Brief Communication

Subgrid scale fluid velocity timescales seen by inertial particles in large-eddy simulation of particle-laden turbulence

Guodong Jin\textsuperscript{a}, Guo-Wei He\textsuperscript{a,}\textsuperscript{*}, Lian-Ping Wang\textsuperscript{b}, Jian Zhang\textsuperscript{a}

\textsuperscript{a}LNM, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, PR China

\textsuperscript{b}Department of Mechanical Engineering, University of Delaware, Newark, DE 19716-3140, USA

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\textbf{1. Introduction}

Large-eddy simulation (LES) has emerged as a promising tool for simulating turbulent flows in general and, in recent years, has also been applied to the particle-laden turbulence with some success (Kassinos et al., 2007). The motion of inertial particles is much more complicated than fluid elements, and therefore, LES of turbulent flow laden with inertial particles encounters new challenges. In the conventional LES, only large-scale eddies are explicitly resolved and the effects of unresolved, small or subgrid scale (SGS) eddies on the large-scale eddies are modeled. The SGS turbulent flow field is not available. The effects of SGS turbulent velocity field on particle motion have been studied by Wang and Squires (1996), Armenio et al. (1999), Yamamoto et al. (2001), Shotorban and Mashayek (2006a,b), Fede and Simoni (2006), Berrouk et al. (2007), Bini and Jones (2008), and Pozorski and Apte (2009), amongst others. One contemporary method to include the effects of SGS eddies on inertial particle motions is to introduce a stochastic differential equation (SDE), that is, a Langevin stochastic equation to model the SGS fluid velocity seen by inertial particles (Fede et al., 2006; Shotorban and Mashayek, 2006b; Berrouk et al., 2007; Bini and Jones, 2008; Pozorski and Apte, 2009). However, the accuracy of such a Langevin equation model depends primarily on the prescription of the SGS fluid velocity autocorrelation time seen by an inertial particle or the inertial particle–SGS eddy interaction timescale (denoted by $\delta T_{lp}$) and a second model constant in the diffusion term which controls the intensity of the random force received by an inertial particle (denoted by $C_0$, see Eq. (7)). From the theoretical point of view, $\delta T_{lp}$ differs significantly from the Lagrangian fluid velocity correlation time (Reeks, 1977; Wang and Stock, 1993), and this carries the essential nonlinearity in the statistical modeling of particle motion. $\delta T_{lp}$ and $C_0$ may depend on the filter width and particle Stokes number even for a given turbulent flow. In previous studies, $\delta T_{lp}$ is modeled either by the fluid SGS Lagrangian timescale (Fede et al., 2006; Shotorban and Mashayek, 2006b; Pozorski and Apte, 2009; Bini and Jones, 2008) or by a simple extension of the timescale obtained from the full flow field (Berrouk et al., 2007).

In this work, we shall study the subtle and non-monotonic dependence of $\delta T_{lp}$ on the filter width and particle Stokes number using a flow field obtained from Direct Numerical Simulation (DNS). We then propose an empirical closure model for $\delta T_{lp}$. Finally, the model is validated against LES of particle-laden turbulence in predicting single-particle statistics such as particle kinetic energy. As a first step, we consider the particle motion under the one-way coupling assumption in isotropic turbulent flow and neglect the gravitational settling effect. The one-way coupling assumption is only valid for low particle mass loading.

* Corresponding author. Tel.: +86 10 82543969; fax: +86 10 82543408.
E-mail addresses: hgw@lnm.imech.ac.cn, guoweihe@yahoo.com (G.-W. He).
2. Governing equations

