

# COORDINATING THE MOTION OF A MANIPULATOR AND A MOBILE BASE IN TREE ENVIRONMENTS

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Abstract: The paper presents a nonholonomic motion planner for mobile manipulators moving in outdoor environment amongst trees. The robot is supposed to move autonomously on flat ground, navigate among the trees and reach a specified target point on the three dimensional space, starting from an arbitrary initial position. The approach is based on a discontinuous feedback law under the influence of a specially designed potential field. The potential field function coordinates the motion of the mobile base and the onboard manipulator. Convergence is shown via Lyapunov's direct method and validity of the approach is verified through simulation examples.

Keywords: Agriculture, Mobile robots, Obstacle avoidance, Manipulators

## 1. INTRODUCTION

Robotic manipulators are commonly attached to a stationary surface, in the close vicinity of the area where they are supposed to operate. This configuration is only appropriate when the robot's desired motion is strictly contained within its workspace. The operational points have to be close to each other, a restriction which considerably limits the range of possible applications. Agricultural tasks, which normally require operating at many distant locations are unavoidably excluded. This restriction is lifted by employing mobile manipulators. Successful employment of mobile manipulators in agriculture has recently been reported in litera-

ture (Mandow *et al.*, 1996; Kondo, 1995; Fujiura *et al.*, 1990; Iida *et al.*, 1996).

One of the main problems concerning the application of a mobile manipulator in agriculture is planning its autonomous motion on the field and coordinating its two main components, namely, the mobile base and the manipulator mounted on top. Due to the kinematic constraints imposed on the wheels of the base, the mobile manipulator is a nonholonomic system. As a consequence, no continuous static feedback motion planning scheme can be applied (Brockett, 1981); it has to be either open-loop, time varying (Murray and Sastry, 1990) or discontinuous (de Wit and Sordalen, 1991). Motion planning for nonholonomic systems has traditionally been divided into two stages: path planing and trajectory planning. This decomposition into two separate consequent

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stages inhibits real time implementation. Particularly for mobile manipulators, motion planning has generally been treated within the framework of optimal control. The complete trajectory is preplanned before execution. On these issues several papers have appeared in literature: Desai and Kumar (Desai and Kumar, 1997) have formulated the problem as an optimal control problem. Pin et. al (Pin *et al.*, 1997) performed local optimization at velocity level. Perrier et. al (Perrier *et al.*, 1998) minimized the total position error using a linearized model. Huang et. al (Huang *et al.*, 1998) decoupled the motion of the vehicle from that of the manipulator and optimized each one using different criteria. Recently, Tanner and Kyriakopoulos (Tanner and Kyriakopoulos, 2000) proposed a closed loop motion planning scheme that achieves convergence and obstacle avoidance based on the potential field method.

Besides the two stage decomposition, optimal control is notoriously known for its computational requirements that render it impractical in real time applications. In this paper the approach followed is based on the use of fast feedback. The system is kinematically steered in real time using position information to navigate itself at each time step. This is made possible with a discontinuous feedback law and a special kind of potential field functions. The primary merit of such an approach is that it merges path planning and motion planning in a robust and fast feedback scheme. Furthermore, it is very practical since it can be combined in a tele-operated scheme where the end-user gives simple commands while the system reconfigures itself automatically without colliding with the environment.

The rest of the paper is organized as follows: Section 2 presents the basic mathematical tools on which our approach is founded and gives a formal problem statement. In section 3 the mathematical framework is briefly introduced, the proposed approach is discussed and its stability properties are established. In section 4 implementation issues are discussed. Section 5 illustrates the efficiency of the approach through a number of non trivial numerical simulations. The authors conclusions are summarized in section 6.

## 2. PROBLEM STATEMENT

Consider the autonomous nonlinear system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

with a possibly discontinuous right hand side.  $f : \mathfrak{R}^n \rightarrow \mathfrak{R}^n$  is measurable and essentially bounded. The Filippov set,  $K[f](x)$ , is formed by the images of the system vector field on a neighborhood of

point  $x$ . It is important that we can neglect a set of measure zero where the vector field is not defined.

For nonsmooth functions  $V$ , the gradient  $\partial V$  is defined as a set of vectors (Shevitz and Paden, 1994). The time derivative of a Lyapunov function for the system can be defined as:

$$\dot{V} \triangleq \bigcap_{\xi \in \partial V(x)} \xi^T K[f](\mathbf{x}).$$

Then  $\frac{d}{dt}V(x)$  exists almost everywhere and

$$\frac{d}{dt}V(\mathbf{x}) \in^{a.e.} \dot{V}$$

Lyapunov's direct method and LaSalle's invariant principle have been extended (Shevitz and Paden, 1994) to the case where either  $\mathbf{f}(\mathbf{x})$  is discontinuous,  $V(\mathbf{x})$  nonsmooth or both. These extensions facilitate the stability analysis of the discontinuous closed loop system by providing the necessary mathematical tools. They allow the system to be treated in the usual fashion in the framework of Lyapunov's direct method and LaSalle's invariant principle.

