NOTES AND CORRESPONDENCE

Comments on "Droplets to Drops by Turbulent Coagulation"

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Riemer and Wexler (2005, henceforth RW05) have utilized the turbulent collision kernel model of Zhou et al. (2001) to study the effect of turbulence on the initiation and development of raindrops from cloud droplets. They concluded that, for a cloud dissipation rate at 300 cm^2 s^{-3}, turbulent coagulation can move 96% of the droplet mass to sizes over 100 μm in radius in 30 min, as compared to only 7% without turbulence. This result shows that turbulence is capable of rapidly transforming droplets to sizes for which the gravitational coagulation can operate effectively, thus overcoming the size-gap bottleneck for rain initiation. Under the assumption that the turbulent collision kernel of Zhou et al. (2001) can be extrapolated to atmospheric Reynolds numbers, RW05 found that the turbulent coagulation kernel is several orders of magnitude larger than the sedimentation kernel for droplets smaller than 100 μm. We believe that the effects of turbulence have been grossly overestimated in RW05 for reasons to be discussed below.

First, we would like to point out an error in RW05 that led to an overestimation of the rms velocity u' by

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a factor of $\sqrt{3}$ and thus an overestimation of the Taylor microscale Reynolds number R_λ by a factor of 3. RW05's estimations were based on the rms velocity u' and the average cloud dissipation rate ε on the in-cloud measurements by MacPherson and Isaac, shown in Table 1 of MacPherson and Isaac (1977). The cloud turbulence is anisotropic and a rough estimate of u' for equivalent isotropic turbulence would be

$$u' = \sqrt{\frac{u'^2 + v'^2 + w'^2}{3}}, \qquad (1)$$

where u', v', and w' are the rms velocities in the three spatial directions. The data shown in Fig. 1 of RW05 appear to be calculated similarly, but without the factor 3 in the denominator of Eq. (1). Therefore, provided that we keep all the other assumptions made in RW05, the correct value of u' at $\varepsilon = 300$ cm^2 s^{-3} would be $3.5/\sqrt{3} = 2.0$ m s^{-1}. Since the Taylor microscale Reynolds number R_λ is calculated as

$$R_\lambda = u'^2 \sqrt{\frac{15}{\nu\varepsilon}}, \qquad (2)$$

it follows that R_λ is overestimated by a factor of 3. Here, ν is the air kinematic viscosity. One may ask what the consequence of this error is on the modeled value of the turbulent collision kernel. We compare in Fig. 1, the turbulent collision kernels as a function of r_2 for $r_1 = 65$ μm and $\varepsilon = 300$ cm^2 s^{-3}, using $u' = 3.5$ m s^{-1} and $u' = 2.0$ m s^{-1}. Note that all notations follow those of

$K_t(r_1, r_2)$ (cm^3/s)

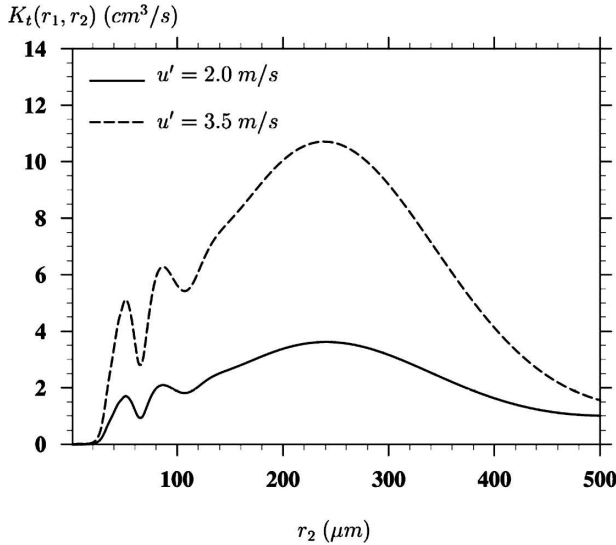
FIG. 1. The turbulent collision kernel $K_t(r_1, r_2)$ as a function of r_2 with $r_1 = 65$ μm and $\varepsilon = 300$ cm^2 s^{-3}.