The study is performed in a forced isotropic turbulent flow field in a periodic box of side $2\pi$. Pseudo-spectral method is used to solve both the Navier–Stokes equations in DNS (Wang et al., 2000) and the filtered Navier–Stokes equations in LES (Yang et al., 2008). The locations and velocities of non-settling heavy particles ($\rho_p \gg \rho_f$) are obtained from the equations of motion as

$$\frac{d\mathbf{x}_p(t)}{dt} = \mathbf{v}_p(t),$$

$$\frac{d\mathbf{v}_p(t)}{dt} = \left[ \mathbf{u}(\mathbf{x}_p(t), t) - \mathbf{v}_p(t) \right] + f,$$

where $x_p(t)$ and $v_p(t)$ are the instantaneous position and velocity of a particle, respectively, $\tau_p$ is the particle Stokes relaxation time, $f$ is the correction factor for nonlinear drag which depends on the particle Reynolds number, $f(Re_p) = 1 + 0.15Re_p^{0.687}$, and $Re_p = |\mathbf{u} - \mathbf{v}_p|d_p/\nu$. As a quick estimate, if we assume that the relative velocity in the definition of $Re_p$ is scaled as the root mean square (rms) velocity, $u_{rms} = 19.32$, and the particle diameter is half of the Kolmogorov length scale, $\eta$, $d_p = 0.5\eta = 0.00675$, the fluid kinematic viscosity is $\nu = 0.0488$ in this study, then the Reynolds number based on the rms velocity is $Re_p = 1.48$. Since the particle Stokes number may be much larger than one in practice, then the correct factor $f$ in Eq. (2) accounts for the nonlinear drag effect. The fluid velocity seen by an inertial particle, $\mathbf{u}(\mathbf{x}_p(t), t)$, is a sum of the resolved fluid velocity seen by the particle, $\mathbf{u}_r(\mathbf{x}_p(t), t)$, and the unresolved SGS fluid velocity seen by the particle, $\mathbf{u}_i(\mathbf{x}_p(t), t) - \mathbf{u}_r(\mathbf{x}_p(t), t)$. The resolved flow is calculated from the filtered Navier–Stokes equations in LES

$$\frac{\partial \mathbf{u}_r}{\partial t} + \mathbf{u}_r \cdot \nabla \mathbf{u}_r = -\frac{1}{\rho} \nabla p + \frac{\partial \mathbf{\tau}_{str}}{\partial t} + \frac{1}{\Re} \rho \mathbf{\tau}_{str} + \mathbf{f},$$

where $\tau_{str} = \mathbf{u}_r \cdot \mathbf{u}_r - \mathbf{u} \cdot \mathbf{u}_r$ is the residual stress tensor, $f$ is a large-scale random forcing which is non-zero only at low wavenumbers in Fourier space $|k| < \sqrt{8}$ (Wang et al., 2000). $\mathbf{u}_r(\mathbf{x}_p(t), t)$ is obtained from the LES flow fields by a six-point Lagrangian interpolation scheme in each direction (Yang et al., 2008). In this study, the Chollet–Lesieur spectral eddy viscosity SGS model is used for the closure of the filtered Navier–Stokes equations (Chollet and Lesieur, 1981; Chollet, 1983)

$$v_p^*(k/k_c) = v_p^*(k/k_c) \sqrt{\frac{E(k_c)}{k_c}}$$

with

$$v_p^*(k/k_c) = C_k^{1/2} \left[ 0.441 + 15.2 \exp(-3.03k/k) \right],$$

where $C_k$ is the Kolmogorov constant and $C_k = 2.09$ from the compensated energy spectrum $E^{-5/3}k^{5/3}E(k)$ in this paper. The value of $C_k$ in our simulation is consistent with that of Kaneda et al. (1999). $E(k_c)$ in Eq. (4) is the value of the energy spectrum function at the cutoff wavenumber $k_c$ and it is dynamically evaluated from the LES fluid field.

