LETTERS

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Dispersion of particles injected nonuniformly in a mixing layer

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The dispersion of fluid elements distributed nonuniformly at the interface of a two-dimensional, temporally evolving mixing layer was studied. The average transverse dispersion of fluid elements was found to be either enhanced or reduced markedly by simply varying the initial number density distribution along the interface. The increase in dispersion is due to the nonuniform stretching of the interface during the growth of the vortical structure. This naturally leads to a conclusion that the dispersion rate of particles in a spatially evolving mixing layer can be controlled by injecting particles nonuniformly in time.

A mixing layer flow resulting from the merging of two parallel streams of different velocities is known to develop organized or coherent structures in the streamwise direction as a result of the nonlinear instability of the vorticity layer at the interface.[1-3] The typical two-dimensional, large-scale evolution of a mixing layer is the roll-up of the vorticity layer at a fundamental wavelength or frequency and the successive pairings of the newly formed vortical structures due to subharmonic perturbations.[1-3] This process leads to the entrainment of the fluid into the layer from both sides of the free streams and a rapid growth of the mixing layer. The interface in the mixing layer is usually stretched at an exponential rate. As such the mixing of the two fluid streams is accelerated. Meanwhile, the interface is also expanded transversely so that particles initially located at the interface are dispersed and mixed with the entrained fluid. In many engineering applications, such as combustion, we are interested in the dispersion rate and concentration distribution of particles that are injected at a certain point into a mixing layer. Many recent studies have investigated the effect of flow characteristics and particle parameters, such as particle inertia, on the dispersion rate.[4-7] In the previous studies the particles were fed into the flow at a uniform rate. In this work numerical simulations were used to illustrate the effect of nonuniform injection on the dispersion rate of particles in a mixing layer. To the author's knowledge, this potentially useful method has not been reported and discussed previously.

In a numerical study it is much easier to consider a temporally evolving mixing layer (T layer). Much of the related information for a spatially evolving mixing layer (S layer) can be inferred with a Galilean transformation. A two-dimensional, incompressible, viscous, temporally evolving mixing layer was simulated with a finite Reynolds number. The Reynolds number, Re, is based on the free-stream velocity and initial vorticity layer thickness. In this study the initial velocity field was assumed to be

$$u(x,z,t=0)\equiv\tanh(z);\quad w(x,z,t=0)\equiv 0.0. \tag{1}$$

Here x and z are coordinates in the streamwise direction and the transverse direction, u and w are fluid velocity components. The initial layer was perturbed with the eigenfunction of linear instability and the resulting flow was computed by direct numerical simulations on a grid resolution of 64×128 in a rectangular box.[8] The initial perturbation amplitude was 0.01. Only the evolution of a fundamental wave with a wave number $\alpha=0.4446$, i.e., the most rapidly growing linear instability mode, was considered in the simulation. Periodic boundary conditions were assumed in the x direction, and uniform, parallel (stress free) velocity fields were assumed at the top and bottom boundaries. The simulation box had a size of $L=2\pi/\alpha=14.13$ in the x direction and $1.25L$ in the z direction; therefore, subharmonic pairing was excluded from the simulations for simplicity.

Our main interest was in the dispersion of nondiffusive tracer particles or Lagrangian fluid elements in the mixing layer. These particles are simply convected by local fluid velocity. As the flow was advanced, the particle location was found with a fourth-order Adams method[9] and a Hermite interpolation.[10]

Consider the average transverse dispersion of particles that originate at the interface $z=0$ at $t=0$. Let $f(x_0)$ be the initial distribution function of particle number density at $z=0$. To compute the average dispersion accurately and efficiently, we only need to know the function $Z(t|x_0)$, which represents the transverse location at time t of a par-

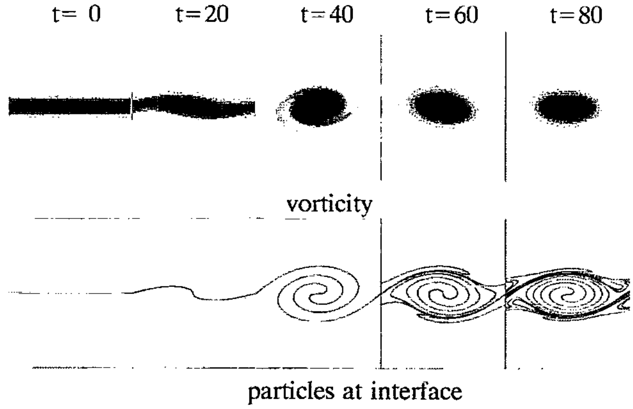

FIG. 1. The evolution of vorticity and interface. Higher gray level represents larger local vorticity value.

