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# FORMATION INPUT-TO-STATE STABILITY

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Abstract: This paper introduces the notion of formation input-to-state stability in order to characterize the internal stability of leader-follower formations, with respect to inputs received by the formation leader. Formation ISS is a weaker form of stability than string stability since it does not require inter-agent communication. It relates group input to internal state of the group through the formation graph adjacency matrix. In this framework, different formation structures can be analyzed and compared in terms of their stability properties and their robustness.

Keywords: Formations, graphs, interconnected systems, input-to-state stability.

## 1. INTRODUCTION

Formation control problems have attracted increased attention following the advances on communication and computation technologies that enabled the development of distributed, multi-agent systems. Direct fields of application include automated highway systems (Varaiya, 1993; Swaroop and Hedrick, 1996; Yanakiev and Kanellakopoulos, 1996), reconnaissance using wheeled robots (Balch and Arkin, 1998), formation flight control (Mesbahi and Hadaegh, 2001; Beard *et al.*, 2000) and sattelite clustering (McInnes, 1995).

For coordinating the motion of a group of agents, three different interconnection architectures have been considered, namely behavior-based, virtual structure and leader-follower. In behavior based approach (Balch and Arkin, 1998; Lager *et al.*, 1994; Yun *et al.*, 1997), several motion premitives are defined for each agent and then the group behavior is generated as a weighted sum of these primary behaviors. Behavior based control schemes are usually hard to analyze formally, although some attempts have been made (Egerstedt, 2000). In leader-follower approaches (Beard *et al.*, 2000; Desai and Kumar, 1997; Tabuada *et al.*, 2001; Fierro *et al.*, 2001), one agent is the leader of the formation and all other agents are required to follow the leader, directly or indirectly. Virtual structure type formations (Tan and Lewis, 1997; Egerstedt and Hu, 2001), on the other hand, usually require a centralized control architecture.

Balch and Arkin (1998) implement behavior - based schemes on formations of unmanned ground vehicles and test different formation types. Yun *et al.* (1997) develop elementary behavior strategies for maintaining a circular formation using potential field methods. Egerstedt and Hu (2001) adopt a virtual structure architecture in which the agents follow a virtual leader using a centralized potential-field control scheme. Fierro *et al.* (2001) develop feedback linearizing controllers for the control of mobile robot formations in which each agent is required to follow one or two leaders. Tabuada *et al.* (2001) investigate the conditions under which a set of formation constraints can be satisfied given the dynamics of the agents and consider the problem of obtaining a consistent group abstraction for the whole formation.

This paper focuses on a different problem: *given a leader-follower formation, investigate how the leader input affects the internal stability of the overall for-*

*mation.* Stability properties of interconnected systems have been studied within the framework of string stability (Swaroop and Hedrick, 1996; Yanakiev and Kanellakopoulos, 1996). String stability actually requires the attenuation of errors as they propagate in the formation. However, sting stability conditions are generally restrictive and generally require inter-agent communication. It is known, for instance (Yanakiev and Kanellakopoulos, 1996) that string stability in autonomous operation of an AHS with constant intervehicle spacing, where each vehicle receives information only with respect to the preceding vehicle, is impossible. We therefore believe that a weaker notion of stability of interconnected system that relates group objectives with internal stability would be useful.

Our approach is based on the notion of input-to-state stability (Sontag and Wang, 1995) and exploits the fact that the cascade interconnection of two input-to-state stable systems is itself input-to-state stable (Khalil, 1996; Krstić *et al.*, 1995). This property allows the propagation of input-to-state gains through the formation structure and facilitates the calculation of the total group gains that characterize the formation performance in terms of stability. Formation ISS is a weaker form of stability than string stability, in the sense that it does not require inter-agent communication and relies entirely on position feedback only (as opposed to both position and velocity feedback) from each leader to its follower. We represent the formation by means of a *formation graph* (Tabuada *et al.*, 2001). Graphs are especially suited to capture the interconnections (Tabuada *et al.*, 2001; Fierro *et al.*, 2001) and information flow (Fax and Murray, 2001) within a formation. The proposed approach provides a means to link the formation leader's motion or the external input to the internal state and the adjacency matrix of the formation. It establishes a method for comparing stability properties of different formation schemes.

The rest of the paper is organized as follows: in section 2 the definitions for formation graphs and formation input-to-state stability (ISS) are given. Section 3 establishes the ISS properties of an leader-follower interconnection and in section 4 it is shown how these properties can be propagated from one formation graph edge to another to cover the whole formation. Section 5 provides examples of two stucturally different basic formation configurations and indicates how interconnection differences affect stability properties. In section 6 results are summarized and future research directions are highlighted.