Similar to the previous studies (Fede et al., 2006), the full fluid velocity seen by an inertial particle is then modeled using an extended, stochastic Langevin equation as

$$\mathbf{u}^* = \left[ \mathbf{u}_r(\mathbf{x}_p(t) + dt, t + dt) - \mathbf{u}_r(\mathbf{x}_p(t), t) \right] - \frac{1}{\delta T_p} \left[ \mathbf{u}^* - \mathbf{u}_r \right] dt + \left[ \frac{4}{3} S_{G_S} \frac{dt}{\delta T_p} \right]^{1/2} \xi,$$

where the superscript * in Eq. (6) denotes the modeled full scale velocity, of which one part is from the resolved velocity in LES, and the other part is from the Langevin equation. The modeled full scale velocity is different from the real full scale velocity from the Navier–Stokes equation in DNS. $\xi$ is a Gaussian random variable of zero mean and unit variance, $S_{G_S}$ is the fluid SGS kinetic energy seen by an inertial particle, which may not be equal to the fluid SGS kinetic energy averaged over the whole space, $S_{G_S}$. An empirical constant is then introduced to relate the two:

$$S_{G_S} = C_0 S_{G_S},$$

where, due to the inertial bias, $C_0$ could depend on the Stokes number. Using the DNS flow field, we can compute both $S_{G_S}$ and $S_{G_S}$ and therefore determine $C_0$. Fig. 1 shows how $C_0$ varies with particle Stokes number, $St_p$, where $St_p$ is defined as $St_p = \tau_p/\tau_f$ and $\tau_f$ is Kolmogorov timescale in DNS flow field, the cutoff location is at $\eta k_f = 0.135$ and $k_f$ is the cutoff wavenumber used in the filtered-DNS. When the Stokes number is very small or very large, particles uniformly distribute in the flow fields, therefore, $C_0$ tends to be 1. However, when the Stokes number is on the order of 1, particles are found preferentially in the regions of low vorticity and high strain rate, leading to significantly lower SGS turbulent kinetic energy. The feature is consistent with the recent observation by Strutt and Lightstone (2006) who showed that there is a net migration of particles towards regions of low kinetic energy. In their idealized model, the flow is composed of two regions of constant turbulent kinetic energy. They found that there is a higher probability for particles to travel into the low kinetic energy region when compared to the region of high kinetic energy.

The above Langevin equation assumes that the SGS fluid flow is isotropic. It ensures that the Lagrangian SGS fluid velocity time scale $\delta T_p$ and the fluid SGS kinetic energy seen by an inertial particle $S_{G_S}$ are both consistently realized.

The equation of motion, Eq. (2), is integrated with a fourth-order Adams–Bashforth method for the particle velocity and then a fourth-order Adams–Moulton method for the particle location, Eq. (1). When the particle inertia is negligibly small, Eqs. (1), (2) and (6) together reduce to the governing equation for fluid particles (Gicquel et al., 2002).

The remaining closure problem is how to evaluate $\delta T_p$, a quantity that requires the consideration of the SGS fluid velocity, the particle Stokes number as well as the filter width.
3. Evaluation of $\delta T_{ip}$

The simplest closure assumption used in recent studies (Fede et al., 2006; Shotorban and Mashayek, 2006b; Bini and Jones, 2008; Pozorski and Apte, 2009) states that $\delta T_{ip}$ is equal to the SGS fluid velocity autocorrelation time $\delta T$ seen by a fluid element. This is true only in the limit that $St \ll 1$. Previously, Wang and Stock (1993) showed that the integral timescale $T_{ip}$ of the fluid velocity seen by an inertial particle can vary from the fluid Lagrangian integral time $T_{i}$ to the fluid Eulerian one-point integral time $T_{E}$ as the particle inertia is increased in the full scale velocity field of an isotropic turbulence. Based on the numerical simulations they proposed that

$$\frac{T_{ip}}{T_{E}} = 1 - \left(1 - \frac{T_{i}}{T_{E}}\right) \frac{1}{(1 + St)^{0.84(1-0.03St)}}. \quad (8)$$

where the particle Stokes number is defined as $St \equiv \tau_{p}/T_{E}$. It is noted that all the parameters in Eq. (8), $T_{i}$, $T_{p}$, and $St$ are based on the full scale velocity field. Berrouk et al. (2007) extended the Wang–Stock model, Eq. (8), in the context of LES by introducing a model for $\delta T_{ip}$ as

