Simulation of interactions between microbubbles and turbulent flows

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Microbubbles formed by small air bubbles in water are characterized as spherical inclusions that are essentially rigid due to the effects of surfactants, and respond to the action of drag forces and added-mass effects from the motion relative to the surrounding fluid. Direct numerical simulations of homogeneous, isotropic turbulence are used to study the effects of the small-scale, dissipation range turbulence on microbubble transport and in particular the average rise velocity of microbubbles. It is found that microbubbles rise significantly more slowly than in still fluid even in the absence of a mean flow, due to a strong interaction with the small-scale vorticity. The way in which microbubbles might modify the underlying turbulence by the variations in their local distribution is discussed for dilute, dispersed systems and some estimates for the enhanced viscous dissipation given.

CONTENTS

INTRODUCTION

Small microbubbles, of diameter 1/2 mm or less, in sea water are generated by a variety of mechanisms such as air entrainment by breaking waves (Thorpe, 1982) turbulence near the free surface or plunging jets (Bonetto and Lahey, 1993). The larger bubbles will tend to rise quickly to the surface leaving a persistent population of smaller microbubbles to interact with the turbulence. The effects of surfactants, present in all but the most carefully controlled flows, is to freeze the surface of microbubble and make it respond as a rigid spherical inclusion. This is illustrated by the experimental data of Detsch (1991) on bubble rise velocities. In these and other turbulent flows containing microbubbles the questions of interest are how the turbulence acts to modify the bubble transport and what is the dynamical interaction between the bubble phase and the flow. Previous work for solid particles in air, such as Squires and Eaton (1991), has shown that the large scale stirring motions in a turbulent flow dominate the dispersion process. However the small scale turbulence has a dominant effect on the relative motion of the particles to the flow, the instantaneous local concentration of the particles and the average settling velocity (Wang and Maxey, 1993). The strongest effects were observed when the terminal fall speed and the inertial response time of the particle were close to the corresponding Kolmogorov velocity and time scales, which characterize the dissipation range.

The question of turbulence modulation is important and has been the focus of much work in the development

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of two-fluid continuum models, such as Elghobashi and Abou-Arab (1983) and Simonin and He (1992). Only recently has it been possible even in a limited sense to perform direct numerical simulations of particle-laden turbulence and the work by Squires and Eaton (1990), Elghobashi and Truesdell (1993) and others has provided new information on which to assess the turbulence models. Experiments by Lance and Bataille (1991) for moderate-sized bubbles in homogeneous turbulence have shown that at low volume void fractions the turbulence is damped by the bubble phase but that turbulence levels may increase as the void fraction is increased. The survey by Gore and Crowe (1991) has shown that a range of effects may be observed depending on the particle or bubble size as well as the void fraction. Clearly there are many interesting physical processes involved which are as yet poorly understood. A major limitation to the development of a theory for bubble-laden flows is a clear microphysical description.

In this paper we present the results of some direct numerical simulations of homogeneous turbulence and the passive transport of microbubbles by the turbulence. The turbulence is shown to reduce significantly the average rise velocity of the microbubbles due to their interactions with the turbulence.

EQUATIONS FOR BUBBLE MOTION

Each microbubble of radius a is represented as a rigid, spherical inclusion due to the influence of surface tension and sufactants in the flow. The position $\mathbf{Y}(t)$ and the velocity $\mathbf{V}(t)$ for the center of a microbubble is obtained by solving the equation of motion for a small, massless particle.

$$0 = -m_F\mathbf{g} + m_F D\mathbf{u}/Dt + \frac{1}{2}m_F\left(D\mathbf{u}/Dt - d\mathbf{V}/dt\right) + 6\pi a\mu(\mathbf{u}(\mathbf{Y}(t),t) - \mathbf{V}(t)). \qquad (1)$$

Here m_F is the mass of the fluid displaced by the spherical bubble, $\mathbf{u}(\mathbf{x},t)$ is the ambient flow field, μ is the dynamic viscosity and $\mathbf{g}$ is the acceleration due to gravity. This equation is based on that for a small sphere given by Maxey and Riley (1983) where the sphere is small compared to the scales on which the flow varies and the relative motion of the bubble to the flow can be approximated as a low Reynolds number unsteady motion. In writing (1) the added-mass term is written in the form given by Auton et al. (1988) using the local Lagrangian fluid acceleration. This equation accounts for the buoyancy of the bubble, the force of the ambient flow itself, the added-mass effects and a quasi-steady viscous drag force due to the relative motion. In this study other effects such as a history term for the unsteady viscous diffusion of vorticity around the sphere are not included so as to focus attention on some of the essential processes.

