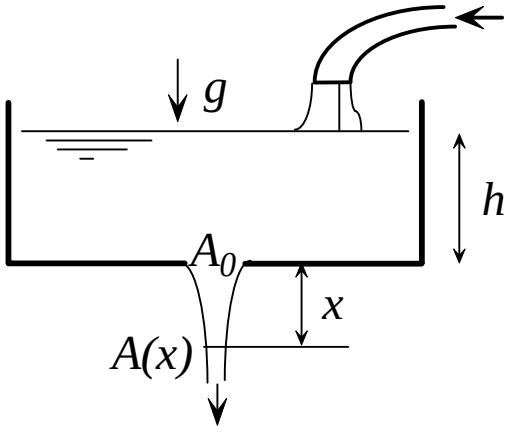
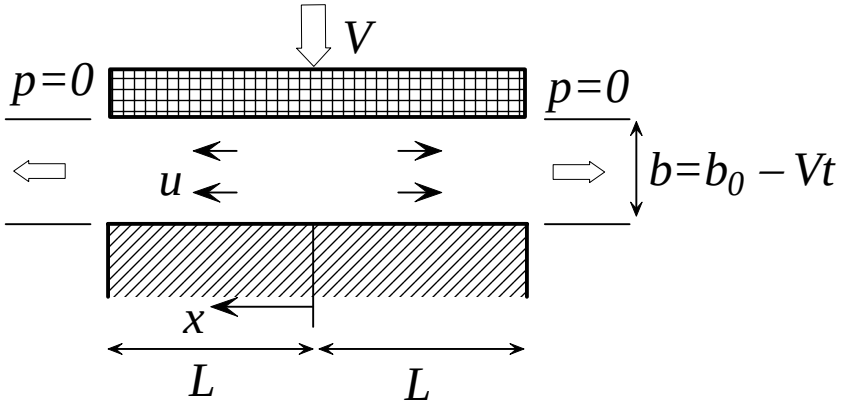
MEEG 630, Intermediate Fluid Mechanics

Homework Set #3

1. A water tank has an orifice at the bottom of the tank. The cross-sectional area of the jet at the point where it leaves the orifice $(\text{at } x = 0)$ is A_0. The height of the water in the tank is h, and this height is kept constant by continuously refilling the tank. The tank cross section is much larger than the orifice area. Neglecting friction, surface tension, and so on. Find the corss-sectional area *A* of the jet as a function of *x.*



2. Two very long parallel plates of length $2L$ are separated by a distance b. The upper plate moves downward at a steady velocity V. A nonviscous and incompressible fluid of density ρ fills the gap between the plates. Fluid is squeezed out between the plates, and since the flow is symmetrical, the velocity parallel to the plate at the center is zero. Assume that $b \ll L$ and that the velocity u parallel to the plates is uniform across the gap but varies with the distance from the center, x, and time t. Treat the flow as being one-dimensional and parallel to the *x*-axis.



(a) Show that the velocity at any point *x* from the center is approximately $u = Vx/b$.

(b) Noting that the gap distance b changes with time and assume that the pressure outside the plates is zero, obtain an expression for the pressure at any point *x* along the plate. Neglect gravity. (Hint: note that the Bernoulli equation for unsteady flow may be written as $\int_a^b \frac{\partial u}{\partial t} ds + \frac{u_b^2}{2} + \frac{p_b}{\rho} = \frac{u_a^2}{2} + \frac{p_a}{\rho}$.)

3. A jet of water with a diameter of 8 cm and a speed of 25 m/s impinges normally on a large stationary flat plate. Find the force required to hold the plate stationary. Compare the average pressure on the plate with the stagnation pressure if the plate is 20 times the area of the jet.

4. Fluid of uniform velocity U_0 and a constant pressure flows over a flat plate. Due to the action of viscosity, the fluid adjacent to the plate is slowed down and at the end of the plate the velocity component parallel to the plate is distributed as $u = U_0 f(y/y_0)$, where y_0 is the distance from the plate at which $u = U_0$. The pressure may be assumed constant. Show that the drag force on the plate per unit width is

$$D = \int_0^{y_0} \rho(U_0 - u)\, u\, dy .$$

5. An incompressible, nonviscous liquid of density ρ and height h flows under the action of gravity through a sluice gate (see the accompanying figure). The depth downstream of the sluice gate is l.

(a) Assume that the flow is steady, estimate the value of upstream velocity V in terms of g, l, h using the Bernoulli equation;

(b) Estimate the force F per unit width in the horizontal direction necessary to hold the gate plate in place in terms of $\rho, g, h,$ and l.