RW05. The curve for $u' = 3.5$ m s^{-1} corresponds exactly to a horizontal cut at $r = 65$ μm of Fig. 2 in RW05.

We observe that the reduction in the u' value causes roughly a factor of 3 reduction in the turbulent collision kernel for $r_2 < 300$ μm. This is expected as, for this small size range, the relative velocity statistics $\langle |w_r(r_1, r_2)| \rangle$ is insensitive to the change in the energy-containing scales [note that $\tau_p(r = 300$ μm$)/T_e = 0.0086 \ll 1$]. However, the radial distribution function $g_{12}(R)$ is assumed to be linearly proportional to R_λ (Zhou et al. 2001), and thus is reduced by a factor of 3. Since the coagulation growth time scale is inversely proportional to the collision kernel, one would expect that it takes roughly three times longer to grow droplets in the early stage by turbulent coagulation than what was reported in RW05.

We note that other observational studies of cloud turbulence suggest that the typical range of rms velocity should be from 0.5 m s^{-1} to 2.0 m s^{-1} (Weil and Lawson 1993; Smith and Jonas 1995; Lottman et al. 2001; Meischner et al. 2001; Furomoto et al. 2003). Therefore, even a value of $u' = 2$ m s^{-1} is considered to be high in clouds, especially in near-adiabatic regions with high liquid water content and weak turbulence (cf. section 3 of Grabowski and Vaillancourt 1999).

Second, there is an inconsistency in the way the collision efficiency is treated. On the one hand, RW05 assumed a collision efficiency of one in calculating the turbulent collision kernel $K_t(r_1, r_2)$. The sedimentation or gravitational kernel $K_s(r_1, r_2)$ given by Eq. (4) of RW05, however, included the gravitational–hydrodynamical collision efficiency from Hall (1980). Namely, the local aerodynamic interactions between droplets

were considered in the gravitational collisions but not in the turbulent collisions. In the size-gap range from roughly 10 to 60 μm in radii, the gravitational collision efficiency can be significantly less than 1 (Klett and Davis 1973; Hall 1980; Wang et al. 2005a). It is also expected that the turbulent collision efficiency for droplets less than 60 μm deviates significantly from 1. While RW05 incorporated in one test the results from Pinsky et al. (1999), we would like to point out that our own recent work (Wang et al. 2005b; Ayala 2005) and the study of Koziol and Leighton (1996) show a much weaker enhancement of collision efficiency by turbulence. Wang et al. (2005b) recently developed a more rigorous hybrid direct numerical simulation approach in which the disturbance flows due to droplets are treated analytically by an improved superposition method (Wang et al. 2005a) and the undisturbed air turbulence is simulated by a pseudospectral method. They find that the enhancement factor on collision efficiency by turbulence, relative to the gravitational collision efficiency, are typically less than two, and are a factor of 2 to 3 less than the enhancement factors implied by the results of Pinsky et al. (1999). At this stage, without a definite, quantitative parameterization of turbulent collision efficiency, a more consistent treatment would be to assume a similar collision efficiency for both gravitational collisions and turbulent collisions. This inconsistent treatment in RW05 obviously tends to exaggerate the effect of turbulence relative to the gravitational coagulation. This affects directly the early growth through the size gap. The level of exaggeration remains to be studied.

Third, we must point out that the model for the radial distribution function $g_{12}(R)$ developed by Zhou et al. (2001) assumed that the turbulent advection dominates the motion of droplets and that the effect of gravitational settling is not important. In the context of cloud droplets, however, the motion of droplets is governed primarily by gravitational settling (Grabowski and Vaillancourt 1999). To illustrate this, we can express the Stokes number (the ratio of droplet response time to the airflow Kolmogorov time) and the nondimensional settling velocity (the ratio of terminal velocity to the Kolmogorov velocity) as

$$\alpha = \frac{\tau_p(r)}{\tau_k} = \frac{2}{9} \frac{\rho_p}{\rho} \frac{\varepsilon^{0.5}}{\nu^{1.5}} r^2$$

$$\approx 5.49 \times 10^{-4} \times [r(\mu m)]^2, \tag{3}$$

$$S_v = \frac{v_T(r)}{v_k} = \frac{2}{9} \frac{\rho_p}{\rho} \frac{g}{\nu^{1.25} \varepsilon^{0.25}} r^2$$