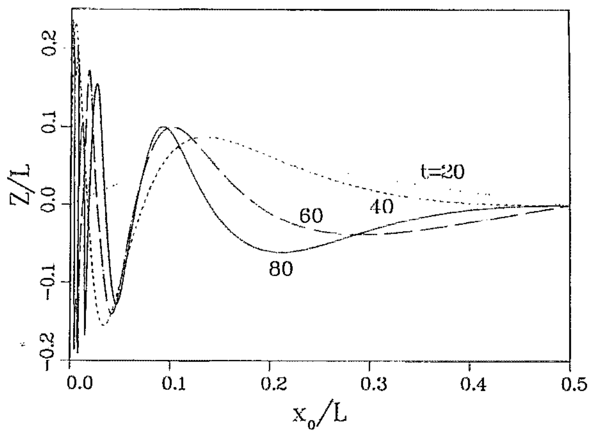

FIG. 2. The transverse location of a particle as a function of x_0.

ticle that was located at $(x_0,0)$ at $t=0$. The average mean-square transverse displacement (MSD) is

$$\langle Z^2(t)\rangle = \int_0^L Z^2(t|x_0)f(x_0)dx_0. \tag{2}$$

Since $Z(t|x_0)$ is antisymmetric about $x_0=L/2$, we only need to know $Z(t|x_0)$ for $0\leqslant x_0\leqslant L/2$. In the simulation, N particles were placed on the left half of the interface with their initial locations given by

$$X^k(0)\equiv x_0^k=\frac{L}{2}\cdot\frac{k-1}{N-1}\cdot\exp\left[-\frac{L}{2}\cdot\left(1-\frac{k-1}{N-1}\right)\right], \quad Z^k(0)=0; \tag{3}$$

where $k=1,2,\ldots,N$. Therefore, this distribution places more particles initially near the edge of the box at $x=z=0$ (the saddle point) as compared to the center $x=L/2$, $z=0$. MSD was then determined by discretizing (2) with a midpoint formula as

$$\langle Z^2(t)\rangle = \sum_{k=1}^{N-1} \frac{[Z(t|x_0^k)+Z(t|x_0^{k+1})]}{2}\cdot f\left(\frac{x_0^k+x_0^{k+1}}{2}\right)\cdot(x_0^{k+1}-x_0^k). \tag{4}$$

It is now clear that the distribution for x_0^k, Eq. (3), determines the way of discretizing Eq. (2) and should not affect the results for MSD, which depend on $f(x_0)$.

Figure 1 shows the vorticity field and the interface at five different times for a typical simulation with Re=300. The roll-up process involves the stretching and tilting of the initial vorticity layer into a spiral and the accumulation of vorticity at the center. After the first complete roll-up at $t\approx 40$, an elliptical core region with high vorticity is formed. The core seems to wobble slightly during its establishment, which is related to the relaxation process known as nutation.[3] The development of the T-layer instability is in agreement with earlier studies.[1–3] The semiheight of the cat's eye defined by the streamlines passing through the saddle points increases rapidly during the roll-up process and reaches its maximum at $t\approx 40$. It oscillates with time afterward. The interface shown in Fig. 1 is actually a plot of 20 000 points representing the locations of 20 000 particles. The locations for half of the particles with initial locations given by Eq. (3) ($N=10\,000$) were computed and those for the other half were plotted using symmetry. If a particle was outside the computation box, its X value was truncated by taking the modulo with respect to L so that the location of the particle could be plotted in the box. The points form continuous curves (visually) for all times because of the choice of the initial distribution, Eq. (3), and because a large number of points were used. In the central region the interface appears as a spiral with more and more turns as time increases. The structure becomes more complicated near the edge of the core region, with many different size lobes appearing at long times. The particles in the lobe region experience larger transverse dispersion as compared to those in the spiral region. These particles originate from a small portion of the interface near the saddle point, as indicated in Fig. 2, where $Z(t|x_0)$ is plotted as a function of x_0 at four different times. The four curves in Fig. 2 have similar structure for most of the x_0, say, $0.05<x_0<0.95$, but oscillate near $x_0=0$ for large times. Namely, $Z(t|x_0)$ is a very sensitive function of x_0 for small x_0 at long times due to continuous stretching of the interface near the saddle point. In fact, since the saddle point is an unstable equilibrium point, a particle initially located near the point, no matter how close, will eventually escape from the local region. The maximum $|Z(t|x_0)|$ occurs at a smaller x_0 as time increases. Overall, the amplitude of oscillation for $Z(t|x_0)$ increases as x_0 decreases for $0\leqslant x_0\leqslant L/2$. Therefore, we see, at least qualitatively, that larger dispersion can be obtained by placing more particles near the saddle points.

