MEEG 630, Intermediate Fluid Mechanics

Homework Set #12: Compressible flow

1. Because both the pressure and density increase across a normal shock wave, the wave itself can be viewed as a thermodynamic device that compresses the gas. Show the following relationship between the pressure ratio and the density ratio across a normal shock:

$$\frac{p_2}{p_1} = \frac{\left(\frac{g+1}{g-1}\right)\frac{r_2}{r_1} - 1}{\left(\frac{g+1}{g-1}\right) - \frac{r_2}{r_1}}$$

Plot p_2 as a function of r_2 for air with initial condition $p_1 = 1$ atm and $r_1 = 1.2$ kg/m^3. On the same plot, draw an isentropic compression curve with the same initial conditions. Comment on your results.

2. In an adiabatic flow of air through a duct, the conditions at two points are

$$u_1 = 250\ m/s,\ \ T_1 = 300\ K,\ \ p_1 = 200\,kPa$$
$$u_2 = 300\ m/s,\ \ p_2 = 150\,kPa$$

Show that the loss of stagnation pressure is nearly 34.2 kPa. What is the entropy increase?

3. A shock wave generated by an explosion propagates through a still atmospheric. If the pressure downstream of the shock wave is 700kPa, estimate the shock speed and the flow velocity downstream of the shock.

4. The air in an automobile tire has a pressure and temperature of 345 kPa and 294 K. A hole of 0.05-in diameter is accidentally punched in the tire and assumes the shape of a convergent nozzle. Neglecting frictional effects, find the discharge rate from the hole (the atmospheric pressure is 1 atm). What would the jet velocity and Mach number be if the hole were correctly shaped for an isentropic expansion to atmospheric pressure?

5. A wedge has a half angle of 50^o. Moving through the air, can it ever have an attached shock? What if the half-angle were 40^o?

6. Air at standard atmospheric conditions is flowing over a surface at a Mach number of $M_1 = 2$. At a downstream location, the surface takes a sharp inward turn by an angle of 20^o. Find the wave angle s and the downstream Mach number. Repeat the calculation by using the weak shock assumption and determine its accuracy by comparing with the first method.

7. A flat plate is inclined at 10^o to an air stream moving at $M_\infty = 2$. If the chord length is $b = 3$ m, find the lift and wave drag per unit span.

8. A thin wedge with a blunt base is placed in a uniform supersonic air flow with $M = 2$. When the sharp end of the wedge is forward, the pressure at point A is measured to be 82 kPa, With the wedge turned blunt end forward in the same flow, a detached bow shock is formed, and the pressure recorded at point B is 410 kPa. Find the free stream pressure p_∞ and the wedge angle q.