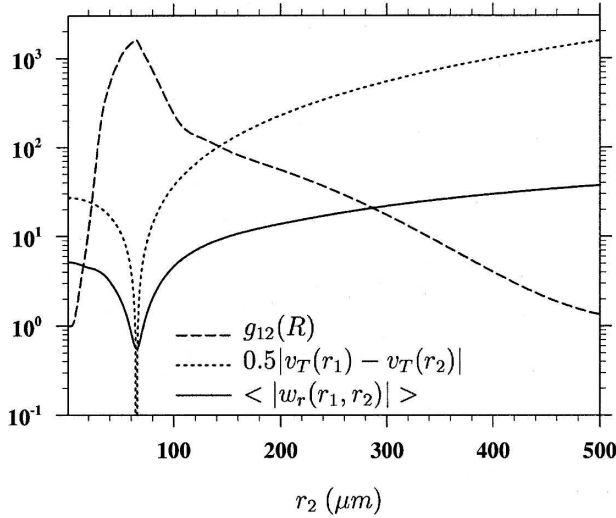$$\approx 4.79 \times 10^{-3} \times [r(\mu m)]^2, \tag{4}$$

FIG. 2. The radial relative velocity (cm s^{-1}) and the radial distribution function for the same case as shown in Fig. 1, based on the model of Zhou et al. (2001). The effective radial relative velocity due to the sedimentation alone is also plotted for comparison.

FIG. 3. DNS results of the radial distribution function for geometric collision of cloud droplets, data taken from Ayala (2005), where $r_1 = 30$ μm, $\varepsilon = 400$ cm^2 s^{-3}, and $R_\lambda = 72.4$. The line represents the predicted value by the model of Zhou et al. (2001) for nonsettling particles.

where we assume $\rho_p = 1$ g cm^{-3}, $\rho = 0.001$ g cm^{-3}, $\nu = 0.17$ cm^2 s^{-1}, $\varepsilon = 300$ cm^3 s^{-2}, and $g = 980$ cm s^{-2}. Here the Stokes drag law is assumed (see below for a discussion of this assumption). The above equations imply that the nondimensional settling rate is always one order of magnitude larger than the Stokes number. For a given dissipation rate, the inertial parameter governing the preferential concentration is not independent of the nondimensional settling. Therefore, as far as the interactions of clouds droplets with small-scale turbulent motion are concerned, the gravity dominates and the turbulent motion is a secondary effect. This is demonstrated by comparing the predicted radial relative velocity due to turbulence with the effective radial relative velocity due to differential settling, as shown in Fig. 2. We note that, in the work of Zhou et al. (2001), the model for $\langle |w_r(r_1, r_2)| \rangle$ is more reliable than that of $g_{12}(R)$. The turbulent radial relative velocity is typically at least one order of magnitude smaller than the effective radial relative velocity due to the sedimentation alone. Therefore, the strong effect of turbulence shown in RW05 relies heavily on the specific model used for the radial distribution function.

There are two issues regarding the extrapolation of the $g_{12}(R)$ model by Zhou et al. (2001) to cloud droplets. The first is the specific assumption of linear dependence on R_λ, which RW05 had discussed extensively. The second issue is the effect of sedimentation on the radial distribution function. In part, RW05 recognized the effect of sedimentation on the preferential concentration of monodisp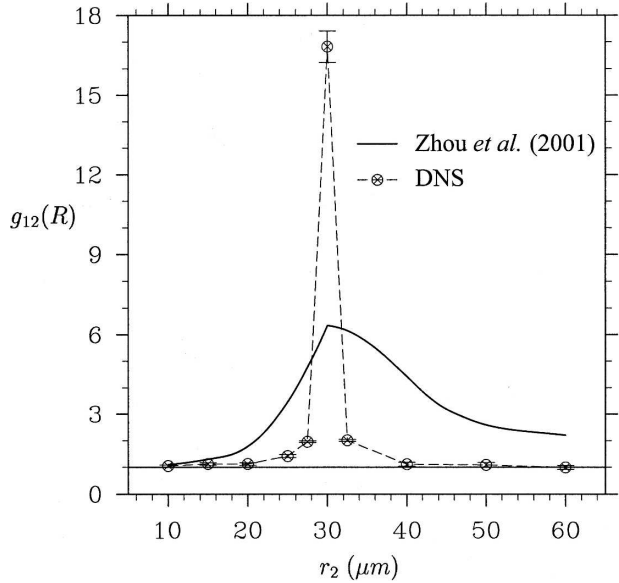erse suspensions (i.e., g_{ii}) by citing the work of Wang and Maxey (1993). The actual effect of sedimentation for the case of moderate α but large S_v is not fully understood.

