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Brief Paper

Backstepping for nonsmooth systems☆

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Abstract

The paper presents a constructive control design for integrator backstepping in nonsmooth systems. The approach is based on non smooth analysis and Lyapunov stability for nonsmooth systems and is similar in spirit with the robust control designs that have appeared in literature, but is applicable to a larger class of systems. The backstepping controller is first applied to the case of a unicycle driven by a new discontinuous kinematic controller yielding global asymptotic convergence with bounded inputs. Then it is used to implement a sliding mode controller in a hybrid system. Simulations results not only verify the convergence properties but also reveal the ability of the new backstepping controller to suppress chattering.

Keywords: Backstepping; Nonholonomic systems; Nonsmooth differential equations; Discontinuous control

1. Introduction

Backstepping is among the most important nonlinear control design techniques with numerous applications. This work is motivated by the problem of stabilizing nonholonomic systems, a class of systems that cannot be stabilized by smooth static state feedback laws (Brockett, 1981). For this class, backstepping has been used either in the cases where the controller is smooth time-varying (Fierro & Lewis, 1995; Jiang & Nijmeijer, 1998; Morin & Samson, 1996; Kolmanovsky & McClamroch, 1995b) or within the regions where the discontinuous controller is smooth (Jiang, 2000).

The literature is rich in work on nonholonomic stabilization (Kolmanovsky & McClamroch, 1995a). As pointed out by Kim and Tsiotras (2000), the majority of time invariant nonholonomic control laws are based on kinematic models (Canudas de Wit & Sordalen, 1992; Astolfi, 1996; Bloch & Drakunov, 1996; Yang & Kim, 1999). Stabilization of dynamic models for nonholonomic systems has also been addressed in Campion, d'Andrea Novel, and Bastin (1991), Reyhanoglu and McClamroch (1992), Jiang (2000), Lin, Pongvuthithum, and Quian (2002), Laiou and Astolfi (1999), Kolmanovsky and McClamroch (1995b), M'Closkey and Murray (1994). A common problem in discontinuous strategies is unboundedness of inputs around the discontinuity manifold and possible appearance of chattering, both of which are treated with various techniques (Astolfi, 1996; Luo & Tsiotras, 2000; Tsiotras & Luo, 1997; Jiang, 2000). Backstepping has been used in translating kinematic controllers into equivalent dynamic ones (Kolmanovsky & McClamroch, 1995b; Fierro & Lewis, 1995; Jiang & Nijmeijer, 1998) but this has only been done for the time-varying case. Kolmanovsky and McClamroch (1995b) extend time-periodic smooth kinematic controllers to dynamic ones using integrator backstepping and the nonsmooth dynamic extension of M'Closkey and Murray (1994). In the latter case, the procedure applies to homogeneous feedback control laws which are smooth everywhere except for the origin. While the homogeneity assumption can be relaxed, it is not clear if the method is still applicable when the nonsmooth region is not restricted to the origin.

Our approach can be used to implement any type of nonholonomic kinematic controller through acceleration inputs. For the smooth case, it recovers the classic backstepping

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designs. For the discontinuous case, we offer the first backstepping methodology, since related work (Freeman & Kokotović, 1996) requires at least local Lipschitz continuity. In Freeman and Kokotović (1996), elements of generalized gradient sets are treated as bounded disturbances. Our backstepping controller generally requires less control effort. The proposed methodology is applied to the problem of stabilization of a dynamic model of a mobile robot, yielding a globally asymptotically stable dynamic controller. Contrary to alternative methodologies, this controller bounded in the neighborhood of the discontinuities. The method is then used to implement a sliding mode controller in a hybrid electronic throttle control system. Implementation shows that the proposed backstepping methodology can successfully suppress chattering phenomena arising in switching controllers. Such behavior has also been observed in Freeman and Kokotović (1993), although in this case controllers were smooth.