## 2. FORMATION GRAPHS

A formation is being modeled by means of a *formation graph*. The graph representation of a formation allows a unified way of capturing both the dynamics of each agent and the inter-agent formation specifications. All agent dynamics are supposed to be expressed by linear, time invariant controllable systems. Formation specifications take the form of reference relative positions between the agents, that describe the shape of the formation and assign roles to each agent in terms of the responcibility to preserve the specifications. Such an assignment imposes a leader-follower relationship that leads to a decentralized control architecture. The assignment is expressed as a directed edge on the formation graph (Figure 1).

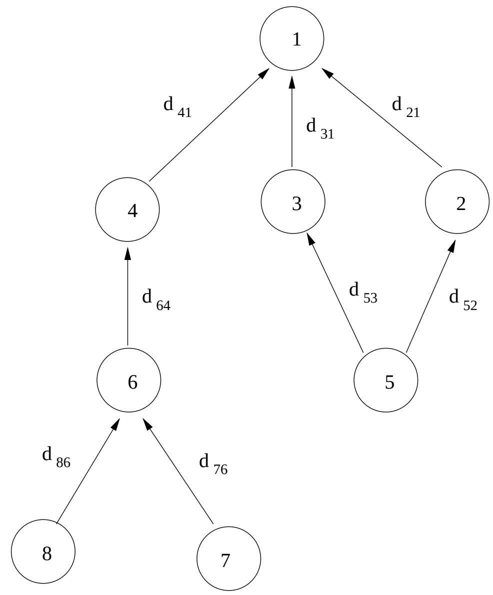


Fig. 1. An example of a formation graph

*Definition 2.1.* (Formation Graph). A formation graph $F = (V, E, D)$ is a directed graph that consists of:

- A finite set $V = \{v_1, \dots, v_l\}$ of $l$ vertices and a mapping $v_i \mapsto T\mathbb{R}^n$ that assignes to each vertice an LTI control system describing the dynamics of a particular agent:
$$\dot{x}_i = A_i x_i + B_i u_i$$
where $x_i \in \mathbb{R}^n$ is the state of the agent accociated with vectice $v_i$, $u_i \in \mathbb{R}^m$ is the agent control input and $A_i \in \mathbb{R}^{n\times n}, B_i \in \mathbb{R}^{m\times m}$ is a controllable pair of matrices.
- A binary relation $E \subset V \times V$ representing a leader-follower link between agents, with $(v_i, v_j) \in E$ whenever the agent associated with vectice $v_i$ is to follow the agent of $v_j$.
- A finite set of formation specifications $D$ indexed by the set $E$, $D = \{d_{ij}\}_{(v_i,v_j)\in E}$. For each edge $(v_i, v_j)$, $d_{ij} \in \mathbb{R}^n$, denotes the desired relative distance that the agent associated with vectice $v_i$ has to maintain from the agent associated with agent $v_j$.

Our discussion specializes in acyclic formation graphs. This implies that there can be at least one agent $v_L$ that can play the role of a leader (i.e. a vectice with no outgoing arrow). The input of the leader can be used to control the evolution of the whole formation. The graph is ordered starting from the leader and following a breadth-first numbering of its vertices.

For every edge $(v_i, v_j)$ we associate an error vector that expresses the deviation from the specification prescribed for that edge:

$$z_{ij} \triangleq x_j - x_i - d_{ij} \in \mathbb{R}_{n_i j}$$

The formation error $z$ is defined as the augmented vector formed by concatenating the error vectors for all edges $(v_i, v_j) \in E$:

$$z \triangleq \left[z_e\right]_{e \in E}$$

A natural way to represent the connectivity of the graph is by means of the adjacency matrix, $A$. We will therefore consider the mapping $E \to \mathbb{R}^{l \times l}$ that assigns to the set $E$ of ordered vertice pairs $(v_i, v_j)$ the adjacency matrix $A_E \in \mathbb{R}^{l \times l}$.