Furthermore, there is another related effect of sedimentation, namely, the effect of sedimentation on the concentration correlation coefficient ρ_{12}. In our recent study (Wang et al. 2005b; Ayala 2005), we find that the strong sedimentation of cloud droplets can dramatically reduce the correlation coefficient and thus the value of $g_{12}(R)$ for cross-size collisions. An example of the results from our recent direct numerical simulations (DNS) is shown in Fig. 3, which shows that 1) the model of Zhou et al. (2001) may overpredict $g_{12}(R)$ by a factor of 2 to 3 for bidisperse collisions and 2) the effect of preferential concentration on $g_{12}(R)$ diminishes much more quickly as r_2/r_1 deviates from 1. A qualitative physical interpretation of this faster decorrelation due to sedimentation is the reduction of mutual interaction time of the particle pair with a same turbulence eddy. Without sedimentation, the pair can be centrifuged out of a given eddy and be found near the perimeter of the eddy. With strong sedimentation, both droplets in the pair can settle down and leave the eddy, leaving a much less chance for them to be located in a same region of the eddy. This reduction of radial distribution function by sedimentation can have a significant effect on the turbulent collision kernel, although the quantitative assessment remains to be done.

In Fig. 3, the DNS value for the monodisperse radial distribution function is larger than the value from the model prediction of Zhou et al. (2001). This is due to the fact that the DNS calculations of Zhou et al. (2001) assumed $R = \eta$ and that the DNS calculations of Ayala (2005) made use of realistic cloud conditions with $R \ll \eta$. The dependence of the monodisperse radial distribution function on the separation R has been studied by Reade and Collins (2000) and Collins and Keswani (2004). In general, the monodisperse radial distribution function follows power-law scaling on R with a negative exponent and as such increases with decreasing R for $R < \eta$. This aspect causes the underprediction of the monodisperse radial distribution function by the model of Zhou et al. (2001).

We shall also point out that the strong sedimentation of cloud droplets can also alter the value of the radial relative velocity $\langle |w_r(r_1, r_2)| \rangle$. Sedimentation or the cross-trajectory effect decreases the fluctuating motion of droplets due to turbulence or the rms velocity of droplets, but at the same, it also decreases the cross-correlation coefficient of the pair velocities. The net effect of sedimentation on $\langle |w_r(r_1, r_2)| \rangle$ may be relatively weak.

Finally, there is an important assumption in the model of Zhou et al. (2001) that was not discussed in RW05. Zhou et al. (2001) assumed a linear Stokes drag in their DNS calculations, besides the assumption of no gravitational settling and the limitation of low flow Reynolds number. Because of the strong sedimentation of water droplets, the Stokes drag is valid only for droplets less than 30 μm in radius (Pruppacher and Klett 1997). This is also demonstrated in Fig. 4, in which we compare the terminal velocity based on the Stokes drag with the observed terminal velocity based on the standard correlation given in Pruppacher and Klett (1997). Also shown is the terminal velocity based on the following nonlinear drag law (Clift et al. 1978):

$$-6\pi\mu r(\mathbf{v} - \mathbf{u})\left[1 + 0.15\left(\frac{2r\rho|\mathbf{v} - \mathbf{u}|}{\mu}\right)^{0.687} \right], \quad (5)$$

where $\mathbf{u}$ and $\mathbf{v}$ are the airflow velocity and the droplet velocity, respectively. Figure 4 shows that the above nonlinear drag yields a good prediction of the terminal velocity. Also shown in Fig. 4 is the droplet Reynolds number based on the terminal velocity computed with Eq. (5). For droplets larger than 100 μm in radii, the model of Zhou et al. (2001) is not expected to be applicable due to droplet wake and other unsteady effects.