We now proceed to compute MSD directly for a given $f(x_0)$. Consider the following modified normal distribution:

$$f\left(y\equiv\frac{x_0}{L}\right)=\frac{1}{\text{erf}(1/2\sqrt{2}\sigma)}\cdot\frac{1}{\sigma\sqrt{2\pi}}\cdot\exp\left(-\frac{y^2}{2\sigma^2}\right), \quad \text{for } 0\leqslant y<0.5;\quad 0, \text{ otherwise.} \tag{5}$$

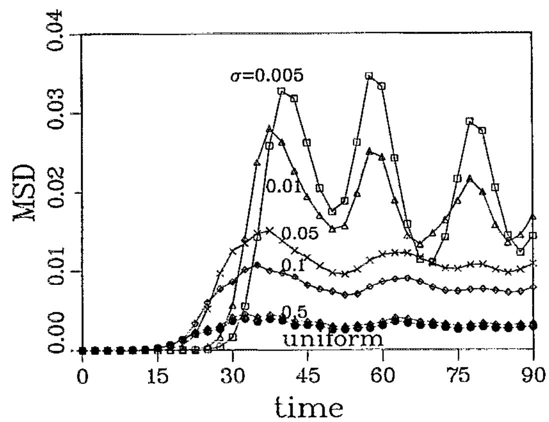

FIG. 3. Average mean-square transverse displacement as a function of time.

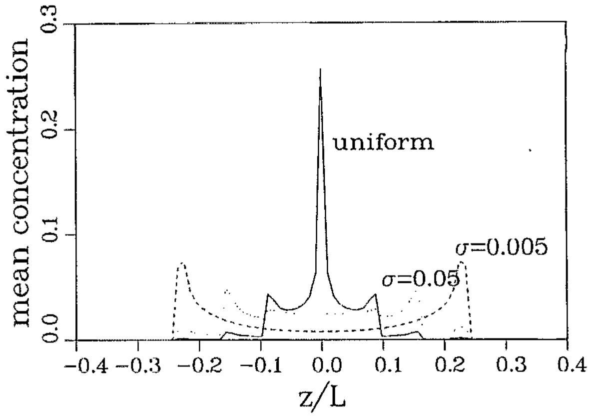

FIG. 4. The particle concentration at time $t=40$ for different σ.

This distribution allows for relatively more particles to be distributed near the saddle points. The parameter σ controls the degree of nonuniformity of the distribution. In Fig. 3 we present the MSD as a function of time for different σ. Clearly the long-time MSD is drastically increased (possibly by a factor of 10) by making $f(x_0)$ more nonuniform.

In addition, the concentration at a particular time was computed by dividing the computation box into many thin horizontal strips of width δ. The x-averaged concentration for a particular strip is defined as the number of particles found in the strip divided by the total number of particles. Figure 4 shows the concentration at $t=40$ for three different σ values. For an initially uniform distribution, the concentration is very nonuniform and has a peak at the center. For small σ, the concentration appears to be quite uniform over a wide range of the core region and the maximum concentration occurs at the edge of the core. It is possible to obtain a uniform concentration distribution extended over the whole mixing layer at a certain time by choosing a proper $f(x_0)$. The concentration fluctuations can also be reduced similarly.

The above findings have important implications to a spatially evolving mixing layer. First, the transverse dispersion of particles in a mixing layer can be easily adjusted by seeding particles nonuniformly in time according to the phase of the evolution of the vortical structure. Second, the method can be used to achieve a more uniform transverse concentration distribution and possibly smaller concentration fluctuations, i.e., a better *mixedness* of the particles. The two implications made here are desirable for many engineering applications, such as a combustion process involving particulate fuel. An improved combustion efficiency may be achieved if the seeding rate of fuel particle is adjusted according to the phase of the driving flows.

Although the subharmonic pairings or three-dimensional features of the layer were not considered in the simulation, the spirit of the method presented can be applied to more complex mixing layer flows. For example, the dispersion of particles in the region where the flow is dominated by subharmonic pairing can be controlled by injecting particles nonuniformly according to the phase of subharmonic waves. In fact, similar results can be expected for other flows with organized structures, such as jets and wake flows, as well as for particles with finite inertia. Numerical tests show that particles injected at other transverse locations relative to the interface can be similarly controlled, but with a phase shift added to the nonuniform distribution. A related study on fluid transport and mixing using material lines, which addresses some of these issues from a different perspective, will be reported in a separate publication.

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