$$\delta T_{ip} = \frac{\delta T_{i}}{\beta} \left[1 - \frac{(1 - \beta)}{(1 + St)^{0.84(1-0.03St)}}\right]. \quad (9)$$

where $St$ is the particle Stokes number, defined as $St \equiv \tau_{p}/\delta T_{ip}$, $\beta$ is the ratio of $\delta T_{i}$ to the fluid Eulerian SGS integral time $\delta T_{E}$. It is important to point out that all the parameters in Eq. (9), $\delta T_{i}$, $\beta$, and $St$ are based on the subgrid scale velocity field. They further assumed that

$$\beta \equiv \frac{\delta T_{i}}{\delta T_{E}} = \frac{T_{i}}{T_{E}}. \quad (10)$$

Eq. (10) shows that the ratio of $\delta T_{i}$ to $\delta T_{E}$ in the SGS flow field is equal to the ratio of $T_{i}$ to $T_{E}$ in the full scale flow field. We will show later that Eq. (10) does not hold for the SGS velocity field. However, when the correct value of $\beta$ is used, the extended Wang–Stock model, Eq. (9), can still be applied to capture approximately the dependence of $\delta T_{ip}$ on the Stokes number, see Fig. 5.

To evaluate $\delta T_{ip}$, we make use of the flow field $u(x,t)$ obtained from DNS. The key flow parameters of the DNS flow field at 256$^3$ grid resolution are: Taylor microscale Reynolds number $Re_{\tau} = 102.1$, Kolmogorov time $\tau_{K} = 0.0037$, Kolmogorov velocity $\nu_{K} = 3.63$, fluid Eulerian integral time $T_{E} = 0.050$ and fluid Lagrangian integral time $T_{i} = 0.037$. The filtered velocity $u(x,t)$ is calculated by truncating the high-wavenumber Fourier modes above a cutoff wavenumber $k_{c}$. The subgrid scale velocity is then

$$u'(x,t) = u(x,t) - \bar{u}(x,t). \quad (11)$$

The cutoff wavenumber $k_{c}$ is varied to obtain the SGS velocity fields with different filter widths (Fede and Simonin, 2006).

At the limit of a very small Stokes number, an inertial particle behaves as a fluid particle. Therefore, we first study the characteristics of the correlation functions and integral timescales of the full fluid velocity and the SGS fluid velocity. The difference between the Eulerian and Lagrangian correlations of the full or unfiltered fluid velocity field, $R_{E}(\tau)$ and $R_{L}(\tau)$, has been discussed in many studies (Kaneda and Gotoh, 1991; Yeung, 2001). The Eulerian fluid velocity temporal correlation can be calculated as

$$R_{E}(\tau) = \frac{\langle u_{i}(x_{0}, t_{0})u_{i}(x_{0} + \tau, t_{0} + \tau) \rangle}{\langle u_{i}(x_{0}, t_{0})u_{i}(x_{0}, t_{0}) \rangle}, \quad (12)$$

and the Eulerian integral timescale is then

$$T_{E} = \int_{0}^{\infty} R_{E}(\tau)d\tau. \quad (13)$$

The Lagrangian fluid velocity autocorrelation can be calculated as

$$R_{L}(\tau) = \frac{\langle u_{i}(x_{0}, t_{0})u_{i}(x_{0} + \tau, t_{0} + \tau) \rangle}{\langle u_{i}(x_{0}, t_{0})u_{i}(x_{0}, t_{0}) \rangle}, \quad (14)$$

and the Lagrangian integral timescale is

$$T_{L} = \int_{0}^{\infty} R_{L}(\tau)d\tau. \quad (15)$$

Fig. 2 shows the Eulerian and Lagrangian autocorrelations for the full velocity field $u(x,t)$. Also shown are the representative Eulerian and Lagrangian velocity autocorrelations for the SGS velocity field $u'(x,t)$ at $\eta_{k} = 0.135$. The SGS Eulerian fluid velocity autocorrelation can be calculated as