2. Mathematical framework

For differential equations with piecewise continuous right hand sides, solutions are defined in terms of a differential inclusion $\mathbf{F}(t,x)$ by Filippov (1988). Here, we will use a constructive characterization of the differential inclusion $\mathbf{F}(t,x)$ (Paden & Sastry, 1987):

Consider the differential equation:

$$\dot{x} = f(t,x), \tag{1}$$

with discontinuous right hand side, in which f is measurable and essentially locally bounded. Then, there exists $\mathbf{N}_f \subset \mathbb{R}^n$, $\mu\mathbf{N}_f = 0$ such that $\forall \mathbf{N} \subset \mathbb{R}^n$, $\mu\mathbf{N} = 0$, $\mathbf{F}(x) \triangleq \overline{\mathrm{co}}\{\lim f(x_i) \,|\, x_i \to x, x_i \notin \mathbf{N}_f \cup \mathbf{N}\}$.

For nonsmooth functions the notions of directional derivative and gradient are generalized as follows:

Definition 1 (Clarke, 1983). Let f be Lipschitz near a given point x, and let v be any other vector in [a Banach space] $\mathbf{X}$. The generalized directional derivative of f at x in the direction v, denoted $f^\circ(x;v)$, is defined as follows:

$$f^\circ(x;v) \triangleq \limsup_{\substack{y\to x\\ t\to 0}} \frac{f(y+tv)-f(y)}{t},$$

where y is a vector in $\mathbf{X}$ and t is a positive scalar.

Definition 2 (Clarke, 1983). Let f be Lipschitz near x. The generalized gradient of f at x, denoted $\boldsymbol{\partial} f(x)$, is the subset of [the dual space of $\mathbf{X}$,] $\mathbf{X}^*$, given by:

$$\boldsymbol{\partial} f(x) \triangleq \{\zeta \in \mathbf{X}^*\colon f^\circ(x;v) \geqslant \langle \zeta, v\rangle, \forall v \in \mathbf{X}\}.$$

In the case where the space is finite dimensional there is a special characterization of the generalized gradient which facilitates its calculation:

Theorem 1 (Clarke, 1983). *Let f be Lipschitz near x, $\boldsymbol{\Omega}_f$ the set of points where f is non differentiable, and $\mathbf{S}$ any set of Lebesgue measure zero in $\mathbb{R}^n$. Then*

$$\boldsymbol{\partial} f(x) = \overline{\mathrm{co}}\left\{\lim_{x_i\to x} \nabla f(x_i)\colon x_i \notin \mathbf{S}, x_i \notin \boldsymbol{\Omega}_f\right\}.$$

The algebra of generalized gradients usually involves inclusions. To turn inclusions into equalities we need the assumption of regularity:

Definition 3 (Clarke, 1983). The function f is called regular at x if: (i) for all v, the usual one-sided directional derivative $f'(x;v)$ exists, and (ii) for all v, $f'(x;v) = f^\circ(x;v)$.

The previous definitions allow us to present the notion of generalized time derivative of a non smooth function:

Theorem 2 (Shevitz & Paden, 1994). *Let $x(\cdot)$ be a Filippov solution to (1) on an interval containing t and $V:\mathbb{R}\times\mathbb{R}^n \to \mathbb{R}$ be a Lipschitz and in addition, regular function. Then $V(t,x(t))$ is absolutely continuous, $(\mathrm{d}/\mathrm{d}t)V(t,x(t))$ exists almost everywhere and*

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t,x(t)) \in^{\text{a.e.}} \dot{\tilde{\mathbf{V}}}(t,x),$$

where $\dot{\tilde{\mathbf{V}}}(t,x) \triangleq \bigcap_{\xi\in\boldsymbol{\partial} V(t,x(t))} \xi^{\mathrm{T}}(\mathbf{F}(t,x(t)),1)^{\mathrm{T}}$.

Shevitz and Paden (1994) also showed that there exists a nonsmooth equivalent to the well known Lyapunov's direct method (Shevitz & Paden, 1994).

