

MEEG 630, Intermediate Fluid Mechanics

Homework Set #4: Vorticity Dynamics

1. A tornado can be idealized as a Rankine vortex with a core of diameter 30 m. The gage pressure ($p - p_{\text{atm}}$) at a radius of 15 m is -2000 N/m^2 .

(a) Show that the circulation around any circuit surrounding the core is $5485 \text{ m}^2/\text{s}$.

(b) Such tornado is moving at a speed of 25 m/s relative to the ground. Find the time required for the gage pressure to drop from -500 N/m^2 to -2000 N/m^2 . Neglect compressibility effects and assume an air temperature of 25°C . [Note that the tornado causes a sudden decrease of the local atmospheric pressure. The damage to structures is often caused by resulting pressure excess on the inside of the walls, which can cause a house to explode.]

2. The velocity field of a flow in cylindrical coordinates (r, θ, z) is

$$u_r = 0, \quad u_\theta = arz, \quad u_z = 0$$

where a is a constant.

(a) Show that the vorticity components are

$$\omega_r = -ar, \quad \omega_\theta = 0, \quad \omega_z = 2az$$

(b) Verify that $\nabla \cdot \vec{\omega} = 0$.

(c) Sketch the streamlines and vorticity lines in an r - z plane. Show that vorticity lines are given by $zr^2 = \text{constant}$.

3. Burgers vortex in cylindrical coordinates has the velocity components as

$$u_r = -ar, \quad u_\theta = \frac{\Gamma}{2\pi r} \left(1 - \exp\left(-\frac{ar^2}{2\nu}\right) \right), \quad u_z = 2az$$

where a and Γ are constant and ν is kinematic viscosity.

(a) Compute the vorticity field for this flow;

(b) Compute each term in the vorticity equation and verify that the vorticity equation is satisfied.

(c) Draw a few vorticity lines and streamlines for this flow.

4. Stuart vortices are an infinite row of vortices that undergo inviscid motion. The flow is steady and two-dimensional and is described by a streamfunction of the form:

$$\psi = \ln(\cosh y + a \cos x)$$

where a is a constant.

(a) Sketch streamlines of this flow for $a = 0$, $a = 0.5$, and $a = 1$, respectively. (you may use a graphics package to do the plots.)

(b) Verify that this flow satisfies the inviscid vorticity equation, namely,

$$\vec{u} \cdot \nabla \omega_z = 0.$$

5. Consider the flow in a 90° angle, confined by the walls at $\theta = 0$ and $\theta = 90^\circ$. Consider a vortex line passing through (x, y) and oriented parallel to the z -axis. Show that the vortex path is given by

$$\frac{1}{x^2} + \frac{1}{y^2} = \text{constant}.$$

Also do your best to sketch the streamlines for the resulting flow field.