Equation (1) may be rewritten as

$$d\mathbf{V}/dt = 3D\mathbf{u}/Dt + (\mathbf{u}(\mathbf{Y}(t),t) - \mathbf{V}(t) + \mathbf{Q})/\tau_B \qquad (2)$$

where

$$\tau_B = m_F/12\pi a\mu, \qquad (3)$$

$$\mathbf{Q} = -2\tau_B\mathbf{g}, \qquad (4)$$

are respectively the inertial response time of the bubble and the terminal rise velocity for still fluid. These two parameters characterize the bubble response. A similar equation to (2) would have results from considering a surfactant-free bubble at higher Reynolds numbers where Levich's estimate for the drag force is $12\pi a\mu(\mathbf{u}(\mathbf{Y}(t),t) - \mathbf{V}(t))$.

The added-mass concept has been found to be accurate even in viscous flows at finite Reynolds numbers according to the direct numerical simulations of Rivero et al. (1991) and Chang and Maxey (1994). The study by Auton et al. (1988) showed that for inviscid flows the action of vorticity in the flow would generate a lift force proportional to $m_F\boldsymbol{\omega} \times (\mathbf{u}(\mathbf{Y}(t),t) - \mathbf{V}(t))$, where $\boldsymbol{\omega}$ is the fluid vorticity. There has been substantial discussion of this in the literature for both viscous and inviscid flows, but recent experiments for small bubbles by Sridhar and Katz (1993) suggest that the standard formulae give substantial overestimates. We exclude the effects of lift forces.

FLOW SIMULATION

The flow $\mathbf{u}(\mathbf{x},t)$ is generated by a direct numerical simulation of homogeneous, isotropic turbulence following procedures described in Ruetsch and Maxey (1991) and Wang and Maxey (1993). The flow is represented by a Fourier series on a periodic domain of side 2π, and the equations of motion

$$\rho D\mathbf{u}/Dt = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f}(\mathbf{x},t), \qquad (5)$$

$$\nabla \cdot \mathbf{u} = 0, \qquad (6)$$

are solved by a pseudo-spectral collocation method. The pressure and fluid density are p and ρ respectively. A random, statistically stationary forcing $\mathbf{f}$ is applied to the flow field at low wavenumbers to maintain the turbulence and balance on average the viscous dissipation. The flow evolution is computed by applying an Adams-Bashforth (second order) scheme to the nonlinear terms and a Crank-Nicholson scheme for the linear terms, the pressure is eliminated through the condition (6) of incompressible flow.

The flow is computed on a grid of 96^3 points with a sustained value of $Re_\lambda = 62$ based on the Taylor microscale λ. Numerically the values of the rms velocity fluctuation $u_0 = 18.0$, $\lambda = 0.474$, the Kolmogorov velocity scale $v_K = 4.73$, the Kolmogorov time scale $\tau_K = 0.00622$, the Kolmogorov length scale $\eta_K = 0.293$ and the ratio of the eddy turnover time scale T_E to τ_K is approximately 16.

MICROBUBBLE TRANSPORT

Once the flow had reached a statistically stationary level, driven by the random forcing, the microbubbles were introduced into the flow at random initial positions with an initial velocity equal to **Q**. The bubble motion was computed by solving (2) numerically with a 4th order Adams-Bashforth scheme and the local fluid velocity **u** or acceleration $D\mathbf{u}/Dt$ were interpolated from their values at the grid points by a partial Hermite spline scheme (Balachandar and Maxey, 1989). The bubbles have no dynamical effect on the flow and simply respond to the local flow conditions. The motion of each bubble may be tracked and from this data quantities such as the mean bubble rise velocity $< \mathbf{V} >$ derived. A combination of ensemble averaging over the bubbles released in each group, time averaging over the later stages of the bubble trajectory, and averaging over the six different groups for different orientations of gravity relative to the turbulent flow was used to obtain the present data.