Theorem 3 (Shevitz & Paden, 1994). *Let (1) be essentially locally bounded and $0 \in \mathbf{F}(t,0)$ in a region $\mathbf{Q} \supset \{t\colon t_0 \leqslant t < \infty\} \times \{x\in\mathbb{R}^n\colon \|x\| < r\}$. Also, let $V\colon \mathbb{R}\times\mathbb{R}^n \to \mathbb{R}$ be a regular function satisfying $V(t,0) = 0$ and $0 < V_1(\|x\|) \leqslant V(t,x) \leqslant V_2(\|x\|)$ for $x \neq 0$ in Q for some V_1, V_2 functions of class $\mathscr{K}$. Then*

(1) *$\dot{\tilde{\mathbf{V}}}(t,x) \leqslant 0$ in $\mathbf{Q}$ implies $x(t) \equiv 0$ is a uniformly stable solution.*
(2) *If there exists a class $\mathscr{K}$ function $\omega(\cdot)$ in $\mathbf{Q}$ with the property $\dot{\tilde{\mathbf{V}}}(t,x) \leqslant -\omega(\|x\|) < 0$ then the solution $x(t) \equiv 0$ is uniformly asymptotically stable.*

LaSalle's invariant principle also generalizes to autonomous nonsmooth systems:

Theorem 4 (Shevitz & Paden, 1994). *Let $\boldsymbol{\Omega}$ be a compact set such that every Filippov solution to the autonomous system $\dot{x} = f(x)$, $x(0) = x(t_0)$ starting in $\boldsymbol{\Omega}$ is unique and remains in $\boldsymbol{\Omega}$, $\forall t \geqslant 0$. Let $V\colon \boldsymbol{\Omega} \to \mathbb{R}$ be a time independent regular function with $v \leqslant 0$, $\forall v \in \dot{\tilde{\mathbf{V}}}$ (if $\dot{\tilde{\mathbf{V}}}$ is the empty set this is trivially satisfied.) Define $\boldsymbol{S} = \{x \in \boldsymbol{\Omega} \,|\, 0 \in \dot{\tilde{\mathbf{V}}}\}$. Then every trajectory in $\boldsymbol{\Omega}$ converges to the largest invariant set, $\mathbf{M}$, in the closure of $\mathbf{S}$.*

3. Stability results

In this section we present our main result, namely the extension of integrator backstepping to nonsmooth systems. We make use of the generalized derivative and gradient, which are introduced in the context of nonsmooth analysis (Clarke, 1983). For the type of nonsmooth systems discussed here, solutions are defined in terms of the Filippov differential inclusion (Filippov, 1988).

Theorem 5. *Consider the system*:

$$\dot{\eta} = f(\eta) + g(\eta)\xi, \tag{2a}$$

$$\dot{\xi} = u, \tag{2b}$$

where $\eta \in \mathbb{R}^n, \xi \in \mathbb{R}^m$. *Assume that the subsystem* (2a) *can be stabilized by a control law* $\xi = \phi(\eta)$ *with* $\phi(0) = 0$, *and that there is a regular* (*possibly nonsmooth*) *locally Lipschitz Lyapunov function* $V(\eta)$ *for which there exists a positive definite, class* $\mathscr{K}_\infty$ *function* $W(\eta)$ *satisfying*:

$$0 < W(\eta) \leqslant d, \quad \forall d \in \mathbf{D} \tag{3}$$

where $\mathbf{D} \triangleq -\bigcap_{\lambda\in\partial V(\eta)} \lambda^{\mathrm{T}}\mathbf{F}(f(\eta)+g(\eta)\phi(\eta))$, *and* $\mathbf{F}(h(x))$ *is the Filippov set of* $\dot{x} = h(x)$. *Then the following law asymptotically stabilizes* (2):

$$u = \zeta + \left(K_z + \mathrm{diag}\left\{\frac{V^\circ(\eta; g(\eta)[\xi-\phi(\eta)])}{\|\xi-\phi(\eta)\|_2^2}\right\}\right)\cdot [\phi(\eta)-\xi], \tag{4}$$

in which $V^\circ(\cdot)$, *is the generalized directional derivative of* V, ζ *is the minimum norm element of the generalized time derivative of* ϕ, $\dot{\boldsymbol{\phi}}(\eta)$, *and* K_z *a positive definite constant matrix.*