$$\delta R_{E}(\tau) = \frac{\langle u_{i}'(x_{0}, t_{0})u_{i}'(x_{0} + \tau, t_{0} + \tau) \rangle}{\langle u_{i}'(x_{0}, t_{0})u_{i}'(x_{0}, t_{0}) \rangle}, \quad (16)$$

and the SGS Eulerian integral timescale is

$$\delta T_{E} = \int_{0}^{\infty} \delta R_{E}(\tau)d\tau. \quad (17)$$

The SGS Lagrangian fluid velocity autocorrelation can be calculated as

$$\delta R_{L}(\tau) = \frac{\langle u_{i}'(x_{0}, t_{0})u_{i}'(x_{0} + \tau, t_{0} + \tau) \rangle}{\langle u_{i}'(x_{0}, t_{0})u_{i}'(x_{0}, t_{0}) \rangle}, \quad (18)$$

and the SGS Lagrangian integral timescale is

$$\delta T_{L} = \int_{0}^{\infty} \delta R_{L}(\tau)d\tau. \quad (19)$$

Here, it is important to point out that $x(t_{0} + \tau)$ in Eqs. (14) and (18) is the location of a fluid particle based on the full velocity field,

$$\frac{dx(t_{0} + \tau)}{d\tau} = u(x(t_{0} + \tau), t_{0} + \tau). \quad (20)$$

For the full velocity field, $R_{E}(\tau)$ decays with $\tau$ more slowly than $R_{L}(\tau)$ at large $\tau$. In contrast, for the SGS velocity field, $\delta R_{E}(\tau)$ drops with $\tau$ more quickly than $\delta R_{L}(\tau)$. These observations imply that $T_{L} > T_{E}$ for the full velocity field ($T_{E}/T_{F} = 0.74$), while $\delta T_{L} < \delta T_{E}$ for the SGS velocity field (in this particular case $\delta T_{L}/\delta T_{E} = 1.83$ at
The SGS Eulerian integral timescale, $\delta T_E$, can be estimated using the SGS integral length scale, $\delta L$, and the rms velocity of the full scale flow field, $u_{rms}$. This is based on the advection or sweeping effect that the decay of the Eulerian correlation at small-scale is dominated by large-scale sweeping. In the inertial subrange, the SGS integral length scale is inversely proportional to the filter width. We plot the variation of the SGS Eulerian integral timescale $\delta T_E$ with respect to the lag time $\tau$ from 0 to $\infty$. Here, $\delta R_{lp}(\tau)$ is calculated as

$$\delta R_{lp}(\tau) = \frac{\langle u'(x_p(t_0), t_0)u'(x_p(t_0 + \tau), t_0 + \tau) \rangle}{\langle u'(x_p(t_0), t_0)u'(x_p(t_0), t_0) \rangle}, \tag{22}$$

It is important to note that the full velocity field is used here to obtain the particle trajectory $x_p(t)$ from Eqs. (1) and (2). A wide range of Stokes numbers ($St = \tau_p/\delta T_E$) and several filter widths ($\eta k_c = 0.135, 0.216$ and $0.284$) were studied. The statistics were computed by tracking $4 \times 10^4$ inertial particles. Fig. 5 plots the variation of $\delta T_{lp}/\delta T_L$ with particle Stokes number $St$ from our DNS results (circle) and a fitted curve (solid line). Several published relations are also shown for comparison (Fede et al., 2006; Shotorban et al., 2006; Berrouk et al., 2007).