In Fig. 1 the values of the change in the average velocity $(< \mathbf{V} > -\mathbf{Q})$ are shown. For the results shown $Q/\nu_K = 1.0$ and the ratio of the timescales τ_B/τ_K varies from 0.2 to 1.8. A bubble with a very rapid response, $\tau_B/\tau_K \ll 1$, will respond immediately to the local flow conditions with velocity $\mathbf{V}(t)$ approximately equal to $(\mathbf{u}(\mathbf{Y}(t), t) + \mathbf{Q})$ at every instant. As the ambient flow is incompressible with zero mean flow the average value of $< \mathbf{u}(\mathbf{Y}(t), t) >$ is zero. So for $\tau_B/\tau_K = 0$ we expect $(< \mathbf{V} > -\mathbf{Q})$ to be zero and the average rise velocity to be the same as in still fluid. As the response time τ_B/τ_K becomes larger there is a consistent decrease in the average rise velocity, so that at $\tau_B/\tau_K = 1.0$ there is a 40% decrease in the average rise velocity. This in contrast to the increase found for the heavy particles in air flows by Wang and Maxey (1993). Another contrast is that over the range of τ_B/τ_K tried there is no indication of a return to $< \mathbf{V} >= \mathbf{Q}$ as τ_B/τ_K increases further. Unlike the heavy particles their are two contributing factors to the change in $< \mathbf{V} >$, which from (2) must satisfy

$$< \mathbf{V} >= \mathbf{Q}+ < \mathbf{u}(\mathbf{Y}(t), t) > +3\tau_B < D\mathbf{u}/Dt > \quad (7)$$

in the long term. Both of the last two terms in (7) are non-zero when averaged following a bubble trajectory and the latter will grow in significance as τ_B increases. This is evident from the results in Fig. 2 which show that $< \mathbf{u}(\mathbf{Y}(t), t) >$ is the dominant factor at low values of τ_B/τ_K.

The rms fluctuations in the bubble velocity relative to the local fluid velocity $< (\mathbf{V} - \mathbf{u})'^2 >^{0.5}$ are shown in Fig. 3; these are computed by subtracting the mean value $< \mathbf{V}(t) - \mathbf{u}(\mathbf{Y}(t), t) >$ from the instantaneous values. Again the level of these velocity fluctuations increases with increases in τ_B/τ_K. These fluctuations are discussed further in the next section.

One of the most noticeable features from these simulations is the very strong local accumulations of microbubbles that develop in response to the instantaneous vorticity distribution. Wang and Maxey (1993) reported simulations similar to those here on a 48^3 grid and for $Re_\lambda = 31$ but with much higher seeding levels, sufficient to estimate the local concentration of microbubbles in the

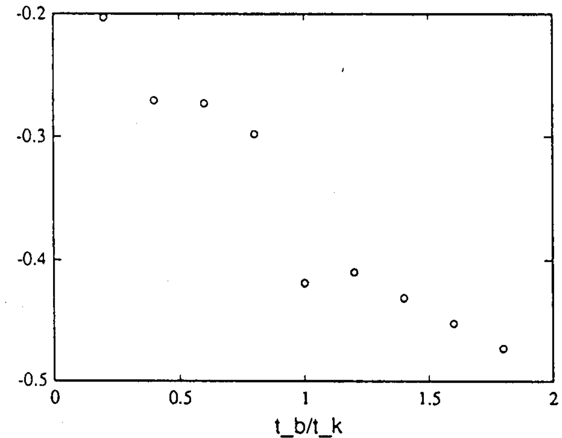

Fig. 1: Change in mean bubble rise velocity $< V - Q >$ $/v_K$ against ratio τ_B/τ_K, for $Q/v_K = 1$.

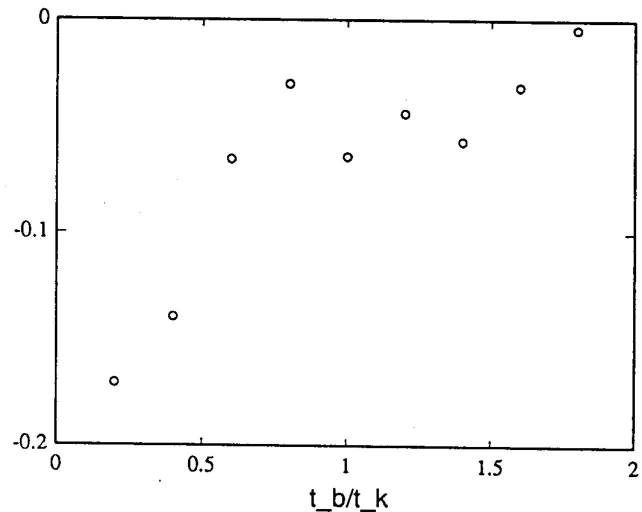

Fig. 2: Bubble mean, $< u(\mathbf{Y}, t) > /v_K$ as time ratio τ_B/τ_K varies.