Proof. The proof structure is adopted from Khalil (1996). By a change of variables: $z = \xi - \phi(\eta)$, $\mathbf{v} = u - \dot{\boldsymbol{\phi}}$, the system (2a)–(2b) can be written as

$$\dot{\eta} = f(\eta) + g(\eta)\phi(\eta) + g(\eta)z,$$

$$\dot{z} = \mathbf{v}.$$

Consider the Lyapunov function candidate: $V_a(\eta,\xi) \triangleq V(\eta) + \frac{1}{2}z^{\mathrm{T}}z$. Then, every element $\mu \in \dot{\mathbf{V}}_a$ satisfies $\mu \leqslant -W(\eta) + \rho + z^{\mathrm{T}}v$, with $\rho \in \bigcap_{\lambda\in\partial V(\eta)} \lambda^{\mathrm{T}}\mathbf{F}(g(\eta)z)$ and $v \in \mathbf{v}$. With $\chi \in \dot{\boldsymbol{\phi}}$, $v = u + \chi$ and substituting yields:

$$\mu \leqslant -W(\eta) - z^{\mathrm{T}}K_z z + \rho - V^\circ(\eta; g(\eta)[\xi-\phi(\eta)]) + z^{\mathrm{T}}(\zeta - \chi).$$

Now, from the definition of the generalized gradient, for all $\rho \in \bigcap_{\lambda\in\partial V(\eta)} \lambda^{\mathrm{T}}\mathbf{F}(g(\eta)z)$ we have that $\rho - V^\circ(\eta; g(\eta)z) \leqslant 0$, and since ζ is the minimum norm element of $\dot{\boldsymbol{\phi}}$, the expression:

$$-W(\eta) - z^{\mathrm{T}}K_z z + \rho - V^\circ(\eta; g(\eta)[\xi-\phi(\eta)]) + z^{\mathrm{T}}(\zeta-\chi)$$

is strictly negative except for the origin $(\eta, z) = (0, 0)$. This implies that every element of $\dot{\mathbf{V}}_a$ is strictly negative. Application of Theorem 3 completes the proof. □

Remark 1. Backstepping a nonsmooth controller yields a differential inclusion. In regions where both $V(t,x)$ and ϕ are differentiable, (4) recovers the known integrator backstepping input (Krstić et al., 1995; Khalil, 1996). At the points of nondifferentiability, if nonempty, $\dot{\boldsymbol{\phi}}$ gives an inclusion. If $\dot{\boldsymbol{\phi}} = \emptyset$ then $\dot{\mathbf{V}}_a = \emptyset$ and the conditions of Theorem 3 are trivially satisfied.

Compared to similar results in Freeman and Kokotović (1996), Theorem 5 gives the backstepping control law explicitly. Not relying on robustness analysis, the control inputs of (4) are less conservative in terms of control effort required. This is due to being able to avoid the overapproximation of the generalized gradient by imposing regularity conditions. Another distinguishing feature of Theorem 5 is that it allows for a discontinuous ϕ; although its generalized time derivative, $\dot{\boldsymbol{\phi}}$ may be locally unbounded, the selection of ζ in (4) ensures that the control inputs are always bounded.

3.1. Example

Consider the double integrator: $\dot{\eta} = \xi$, $\dot{\xi} = u$. Let $\phi(\eta) = -\mathrm{sgn}(\eta)$, defined as $\phi = -1$ for $\eta > 0$, $\phi = 1$ for $\eta < 0$ and $\phi = 0$ for $\eta = 0$. Then, for $V(\eta) = \frac{1}{2}\eta^2$ we have $\dot{V}(\eta) = -|\eta| = W(\eta) < 0$ for $\eta \neq 0$, and $V^\circ(\eta; \xi - \phi) = \eta(\xi - \phi)$. Theorem 5 suggests:

$$u = \zeta + \left(k_z + \frac{V^\circ(\eta; \xi-\phi)}{(\xi-\phi)^2}\right)(\phi - \xi)$$

$$= \zeta + k_z(\phi - \xi) - \eta,$$

for $\xi \neq \phi$. When $\xi = \phi$, we can analytically define it to be so. Since $0 \in \dot{\boldsymbol{\phi}}$, we have $\zeta = 0$ and thus

$$u = -k_z(\mathrm{sgn}(\eta) - \xi) - \eta$$

Fig. 1 shows the vector field of the closed loop system.