Our aim is to investigate the stability properties of the formation with respect to the input $u_L$ of the formation leader. We thus need to define the kind of stability in terms of which the formation will be analyzed:

*Definition 2.2.* (Formation Input-to-State Stability). A formation is called input-to-state stable iff there is a class $\mathscr{K}\mathscr{L}$ function $\beta$ and a class $\mathscr{K}$ function $\gamma$ such that for any initial formation error $z(0)$ and for any bounded inputs of the formation leader $u_L(\cdot)$ the evolution of the formation error satisfies:

$$\|z(t)\| \leq \beta(\|z(0)\|, t) + \gamma\left(\sup_{\tau \leq t} \|u_L\|\right) \qquad (1)$$

By investigating the formation input-to-state stability we establish a relationship between the amplitude of the input of the formation leader and the evolution of the formation errors. This will provide upper bounds for the leaders input in order for the formation shape to be maintained inside some desired specifications. Further, it will allow to characterize and compare formations according to their stability properties.

## 3. EDGE INPUT-TO-STATE STABILITY

In the leader-follower configuration, one agent is required to follow another by maintaining a constant distance, $x_j - x_i = d_{ij}$. If agent $i$ is required to follow agent $j$, then this objective is naturally pursued by applying a follower feedback control law that depends on the relative distance between the agents. For $x_i = x_j - d_{ij}$ to be an equilibrium of the closed loop control system:

$$\dot{x}_i = A_i x_i + B_i u_i$$

it should hold that $A_i(x_j - d_{ij}) \in \mathscr{R}(B_i)$; otherwise the follower cannot be stabilized at that distance from its leader. Suppose that there exists an $e_{ij}$ such that $B_i e_{ij} = -A_i(x_j - d_{ij})$. Then the following feedback law can be used for the follower:

$$u_i = K_i(x_j - x_i - d_{ij}) + e_{ij}$$

leading to the closed loop dynamics:

$$\dot{x}_i = (A_i - B_i K_i)(x_i - x_j + d_{ij})$$

Then the error dynamics of the $i$-$j$ pair of leader-follower becomes:

$$\dot{z}_{ij} = (A_i - B_i K_i) z_{ij} + \dot{x}_j$$

which can be written, assuming that agent $j$ follows agent $k$:

$$\dot{z}_{ij} = (A_i - B_i K_i) z_{ij} + g_{ij} \qquad (2)$$

where $g_{ij} \triangleq -(A_j - B_j K_j) z_{jk}$.

The stability of the follower is thus directly dependent on the matrix $(A_i - B_i K_i)$, the eigenvalues of which can be arbitrarily chosen, and the interconnection term $g_{ij}$. The interconnection term can be bounded as follows:

$$g_{ij} \leq \lambda_M(A_j - B_j K_j) \|z_{jk}\|$$

where $\lambda_M(\cdot)$ is the maximum eigenvalue of a given matrix.

If $K_i$ is chosen so that $A_i - B_i K_i$ is Hurwitz, then the solution of the Lyapunov equation:

$$P_i(A_i - B_i K_i) + (A_i - B_i K_i)^T P_i = -I$$

provides a symmetric and positive definite matrix $P_i$ and a natural Lyapunov function candidate $V_i = x_i^T P_i x_i$ for the interconnection dynamics (2) that satisfies:

$$\lambda_m(P_i)\|x_i\| \leq V_i \leq \lambda_M(P_i)\|x_i\|$$

where $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the minimum and maximum eigenvalue of a given matrix, respectively. For the derivative of $V_i$:

$$\begin{aligned}\dot{V}_i &\leq -\|x_i\|^2 + 2\lambda_M(P_i)\lambda_M(A_j - B_j K_j)\|x_i\|\|z_{jk}\| \\ &\leq -(1-\theta)\|x_i\|^2 \leq 0\end{aligned}$$

for all $\|x_i\| \geq \frac{2\lambda_M(P_i)\lambda_M(A_j - B_j K_j)}{\theta}\|z_{jk}\|$ where $\theta \in (0,1)$. Viewing (2) as a perturbed system:

$$\begin{aligned}\|z_{ij}(t)\| \leq& \left(\frac{\lambda_M(P_i)}{\lambda_m(P_i)}\right)^{\frac{1}{2}} \|z_{ij}(0)\| \mathrm{e}^{-\frac{1-\theta}{2\lambda_M(P_i)}t} \\ &+ \frac{2(\lambda_M(P_i))^{\frac{3}{2}}\lambda_M(A_j - B_j K_j)}{(\lambda_m(P_i))^{\frac{1}{2}}\theta}\|z_{jk}\|\end{aligned}$$