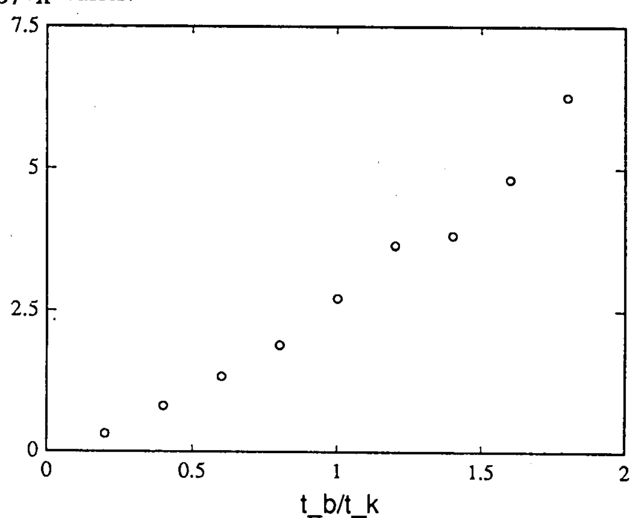

Fig. 3: Rms relative fluctuation $< (\mathbf{V} - \mathbf{u})'^2/v_K$ scaled by v_K as τ_B/τ_K varies.

flow. Most of the bubbles for $\tau_B/\tau_K = 1$ and $Q/v_K = 1$ were found to have collected in the strong vorticity structures that characteristically appear as tubes or distorted sheets in the flow. There were very high correlations between the local concentration, the vorticity level and local minima in the pressure distribution. These correlations were much higher than the corresponding correlation of heavy particles with the strain-rate distribution. Some 90% of the grid points had no bubbles and only 18% of the bubbles were at locations where the concentration $C \leq 10$. A completely uniform distribution of bubbles would correspond to an average concentration of 1.185.

FLOW MODIFICATION

The implication of discussing flow modification is that the volume fraction occupied by the dispersed bubble phase is small and that it makes a small change to the underlying flow, which in some sense closely resembles the original turbulent flow without microbubbles. So we shall restrict attention to very dilute systems with low volume void fractions. This viewpoint is consistent with the way the microbubble transport problem was formulated in the previous sections. The equation of motion (1) for a small spherical particle embedded in an ambient flow $\mathbf{u}(\mathbf{x},t)$ is obtained by analyzing the localized disturbance flow that forms around the particle due to its presence and motion relative to the local flow (Maxey and Riley, 1983). Taking the particle in isolation, the complete flow $\tilde{\mathbf{u}}(\mathbf{x},t)$ is the sum of the ambient flow $\mathbf{u}(\mathbf{x},t)$ and a disturbance for $\mathbf{u}'(\mathbf{x},t)$ such that

$$\tilde{\mathbf{u}}(\mathbf{x},t) \to \mathbf{u}(\mathbf{x},t) \text{ for } |\mathbf{x}-\mathbf{Y}(t)| \gg a \quad (8)$$

and

$$\tilde{\mathbf{u}}(\mathbf{x},t) = \mathbf{V}(t) \text{ on } |\mathbf{x}-\mathbf{Y}(t)| = a, \quad (9)$$

neglecting for the moment the issue of particle rotation. The stress tensor $\tilde{\sigma}_{ij}$ then has contributions σ_{ij} from the ambient flow and σ'_{ij} from the disturbance flow such that the resultant fluid force on the particle or bubble is $\tilde{F}_i = F_i + F'_i$,

$$F_i = \oint_s \sigma_{ij} n_j \, ds \quad (10)$$

$$F'_i = \oint_s \sigma'_{ij} n_j \, ds \quad (11)$$

The ambient flow is defined everywhere, including the interior of the sphere, and $\mathbf{F}$ may be expressed as a volume integral of $\partial\sigma_{ij}/\partial x_j$ over the interior of the sphere. So if $\mathbf{u}(\mathbf{x},t)$ is governed by

$$\rho D\mathbf{u}/Dt = -\nabla p + \mu\nabla^2\mathbf{u} + \rho\mathbf{g}, \quad (12)$$

$\mathbf{F}$ is to first approximation

$$\mathbf{F} = -m_F\mathbf{g} + m_F D\mathbf{u}/Dt. \quad (13)$$

The evaluation of $\mathbf{F}'$ depends on the conditions satisfied by the disturbance flow, whether it is a Stokes flow or not, etc. Within the present context (1) this force is specified as

$$\mathbf{F}' = \frac{1}{2}m_F\left(\frac{D\mathbf{u}}{Dt} - \frac{d\mathbf{V}}{dt}\right) + 6\pi a\mu(\mathbf{u}(\mathbf{Y},t) - \mathbf{V}(t)) \quad (14)$$

which is dependent only on the relative motion of the particle through the fluid.