4. Applications

4.1. Mobile robot stabilization

In this section the control law (4) is used to backstep a discontinuous nonholonomic controller (Tanner & Kyriakopoulos, 2002), in order to stabilize the dynamic model of a mobile robot (Fig 2).

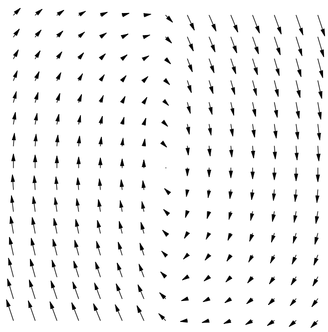

Fig. 1. Vector field of the closed loop system.

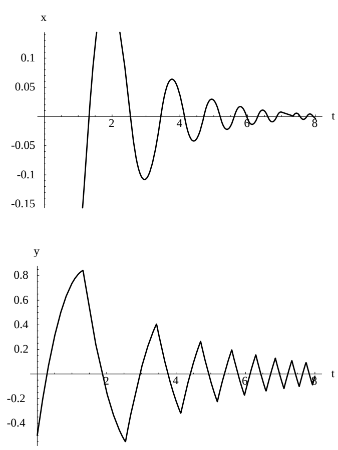

Fig. 2. Solution from initial conditions $(-0.5, -0.5)$.

Consider the dynamic equations of a nonholonomic mobile robot moving on the horizontal plane:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}, \tag{5a}$$

$$\begin{bmatrix}\dot{v}\\ \dot{\omega}\end{bmatrix} = M(x,y,\theta)^{-1}(f - R(x,y,\theta,v,\omega)), \tag{5b}$$

where (x, y) are the cartesian coordinates of the robot, θ its orientation, v and ω its translational and rotational velocities, M is the inertia matrix of the system, R is term containing Coriolis and centrifugal terms and f is the vector of input forces. The choice of input forces: $f = R(q,u) + M(q)w$, linearizes (5b) via feedback and results in w being the new control input. To stabilize (5) we will first design a kinematic controller:

Proposition 1. *The following feedback control law*:

$$v_d = \mathrm{sgn}(x)k_v[(y^2 - x^2)\cos\theta - 2xy\sin\theta], \tag{6a}$$

$$\omega_d = k_\omega(\arctan2(2xy, x^2 - y^2) - \theta), \tag{6b}$$

where k_v and k_ω positive constants, and the sign function is defined as $\mathrm{sgn}(x) = 1,\ x \geqslant 0$ *and* $\mathrm{sgn}(x) = -1,\ x < 0$, *asymptotically stabilizes* (5a) *to the origin.*

Proof. Consider the positive dipolar (Tanner, Loizou, & Kyriakopoulos, 2001) semidefinite function in $\mathbb{R}^2 \setminus \{0\}$ (Fig. 3):

$$V(x,y) = \mathrm{e}^{\frac{-|x|}{x^2+y^2}}.$$

The function is regular everywhere in its domain of definition. The one-sided (for $x > 0$) derivative at $(0, y)$ in the direction of $v = (v_x, v_y)$ is $-v_x/y^2$, equal to the generalized directional derivative at $(0, y)$. Similarly it can be shown for $x < 0$. In all points where $x \neq 0$:

$$\dot{V} = \frac{-\mathrm{sgn}(x)v\mathrm{e}^{\frac{-|x|}{x^2+y^2}}}{(x^2+y^2)^2}[(y^2 - x^2)\cos\theta - 2xy\sin\theta]. \tag{7}$$

