r^3 \cos(3q)$ and $y = r^3 \sin(3q)$.]

2. A two-dimensional source of strength m is located in a rectangular corner formed by two infinite walls. The distance of the source from each wall, h, is given. The fluid is inviscid, incompressible, and the flow is steady.



(a) Find the complex velocity potential $w(z)$ for this flow by the method of images;
(b) Determine the velocity distribution along the wall at y=0 as a function of x.

[Hint: $u(x, y=0) = \left.\frac{dw(z)}{dz}\right|_{y=0}$.]

3. By integrating pressure, show that the drag on a plane half body (with the complex velocity potential $w(z) = U\ z + \frac{m}{2p} \ln z$, as discussed in class) is zero.

4. Take a plane source of strength m at point $(-a,\ 0)$, a plane sink of equal strength at $(a,\ 0)$, and superpose a uniform stream U directed along the x-axis. Show that there are two stagnation points located on the x-axis at points

$$\pm a\left(\frac{m}{p\,aU} + 1\right)^{1/2}$$

Show that the streamline passing through the stagnation points is given by $y = 0$. Verify that the line $y = 0$ represents a closed oval-shaped body, whose maximum width h is given by the solution of the equation

$$h = a \cot\left(\frac{p\,U\,h}{m}\right).$$

The body generated by the superposition of a uniform stream and a source-sink pair is called a Rankine body. It becomes a circular cylinder as the source-sink pair approach each other.

5. A two-dimensional potential vortex with clockwise circulation Γ is located at point $(0, a)$ above a flat plate. The plate coincides with the x-axis. A uniform stream U directed along the x-axis flows over the vortex. Sketch the flow pattern and show that it represents the flow over an oval-shaped body. [Hint: Introduce the image vortex and locate the two stagnation points on the x-axis.]

   If the pressure at $x = \pm\infty$ is p_∞, and that below the plate is also p_∞, then show that the pressure at any point on the plate is given by

$$p - p_\infty = \frac{r\Gamma^2 a^2}{2p^2(x^2 + a^2)^2} - \frac{rU\Gamma a}{p(x^2 + a^2)}$$

   Show that the total upward force on the plate is

$$F = \frac{r\,\Gamma^2}{4p\,a} - r\,U\,\Gamma\,.$$