Our DNS results show that for $St < 0.03$, $\delta T_{lp} \approx \delta T_L$, or the difference between $\delta T_L$ and $\delta T_{lp}$ is negligible. For the particles with very large inertia, $St \approx 100$ or larger, they do not respond to the SGS eddies, thus $\delta T_{lp} \approx \delta T_E$. For a particle with intermediate inertia, $\delta T_{lp}/\delta T_L$ varies with $St$ and this variation is non-monotonic. The ratio first increases with increasing $St$, reaches a maximum, and then decreases to approach the limiting value of $\delta T_E/\delta T_L$. That is because, for some range of $St$, the trajectory of an inertial particle near a SGS eddy tends to deviate from the streamline due to the centrifugal effect or the inertial bias. The inertial particle near a vortex tube experiences a less-curved path along which the SGS fluid velocity does not change so fast and thus the SGS fluid velocity seen by the inertial particle appears to be better correlated. This leads to the ratio $\delta T_{lp}/\delta T_L$ being larger than one (note that $\delta T_L/\delta T_L < 1$). Such a non-monotonic dependence on Stokes number is consistent with the previous numerical results showing that $T_L/\delta T_L$ has a typical mean value of 0.78 and is almost independent of $Re$, (Yeung, 2001).
ber, $St_k$, was also seen in the study of Jung et al. (2008) who studied $T_{ip}$ with unfiltered flow velocity field. A maximum value for $\partial T_{ip}/\partial T$ was observed at $St \sim 0.5$. For larger $St$, the inertial particle is not very responsive to SGS fluid eddies and therefore, $\partial T_{ip}$ gradually approaches $\partial T$. Our simulation results can be fitted by an empirical curve (solid line in Fig. 5) of the form

$$\frac{\partial T_{ip}}{\partial T} = \frac{1}{\beta} \left( \frac{0.444 - 0.7\eta_k}{\eta_k} \right) \exp \left\{ -\left[ \ln \left( \frac{St}{0.5} \right) \right]^2 + 1 - (1 - \beta) \exp \left( -\frac{St}{0.5} \right) \right\},$$

(23)

where $\beta = \partial T_{ip}/\partial T$, $\eta_k$ can be approximately expressed as $\eta_k = \pi/\delta$ and $\delta$ is the length scale of the filter width in physical space (Pope, 2000). This fit was optimized with the results from all three cutoff locations, noting that both $\partial T$ and $\partial T_{ip}$ also depend on $\eta_k$. The above relation captures all the main characteristics of the dependence of $\partial T_{ip}$ on $St$ and $\eta_k$. It is worth pointing out that the first term in the braces represents that the magnitude of the convexity near $St = 0.5$ depends on the filter width $\eta_k$. $St$ when $St$ is small.

Berrouk et al. (2007) used the Wang–Stock model (Wang and Stock, 1993) with $\beta = 0.356$ (dash-dotted line). The model is qualitatively incorrect in the overall trend, implying that their assumption, Eq. (10) should not be used for estimating SGS timescale ratio. However, if we use the the correct value of $\beta = 1.63$ based on the SGS motion when $\eta_k = 0.135$, the extended Wang–Stock model, Eq. (9), can predict the correct limiting behaviors for $St \sim 0$ and $St \to \infty$, and approximately capture the overall trend of the Stokes number dependence (dashed line). As the Wang–Stock model was developed from kinematic random flow fields where the flow dynamics were not considered, the interesting non-monotonic behavior of $\partial T_{ip}/\partial T$ near $St = 0.5$ could not be realized. The dotted line is $\partial T_{ip} = \partial T$ (Fede et al., 2006; Shotorban and Mashayek, 2006b), this assumption is only suitable for very small $St$ (say, $St < 0.05$). Shotorban and Mashayek (2006b) modified the assumption $\partial T_{ip} = \partial T$ by setting $\partial T_{ip}/\partial T = 1/[1 + \left( \tau_p/\tau_{ip} \right)^2]$, their relation is plotted using the dash-dot-dotted line. Their relation would imply that $\partial T_{ip}$ monotonically decreases to zero rather than to $\partial T$, with the increase of $St$, which is unphysical.

4. Application of $\partial T_{ip}$ to the Langevin stochastic model

Finally, we apply the closure model (Eq. (23)) and $C_0$ shown in Fig. 1 in the Langevin stochastic equation (Eq. (6)) in LES to study the particle kinetic energy.