It can easily be verified that the generalized gradient of V on the y-axis ($x = 0$) is the empty set: $\dot{\tilde{\mathbf{V}}}(0, y) = \emptyset$. Substituting for v in (7), we obtain:

$$\dot{V} = \frac{-\mathrm{e}^{\frac{-|x|}{x^2+y^2}}}{(x^2+y^2)^2}[(y^2 - x^2)\cos\theta - 2xy\sin\theta]^2 \leqslant 0$$

For $x = 0$, condition $\xi \leqslant 0, \forall \xi \in \dot{\tilde{\mathbf{V}}}$ is trivially satisfied. The set $\boldsymbol{S} \triangleq \{(x,y,\theta) \,|\, 0 \in \dot{\tilde{\mathbf{V}}}\}$ is given as $\mathbf{S} = \{(x,y,\theta) \,|\, x((y^2 - x^2)\cos\theta - 2xy\sin\theta) = 0\}$. In any point where $(y^2 - x^2)\cos\theta - 2xy\sin\theta = 0$, it is $|\arctan2(2xy, x^2 - y^2) - \theta| = \frac{\pi}{2}$ which means that $\omega \neq 0$. Thus, $\mathbf{S} = \{(x,y,\theta) \,|\, x = 0\}$. In $\mathbf{S}$ we have: $v = k_v y^2 \cos\theta,\ \omega = k_\omega(\arctan2(0, -y^2) - \theta)$. For the invariant set $\mathbf{E} \subset \mathbf{S}$, $y = \theta = 0$ and so it is $\mathbf{E} \equiv \{0\}$. Applying LaSalle's principle for nonsmooth systems (Shevitz & Paden, 1994), the proof is completed. □

Application of Theorem 5 in this case yields the acceleration control inputs:

$$w_1 = -\mathrm{sgn}(x)\left[\frac{\mathrm{e}^{\frac{-|x|}{x^2+y^2}}(v - v_d)^2(x^2 - y^2)\cos\theta}{(x^2+y^2)^2[(v - v_d)^2 + (\omega - \omega_d)^2]} + \frac{2\mathrm{e}^{\frac{-|x|}{x^2+y^2}}(\omega - \omega_d)(v - v_d)xy\sin\theta}{(x^2+y^2)^2[(v - v_d)^2 + (\omega - \omega_d)^2]}\right] - k_{z_v}(v - v_d)$$

$$w_2 = -\mathrm{sgn}(x)\left[\frac{2\mathrm{e}^{\frac{-|x|}{x^2+y^2}}(\omega - \omega_d)^2 xy\sin\theta}{(x^2+y^2)^2[(v - v_d)^2 + (\omega - \omega_d)^2]} + \frac{\mathrm{e}^{\frac{-|x|}{x^2+y^2}}(v - v_d)(\omega - \omega_d)(x^2 - y^2)\cos\theta}{(x^2+y^2)^2[(v - v_d)^2 + (\omega - \omega_d)^2]}\right] - k_{z_\omega}(\omega - \omega_d).$$

The controller is tested in numerical simulations with the parameters chosen as follows: $k_v = 10$, $k_\omega = 3$, $k_{z_v} = 10$,

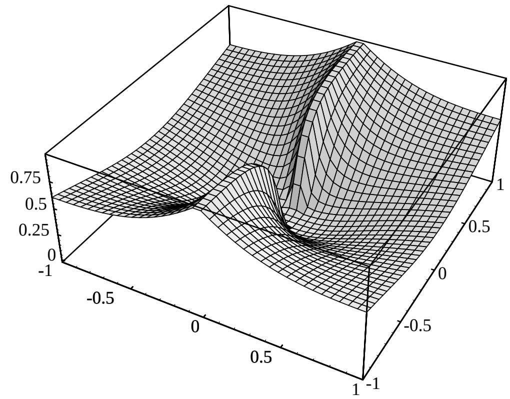

Fig. 3. A dipolar positive semidefinite Lyapunov function.

