

MEEG 630, Intermediate Fluid Mechanics

Homework Set # 2

1. Let a one-dimensional velocity field be $u = u(x, t)$, with $v = 0$ and $w = 0$. The density varies as $\rho = \rho_0(2 - \cos \omega t)$. Find an expression for $u(x, t)$ if $u(0, t) = U$.

2. The velocity of an incompressible flow in two-dimensional channel is

$$u = u_0 \left[1 - \frac{y^2}{h^2} \right], \quad v = 0, \quad \text{for} \quad -1 \leq \frac{y}{h} \leq 1$$

Find the stress normal and tangential to a plane located at $y/h=0.5$ with its normal at a 30° angle to the flow direction. Assume that the local pressure is p and the fluid viscosity is μ .

3. A two-dimensional viscous flow of incompressible fluid of viscosity μ is given by the stream function

$$\psi = -Axy,$$

where A is a constant.

(a) Sketch the streamlines accurately near the point $x = y = 0$. Take A to be positive.

(b) Assuming that there is no body force, integrate the Navier-Stokes equations for this special case and obtain a relationship between the pressure, A , x , and y , and density ρ .

(c) If the average pressure at the point $x = y = 0$ is p_0 , what is the maximum normal stress magnitude at this point (by considering all possible surface orientations)? What is the maximum shear stress magnitude at this point?

4. A rectangular tank containing water with a free surface on top is placed on wheels and is given a constant horizontal acceleration a . Show that, at steady state, the angle made by the free surface with the horizontal is given by $\tan \theta = a/g$.

5. Consider an incompressible horizontal Couette flow, which is the flow between two horizontal plates separated by a distance b . The upper plate is moving parallel to itself at speed U , and the lower plate is stationary. Let the x -axis lie on the lower plate. All flow fields are independent of x . Show that the pressure distribution is hydrostatic and that the solution of the Navier-Stokes equation is

$$u(y) = \frac{Uy}{b}$$

Write the expressions for the stress and strain tensors, and show that the viscous dissipation per unit volume is $\phi = \mu U^2 / b^2$.

Take a rectangular control volume for which the two horizontal surfaces coincide with the plates and the two vertical surfaces are perpendicular to the flow. Evaluate every term in the mechanical energy equation for this control volume, and show that the balance is between the viscous dissipation and the work done in moving the upper plate.

6. Can we neglect the Coriolis force due to the earth rotation $(-2 \rho \vec{\Omega} \times \vec{u})$ for normal wind tunnel experiments? Explain. Here $\vec{\Omega}$ is the angular velocity of the earth.

How about the case of ocean circulation where the velocity scale is 1m/s and the length scale is 10 km?