Fig. 6 compares the ratios of particle kinetic energy to fluid kinetic energy, as a function of Stokes number, $St_k$, using the present closure. For the range of Stokes numbers studied ($St_k = 0.1 \sim 30$), the conventional LES under-predicts the particle kinetic energy due to the missing of SGS fluid turbulence. However, with the increase of particle Stokes number, the relative difference decreases. This is expected since particles with large Stokes number are less affected by the SGS turbulence. It is observed that the particle SGS Langevin model with our $\partial T_{ip}$ formulation greatly improves the accuracy of particle kinetic energy. This result is consistent with the work of (Fede et al., 2006). In Fig. 6, we also include their results. (Fede et al., 2006) performed a priori calculations with the filtered DNS flow field and they focused mostly on particles with relaxation time $\tau_p$ close to the SGS turbulent timescale of the flow $\tau_{ip}$. The resolution of their DNS flow field is $128^3$ and the cutoff location of the filtered-DNS flow field is at $\eta_k = 0.38$. Their a priori calculations also showed a much better prediction when a stochastic Langevin equation is employed. Since we covered a wider range of particle Stokes numbers and used a higher DNS flow Reynolds number, our results show more convincingly that the SGS Langevin model works for small Stokes numbers. The figure also indicates that our new closure model for $\partial T_{ip}$ led to a better overall prediction.

It should be noted that, under the effect of gravity, a heavy particle moves relatively to the surrounding fluid with a drift velocity, crossing the trajectories of fluid elements and interacting with different small-scale eddies. As a result, we expect that the timescale of the SGS velocity seen by the heavy particle decreases as the settling velocity increases. This is known as the crossing trajectory effect for a heavy particle in turbulent flows, except here in the context of LES the interaction is restricted to SGS turbulent eddies. Furthermore, a related effect is that the timescale in the vertical direction is larger than that in the horizontal direction due to the continuity effect in an incompressible turbulent flow. These effects related to the gravity have been studied by Yudine (1959), Csanyi (1963), Reeks (1977), Wells and Stock (1983), and Wang and Stock (1993), amongst others, for the full turbulent flow field. Qualitatively and in the current context of SGS eddies, we expect that the gravity effect is important when the particle settling velocity is comparable to the rms velocity fluctuation of the SGS flow field. The precise dependence of the SGS fluid velocity time scale seen by a particle on the settling velocity requires further investigation.

5. Summary and concluding remarks

In this study, we examined carefully the SGS Lagrangian correlation times seen by both fluid and inertial particles. These SGS Lagrangian statistics show qualitatively different behaviors than the Lagrangian statistics for the full fluid velocity field. For fluid particles, we found that $\partial T$ is larger than $\partial T_e$ for SGS velocity field, whereas $T_e$ is smaller than $T_e$ for the full velocity field. This suggests that it is important to relate the limiting values of $\partial T_{ip}$ to $\partial T_e$ and $\partial T_e$ in the SGS flow field, $T_e$ and $T_e$ should not be used in the context of recovering the SGS contributions to particle motion. Based on the DNS flow field, we computed the SGS fluid velocity correlation time seen by inertial particles, $\partial T_{ip}$, for a wide range of particle Stokes number and several filter widths. It was shown that $\partial T_{ip}$ first increases with Stokes number, reaches a peak when $St$ is on the order one, and then decreases with increasing $St$ number. Based on these results, an empirical model for $\partial T_{ip}$ is proposed to
take into account of the dependence of $\delta T_p$ on Stokes number and filter width. Using the Langevin stochastic equation with our new closure model, we demonstrated that the particle kinetic energy can be much better predicted in LES.

We recognize that in some applications, the gravitational settling may also be important. This could complicate the closure of $\delta T_p$. The more difficult problem in LES of particle-laden flows is the prediction of particle-pair statistics, such as the radial distribution function, the pair relative velocity at contact and the collision rate, which are more sensitive to small-scale eddies. These issues remain to be studied.

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