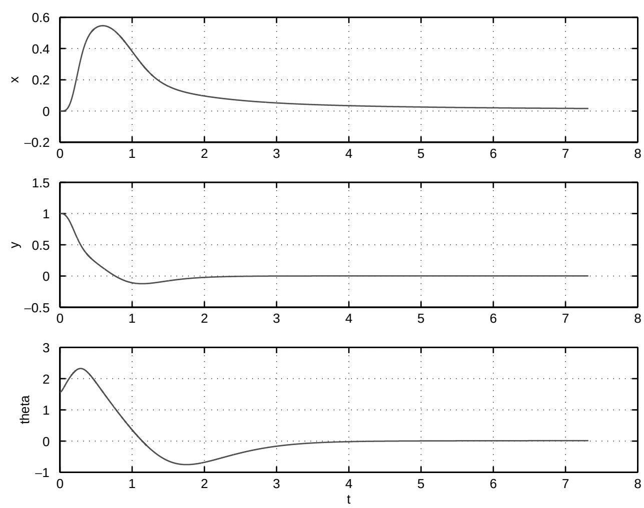

Fig. 4. Trajectories with initial conditions $(0, 1, \pi/2)$.

$k_{z_\omega} = 50$. Fig. 4 gives the trajectories for initial conditions $(x, y, \theta) = (0, 1, \pi/2)$. Note that the nondifferentiability of the Lyapunov-like function at $x = 0$ does not affect performance.

What is worth noting is that the backstepping technique introduced in this paper suppresses chattering through an appropriate choice of control gains. The integrator of (2b) acts as a low pass filter on the reference input, suppressing high frequency switching. In switching control designs, application of backstepping offers simultaneously a method to decompose controller design and alleviate chattering. To illustrate this chattering filtering property, we artificially introduced switching to controller (6). Fig. 5 shows that chattering is suppressed without affecting convergence.

4.2. Electronic throttle control

The electronic throttle control (ETC) system is an embedded control system that regulates the amount of air and fuel that enters into the engine of an automobile. In its original implementation, the throttle is controlled by a PWM driven motor. As such, the system can be modeled as a hybrid system, with discrete modes arising from friction

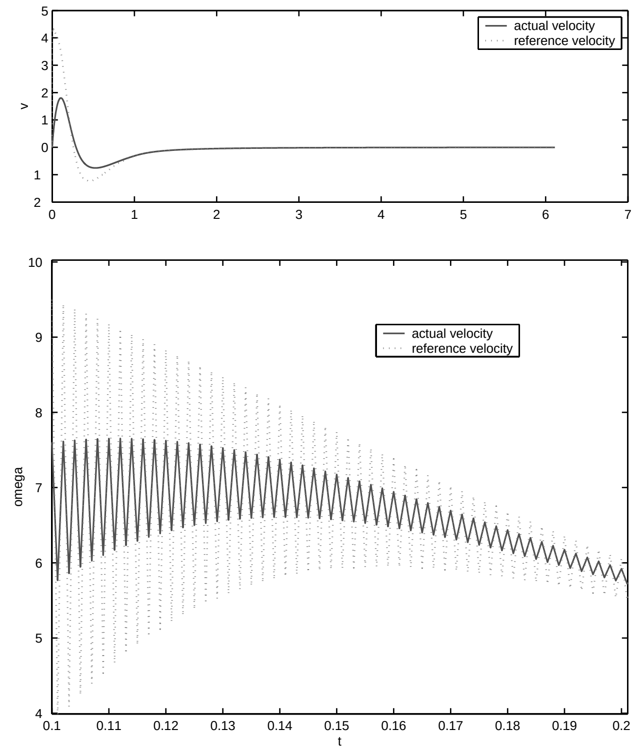

Fig. 5. Imposing chattering: backstepping as a filter.