The local turbulent fluctuations may make the Stokes drag assumption questionable even for droplets less
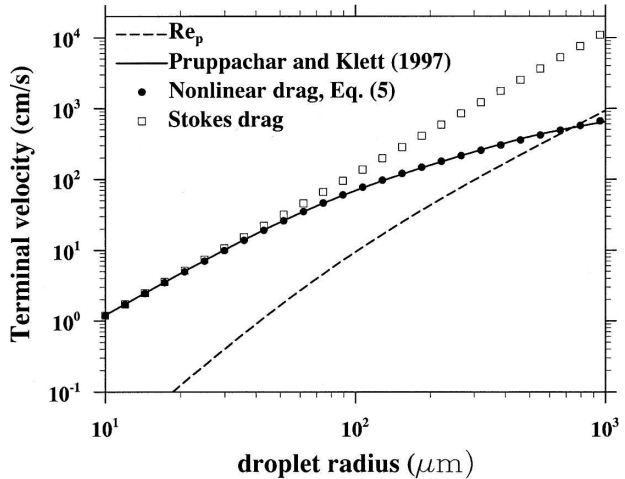

FIG. 4. The still-air terminal velocity v_T as a function of droplet radius, based on the linear Stokes drag, the nonlinear drag, and the standard correlation given in Pruppacher and Klett (1997). Also shown is the droplet Reynolds number based on the nonlinear drag.

than 30 μm in radius. The nonlinear drag reduces the effective Stokes number (e.g., Wang and Maxey 1993) and as such will delay the accumulation effect to larger droplet size. In the work of Ayala (2005), who used the nonlinear drag given by Eq. (5), it was found that nonlinear drag could decrease the turbulent collision kernel for droplets less than 50 μm in radii but increase it for larger droplets.

In summary, we point out three drawbacks in RW05, namely, the overestimation of the rms airflow velocity, the inconsistent treatment of the collision efficiency, and the use of the collision kernel model by Zhou et al. (2001) that was not intended for cloud droplets. All of which, except the power-law scaling of $g_{ii}(R)$ with R, seem to exaggerate the effect of turbulence on a quantitative level. As pointed out in Grabowski and Vaillancourt (1999), the conditions in clouds are very different from the turbulence-dominated case for which most studies on collisions of inertial particles have been carried out, including the work of Zhou et al. (2001) on which the RW05 study was based. In general, we feel that much remains to be done in quantifying the turbulent collision kernel in the context of cloud droplets.

We do, however, agree with the general conclusions of RW05 on a qualitative level; namely, turbulence plays an important role in the initiation and development of rain drops from clouds droplets. We would like to mention that a few recent DNS studies have been designed for the relevant conditions of cloud droplets (Franklin et al. 2005; Wang et al. 2005b,c; Ayala 2005). These studies suggest that 1) turbulence can definitely introduce a moderate enhancement on collision rate

and collision efficiency, and 2) the enhancement is very unevenly distributed, with larger than the average enhancement for $r_2/r_1 \to 1$ and r_2/r_1 (or r_1/r_2) $\to 0$ (Wang et al. 2005b; Ayala 2005). Interestingly, these limiting cases correspond to either the case when the gravitational mechanism is weak or the case that the collision efficiency in the absence of turbulence is very low. Therefore, it may be possible that turbulence can overcome the size-gap bottleneck by a moderate increase of collision kernel on average, supplemented by stronger effects at just the right places. Here the wording "moderate enhancement" means an average enhancement less than a factor of 10 and much less than what were shown in RW05. Turbulence may shorten the growth time for drizzle formation by a factor of 2 to 3, relative to the gravitational coagulation alone, at moderate flow dissipation rate of 400 cm^2 s^{-3} (Wang et al. 2005c). It is somewhat premature at this stage to assess the effects of turbulence quantitatively, without a well-established turbulence kernel applicable to cloud conditions.

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