Equation (3) implies that (2) is input-to-state stable with respect to $\|z_{jk}\|$ as input and

$$\beta_i(r,t) = r\bar{\beta}_i \mathrm{e}^{-\frac{1-\theta}{2\lambda_M(P_i)}t} \qquad (3)$$

$$\gamma_i(r) = \bar{\gamma}_i r \qquad (4)$$

where

$$\bar{\beta}_i = \left(\frac{\lambda_M(P_i)}{\lambda_m(P_i)}\right)^{\frac{1}{2}}$$

$$\bar{\gamma}_i = \frac{2(\lambda_M(P_i))^{\frac{3}{2}}\lambda_M(A_j - B_j K_j)}{(\lambda_m(P_i))^{\frac{1}{2}}\theta}$$

## 4. FROM EDGE STABILITY TO FORMATION STABILITY

An important property of input-to-state stability is that it is preserved in cascade connections. The property allows propagation of ISS properties from one agent to another, all the way up to the formation leader. This procedure will yield the global input gains of the leader and give a measure of the sensitivity of the formation shape with respect to the input applied at the leader.

In the previous section it was shown that under the assumption of pure state feedback, a formation graph edge is input-to-state stable. The gain functions for the cascade interconnection

$$\begin{aligned}\dot{x}_1 &= f_1(t,x_1,x_2,u)\\ \dot{x}_2 &= f_2(t,x_2,u)\end{aligned}$$

are given as:

$$\begin{aligned}\beta(r,t) &= \beta_1(2\beta_1(r,\tfrac{t}{2}) + 2\gamma_1(2\beta_2(r,0)),\tfrac{t}{2})\\ &\quad + \gamma_1(2\beta_2(r,\tfrac{t}{2})) + \beta_2(r,t),\\ \gamma(r) &= \beta_1(2\gamma_i(2\gamma_2(r)+2r),0)\\ &\quad + \gamma_1(2\gamma_2(r)+2r) + \gamma_2(r).\end{aligned}$$

From the general expressions and (3)-(4) the leader $j$ - follower $i$ composite ISS gains become:

$$\begin{aligned}\beta_{ij}(r,t) &= \Big(2\bar{\beta}_i^2 e^{-\frac{1-\theta}{2\lambda_M(P_i)}t} + 4\bar{\gamma}_i\bar{\beta}_i\bar{\beta}_j e^{-\frac{1-\theta}{4\lambda_M(P_i)}t}\\ &\quad + 2\bar{\gamma}_i\bar{\beta}_j e^{-\frac{1-\theta}{4\lambda_M(P_j)}t} + \bar{\beta}_j e^{-\frac{1-\theta}{2\lambda_M(P_j)}t}\Big) r\\ \gamma_{ij}(r) &= \Big(4\bar{\beta}_i\bar{\gamma}_i\bar{\gamma}_j + 4\bar{\beta}_i\bar{\gamma}_i + 2\bar{\gamma}_i\bar{\gamma}_j + 2\bar{\gamma}_i + \bar{\gamma}_j\Big) r\end{aligned}$$

Based on the above we define:

$$\begin{aligned}\bar{\beta}_{ij} &\triangleq \beta_{ij}(1,0) = 2\bar{\beta}_i^2 + 4\bar{\gamma}_i\bar{\beta}_i\bar{\beta}_j + 2\bar{\gamma}_i\bar{\beta}_j + \bar{\beta}_j\\ \bar{\gamma}_{ij} &\triangleq \gamma_{ij}(1) = 4\bar{\beta}_i\bar{\gamma}_i\bar{\gamma}_j + 4\bar{\beta}_i\bar{\gamma}_i + 2\bar{\gamma}_i\bar{\gamma}_j + 2\bar{\gamma}_i + \bar{\gamma}_j\end{aligned}$$

Although the $\beta_{ij}$ function for any connected $i$, $j$ vertices on the formation graph has to be calculated through successive compositions, the $\gamma_{ij}$ function can be algorithmically calculated:

Let $a_i$ be the $i$ row of the $n \times n$ adjacency matrix $A$ of the formation graph, $F = (V,E,D)$. Define:

$$\bar{\beta}^0 \triangleq \left[\bar{\beta}_1 \ \cdots \ \bar{\beta}_n\right]^T, \quad \bar{\gamma}^0 \triangleq \left[\bar{\gamma}_1 \ \cdots \ \bar{\gamma}_n\right]^T$$

and for the $k+1$ iteration let

$$\begin{aligned}\bar{\beta}^{k+1} &\triangleq \left[\bar{\beta}_1^{k+1} \ \cdots \ \bar{\beta}_n^{k+1}\right]^T,\\ \bar{\gamma}^{k+1} &\triangleq \left[\bar{\gamma}_1^{k+1} \ \cdots \ \bar{\gamma}_n^{k+1}\right]^T\end{aligned}$$

be given recursively as:

$$\bar{\gamma}^{k+1} = \bar{\gamma}^k + n_{n-k}c_\gamma^k, \quad \bar{\beta}^{k+1} = \bar{\beta}^k + n_{n-k}c_\beta^k$$

where

$$n_{n-k} = [\underbrace{0 \ \cdots \ 0}_{n-k-1} \ 1 \ \cdots \ 0]^T,$$

$$\begin{aligned}c_\gamma^k &= 2(2a_{n-k}\bar{\beta}^k a_{n-k}\bar{\gamma}^k\bar{\gamma}_{n-k}^k + 2a_{n-k}\bar{\beta}^k a_{n-k}\bar{\gamma}^k\\ &\quad + a_{n-k}\bar{\gamma}^k\bar{\gamma}_{n-k}^k + a_{n-k}\bar{\gamma}^k),\\ c_\beta^k &= 2(2a_{n-k}\bar{\beta}^k a_{n-k}\bar{\gamma}^k\bar{\beta}_{n-k}^k + 2a_{n-k}\bar{\gamma}^k\bar{\beta}_{n-k}^k\\ &\quad + a_{n-k}\bar{\beta}^k)\end{aligned}$$

The algorithm terminates in at most $n-1$ steps, since this is maximum path length in a directed acyclic graph with $n$ vectices. It results in a $\bar{\gamma}$ vector containing the formation ISS input gains of each agent. These are the gains that express the sensitivity of the formation stability with respect to the dynamic interaction at each agent due to interconnection. Thus if agent $i$ follows agent $j$, then the steady state of the formation errors, $(z_k)_{ss}$ of all edges $k$ between the descendants of agent $i$ is bounded:

$$\|(z_k)_{ss}\| \leq \bar{\gamma}_i \|z_{ij}\|$$

The steady state error for the whole formation will then be bounded by a linear function of the norm of the leader's velocity:

$$\|z_{ss}\| \leq \bar{\gamma}_1 \|\dot{x}_1\|$$

## 5. EXAMPLES

Consider the case of three agents where one is assigned to be a leader of the formation and the other two have to be followers. Denote the leader by $v_1$ and the followers by $v_i$, $i = 2,3$. Suppose that the task is for $v_2$ to follow $v_1$ keeping a constant distance $d$ and $v_3$ to follow $v_2$ keeping the same constant distance $d$ (Figure 2).

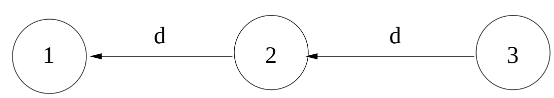


Fig. 2. Serial interconnection

A simple one dimensional model of the formation could be given as:

$$\begin{aligned}\dot{x}_1 &= u\\ \dot{x}_2 &= k(d + x_1 - x_2)\\ \dot{x}_3 &= k(d + x_2 - x_3)\end{aligned}$$

where $x_i$, $i = 1,2,3$ are each agent absolute coordinates, $u$ is the leader's velocity, $d$ is the inter-agent desired distance, and $k$ is the gain of the feedback law which is based on relative position measurements. By a change of coordinates,

$$z_1 = x_1$$
$$z_2 = x_2 - x_1 - d$$
$$z_3 = x_3 - x_2 - d$$

the formation equations can be written as:

$$\dot{z}_1 = u$$
$$\dot{z}_2 = -kz_2 - u$$
$$\dot{z}_3 = -kz_3 + kz_2$$

For the $1-2$ interconnection, a Lyapunov function candidate could be:

$$V_2(z_2) \triangleq \frac{1}{2} z_2^2$$

Obviously, $\frac{1}{2}|z_2|^2 \le V_2(z_2) \le \frac{1}{2}|z_2|^2$. Then,

$$\dot{V}_2(z_2) = -kz_2^2 + z_2 u$$

Select any $\theta \in (0,1)$ and write $\dot{V}_2$ as

$$\dot{V}_2(z_2) = -k(1-\theta)z_2^2 - k\theta z_2^2 + z_2 u \le -k(1-\theta)z_2^2,$$

for $|z_2| > \frac{\sup_{\tau\le t}|u|}{k\theta}, \theta \in (0,1)$. Then it follows that,

$$|z_2| \le |z_2(0)|\mathrm{e}^{-\sqrt{2k(1-\theta)t}} + \frac{\sup_{\tau\le t}|u|}{k\theta}$$
$$= \beta_2(|z_2(0)|,t) + \gamma_2(\sup_{\tau\le t}|u|)$$