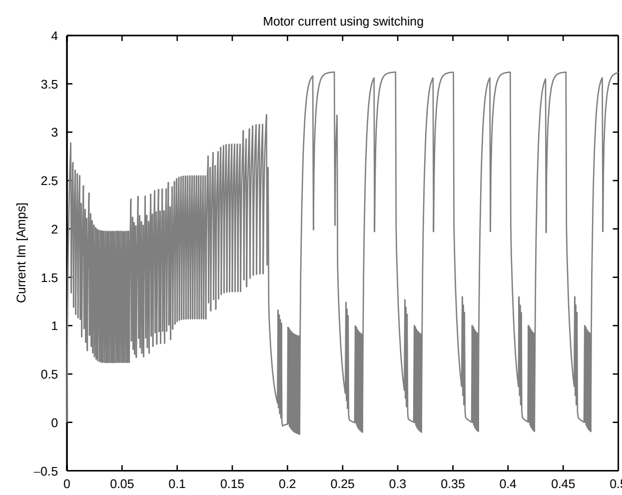

Fig. 6. Motor current under the switching control scheme.

phenomena and changes in the actuator circuitry upon reception of a voltage pulse. The system switches between `ON` and `OFF` modes depending on whether it receives a pulse from the PWM generator. The switching logic is determined by a condition on the motor current which is based on a sliding mode controller design for the throttle dynamics: if $i_m < i_m^d$ then switch from `OFF` to `ON`; if $i_m \geqslant i_m^d$ switch from `ON` to `OFF`, where i_m^d is the sliding mode control input designed for the throttle dynamics. Originally, the sliding mode controller was implemented by switching the motor on and off. This causes significant chattering in motor current (Fig. 6).

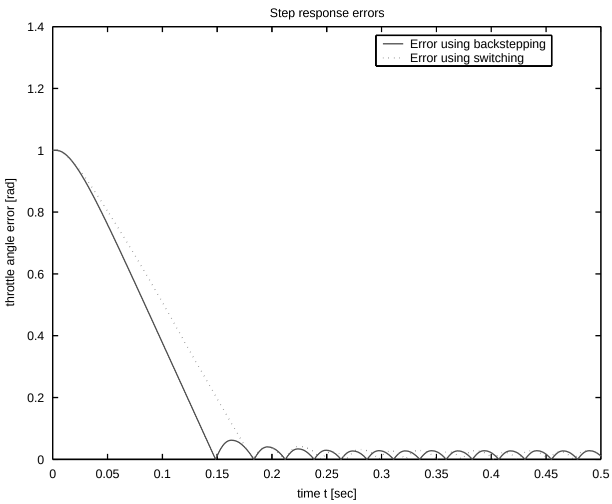

Fig. 7. Position errors under the sliding mode controller with the two implementations.

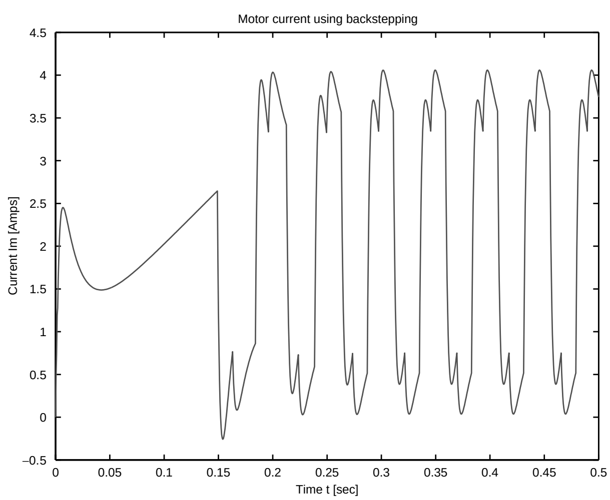

Fig. 8. Motor current under the backstepping controller.

For that reason, we investigate implementing the same sliding mode controller for the throttle subsystem by continuously regulating the voltage of the motor. This will involve backstepping the sliding mode controller through the motor current dynamics (the voltage remains always bounded.) We tune the backstepping gain K_z so that errors are comparable in the two implementations (Fig. 7). In the backstepping implementation, however, chattering in the motor current is eliminated (Fig. 8).

5. Concluding remarks

We present an extension of integrator backstepping to nonsmooth systems. The result is based on nonsmooth analysis and Lyapunov stability for nonsmooth systems. Backstepping of nonsmooth control laws can also be used for chattering suppression at the expense of convergence speed. The potential of this approach is demonstrated in the stabilization problem of a nonholonomic dynamic model of a mobile robot, and in the sliding mode controller for an electronic throttle control system.

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