MEEG 630, Intermediate Fluid Mechanics

Homework Set #11: Turbulent Flows

1. Derive the following transport equation for the turbulent fluxes of temperature, $\overline{u_i' T'}$, using the Navier-Stokes equations and the energy equation under the Boussinesq approximation:

$$\frac{\partial \overline{u_i' T'}}{\partial t} + U_j \frac{\partial \overline{u_i' T'}}{x_j} = \frac{\partial\left(- \overline{u_i' u_j' T'}\right)}{x_j}$$
$$- \overline{u_i' u_j'} \frac{\partial \overline{T}}{x_j} - \overline{T' u_j'} \frac{\partial U_i}{x_j}$$
$$+ g\, a\, \overline{T'^2}\, d_{i3} - \frac{1}{r} \overline{T' \frac{\partial p'}{\partial x_j}}$$
$$+ n\, \overline{T' \frac{\partial^2 u_i'}{\partial x_j \partial x_j}} + \frac{k}{r C_p} \overline{u_i' \frac{\partial^2 T'}{\partial x_j \partial x_j}}$$

Here u_i' and T' are the velocity fluctuations and temperature fluctuations, respectively. Also do your best to interpret each term in the above equation.

2. Measurements in an atmosphere at $20\,^oC$ show an rms vertical velocity of $w_{rms} = 1\ m/s$ and an rms temperature fluctuation of $T_{rms} = 0.1^oC$. If the correlation coefficient between the vertical velocity and temperature fluctuations is 0.5, calculate the vertical turbulent heat flux $r\ C_p\ \overline{w' T'}$.

3. A horizontal smooth pipe 20 cm in diameter carries water at a temperature of $20\,^oC$. The drop of pressure is $dp/dx = 8\,N/m^2$ per meter. Assuming turbulent flow, verify that the thickness of the viscous sublayer is about 0.25 mm.

4. Derive the logrithmic velocity profile for a smooth wall
$$\frac{U}{u_*} = \frac{1}{k} \ln \frac{y\, u_*}{n} + 5.0$$

be starting from $U = \frac{u_*}{k} \ln\ y + \text{constant}$ and matching the profile to the edge of the viscous sublayer at $y = 10.7 n / u_*$.

5. Consider a one-dimensional turbulent dispersion of particles issuing from a point source. Assume a Gaussian Lagrangian correlation function of particle velocity
$$R_L(t) = \exp\left(- \frac{t^2}{t_c^2} \right),$$

where t_c is a constant. What is the integral time scale? Using the Taylor theory, estimate the eddy diffusivity at large times, given that the rms fluctuation velocity is 1m/s and $t_c = 1\ s$.