The ISS input-gain function for agent $v_2$ is

$$\gamma_2 = \frac{\sup_{\tau\le t}|u|}{k\theta}$$

Similarly, for agent $v_3$ a Lyapunov function candidate could be:

$$V_3(z_3) \triangleq \frac{1}{2} z_3^2$$

and its time derivative would then be

$$\dot{V}_3(z_3) = -kz_3^2 + kz_3 z_2$$

For $|z|_3 > \frac{\sup_{\tau\le t}|z_2(\tau)|}{\theta}$ it follows that

$$\gamma_3 = \frac{\sup_{\tau\le t}|z_2(\tau)|}{\theta}$$

Then the formation, as a cascade connection of the subsystems of agents $v_2$ and $v_3$, is input-to-state stable with

$$\gamma(\sup_{\tau\le t}|u|) = \frac{6 + 6k\theta + \theta}{k\theta^2} \sup_{\tau\le t}|u|$$

In the second case, both agents $v_2$ and $v_3$ are to follow the leader at designated distances that are $d$ for $v_2$ and $2d$ for $v_3$. The formation shape is the same as in the previous case, however the interconnections are different as reflected in the different formation graphs (Figure 3).

The formation equations in this case are:

$$\dot{x}_1 = u$$
$$\dot{x}_2 = k(d + x_1 - x_2)$$
$$\dot{x}_3 = k(2d + x_2 - x_3)$$

which by a change of variables:

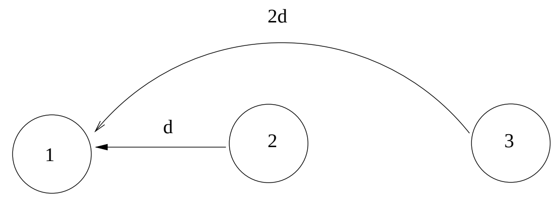


Fig. 3. Parallel interconnection

$$z_1 = x_1$$
$$z_2 = x_2 - x_1 - d$$
$$z_3 = x_3 - x_2 - 2d$$

For the same Lyapunov candidate functions used in the previous case, it follows that:

$$\beta_2 = |z_2(0)|\mathrm{e}^{-\sqrt{2(1-\theta)t}}, \quad \gamma_2 = \frac{1}{k\theta}\sup_{\tau\le t}|u|$$
$$\beta_3 = |z_3(0)|\mathrm{e}^{-\sqrt{2(1-\theta)t}}, \quad \gamma_3 = \frac{1}{k\theta}\sup_{\tau\le t}|u|$$

and the input gain for the formation is:

$$\gamma(\sup_{\tau\le t}|u|) = \frac{2}{k\theta}\sup_{\tau\le t}|u|$$

It can be shown analytically that the second formation can outperform the first in terms of the magnitude of relative errors with respect to the leader's velocity. Specifically, if we denote by $\gamma_s$ the input-to-state gain of the first interconnection connection and by $\gamma_p$ the input-to-state gain of the second interconnection,

$$\frac{\gamma_s}{\sup_{\tau\le t}|u|} = \frac{6 + 6k\theta + \theta}{k\theta^2} \ge \frac{6\theta + 6k\theta + \theta}{k\theta^2}$$
$$= \frac{6k\theta + 7\theta}{k\theta^2} = \frac{6k+7}{k\theta} \ge \frac{7}{k\theta} \ge \frac{2}{k\theta} = \frac{\gamma_p}{\sup_{\tau\le t}|u|}$$

## 6. CONCLUSIONS

In this paper, the notion of formation input-to-state stability has been introduced. This form of stability can be used to characterize the internal state of a formation that has a leader-follower achitecture, and establishes a link between the motion of the leader of the formation or its external input and the shape of the formation. Formation ISS is a weaker form of stability than string stability, in the sense that it does not require inter-agent communication and relies entirely on position feedback only (as opposed to both position and velocity feedback) from each leader to its follower. Moreover, it establishes a link between the formation internal state and the outside world. In the proposed framework, different formation structures can be analyzed and compared in terms of their stability properties.

Future work is directed towards investigating the effect of (limited) inter-agent communication on formation stability and consistent ways of group abstractions that are based on the formation ISS properties.

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