The modification, or first correction, to the ambient flow requires the inclusion of a point-force term $\mathbf{F}'\delta(\mathbf{x}-\mathbf{Y}(t))$ to conserve momentum

$$\rho D\mathbf{u}/Dt = -\nabla p + \mu\nabla^2\mathbf{u} + \rho\mathbf{g} - \mathbf{F}'\delta(\mathbf{x}-\mathbf{Y}(t)). \quad (15)$$

This point-force term may be viewed as the far field effect, at large distances from the spherical particle compared to its radius, that is transmitted by the disturbance flow from the particle surface to the ambient flow. This is for example the case for a steady Stokes flow. The term in $\mathbf{F}$ is not included since this is already specified by the ambient flow itself. This process may be repeated for each particle or bubble in turn leading to a sum over different point-force corrections. The ambient flow $\mathbf{u}$ will be modified by each of these particles so that while they may not interact directly through their localized disturbance flows, they may do so in a cumulative sense via the modified ambient flow.

For a microbubble of zero mass the net force is zero and the two contributions $(\mathbf{F}+\mathbf{F}')$ sum to zero. The feedback to the ambient flow on the right of (14) is thus a term $+\mathbf{F}\delta(\mathbf{x}-\mathbf{Y}(t))$ given by (12). The feedback consists of a localized buoyancy due to the presence of the bubble and an acceleration term $m_F D\mathbf{u}/Dt$ representing the fact there is a spherical inclusion in the ambient flow.

It is helpful to consider the kinetic energy budget in understanding the dynamial interactions. On the surface the rate of working by the flow on the particle is $\mathbf{V}\cdot(\mathbf{F}+\mathbf{F}')$ from (10)–(11). In general for a particle of mass m_p this would go into the change of kinetic energy $1/2m_p\mathbf{V}^2$ and potential energy $-m_p\mathbf{g}\cdot\mathbf{x}$ of the particle. Correspondingly from (15) the kinetic energy of the ambient flow is changed by the rate of working $-\mathbf{u}(\mathbf{Y}(t),t)\cdot\mathbf{F}'$, taking into account the physical meaning of the delta function. The sum of these rates of working shows there is a net change in the energy. The contribution from $\{-(\mathbf{u}-\mathbf{V})\cdot\mathbf{F}'\}$ gives a net transfer of energy into the kinetic energy of the disturbance flow motion, represented here by the added-mass effect, aand to balance the rate of viscous dissipation of kinetic energy in the disturbance flow. The term in $\mathbf{V}\cdot\mathbf{F}$ is the rate of working due to buoyancy and involves the relative acceleration $(-\mathbf{g}+D\mathbf{u}/Dt)$. It must be remembered that buoyancy is transmitted to the bubble by surface stresses, and for example a fluid in free fall would exert no buoyancy force.

The viscous dissipation associated with the Stokes drag is irreversible and represents an enhanced dissipation for the turbulence ϵ_B due to the bubble phase. For a single bubble an estimte of the dissipation can be found form the data on $< (\mathbf{V}-\mathbf{u})'^2 >$ shown in Fig. 3. If the number of bubbles in a volume $(\eta_K)^3$ is n, locally the ratio of ϵ_B/ϵ is

$$\epsilon_B/\epsilon = 6\pi n(a/\eta_K) < (\mathbf{V}-\mathbf{u})'^2 > /v_K^2. \quad (16)$$

From the data this may be substantial especially in view of the locally strong accumulations of microbubbles that may occur. The value of (a/η_K) is directly related to τ_B/τ_K through (3) and is $3(\tau_B/\tau_K)^{1/2}$. Obviously our interest is in bubbles where $\tau_B/\tau_K \leq 1$, but this fact also illustrates the importance of bubble size.

The results and ideas here are by no means the last word but we hope at least clarify some important issues.

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