The problem at hand is to plan the motion of a mobile manipulator between any two arbitrary configurations, coordinating the mobile robot and the attached manipulator. The robot approaches a particular point in a tree environment, without colliding with the surrounding trees. The mobile manipulator is assumed to move on a flat surface, although this assumption can be relieved once specific information on the ground surface is available to allow its modeling. Both the mobile base and the manipulator are modeled kinematically. In such a framework, the mobile base is modeled as a unicycle while the manipulator will be trivially described as a parallel series of integrators. A joint configurations space description of the manipulator can accommodate obstacles.

An additional desired feature for the planning scheme is feedback. This would enable the rejection of disturbances and/or noise and the reduction of computational burden which is typical of optimal control methodologies.

The problem can be formally stated as follows:

Define  $\mathbf{z} \triangleq [x \ y \ \theta \ \mathbf{q}]^T$  where  $x, y$  denote the position of the mobile platform on the plane on which it is moving,  $\theta$  is its orientation, and  $\mathbf{q} \triangleq [q_1 \ \dots \ q_m]^T$  are the manipulator joint position vector. Then consider the system:

$$\dot{x} = v \cos \theta \tag{1a}$$

$$\dot{y} = v \sin \theta \tag{1b}$$

$$\dot{\theta} = \omega \tag{1c}$$

$$\dot{\mathbf{q}} = \mathbf{u} \tag{1d}$$

Given an obstacle free configuration space,  $C$ , and two arbitrary points  $\mathbf{z}_0, \mathbf{z}_f$  lying in the same con-

nected component of  $C$ , define a feedback scheme  $U : \mathbf{q} \mapsto [v \ \omega \ \mathbf{u}]$  such that the resulting trajectory  $\mathbf{r} : [0, T] \rightarrow C$ , satisfies  $\mathbf{r}(0) = \mathbf{z}_0$ ,  $\mathbf{r}(T) = \mathbf{z}_f$ .

The configuration space of the system is the manifold  $\mathbb{R}^2 \times \mathbf{S}^{m+1}$ . The obstacle free configuration space,  $C$ , is formed by removing from the configuration space the obstacle regions.

### 3. APPROACH TO SOLUTION

#### 3.1 Dipolar Navigation Functions

For obstacle and singularity avoidance potential fields provide a conceptually appealing option. Navigation functions are positive definite smooth functions, attaining a maximum at the boundary of the free configuration space, and vanishing only at the origin. Their gradient does not vanish except for the origin and perhaps a countable set of isolated points -therefore no local minima are created. Navigation functions serve as natural Lyapunov function candidates. The mathematical requirements for the existence of navigation functions are not extremely strict: every smooth connected and compact manifold with boundary admits a navigation function (Rimon and Koditschek, 1992).

Potential-based motion planning methods assume that the system can be described as a single point in the configuration space. Real robots however are generally multilink mechanisms consisted of rigid bodies. In order to shrink the robot to a point we have to account for its volume and increase the obstacles' volume accordingly (Latombe, 1991).

Contrary to a previous approach of the authors (Tanner and Kyriakopoulos, 2000), this procedure is now facilitated by the use of appropriate contraction mapping transformations. Both the robot and the obstacles are reduced to single points in the interior or at the boundary of the configuration space, respectively. A chain of ellipsoid-shaped volumes is first transformed to a sphere through consecutive *purging* transformations. Each ellipsoid-shaped volume in the chain is 'absorbed' into its "parent" link until the whole chain is reduced to a sphere. In this spherical space (Rimon and Koditschek, 1992) where the robot and the obstacles are represented by spheres, the motion planning problem for a non-zero volume robot can be reduced to an equivalent problem where the robot is reduced to a point and the obstacles are enlarged by the robot radius. The obstacle-free configuration spaces are homeomorphic. In this way the construction of the 'grown' obstacles is greatly facilitated.

The general form of the navigation function used is (Rimon and Koditschek, 1992):

$$V(\mathbf{z}) = \frac{\|\mathbf{z} - \mathbf{z}_f\|^2}{[\|\mathbf{z} - \mathbf{z}_d\|^{2k} + \beta(\mathbf{z})]^{1/k}}$$

where  $\beta(\mathbf{z})$  is an obstacle dependent function and  $k$  is a tuning parameter.

In (Tanner and Kyriakopoulos, 2000) a new type of navigation function was introduced: the *dipolar navigation function*. Dipolar navigation functions are constructed such that the flows of their potential field are tangent to the  $x$  axis at the origin. Stabilization of the mobile robot's orientation is achieved implicitly, by aligning the robot's trajectories with the potential flows.

It must be noted that it is always possible to generate a smooth dipolar potential field (e.g. by artificially increasing the potential along  $y$  axis.) A possible candidate could be:

$$V(\mathbf{z}) = \frac{\|\bar{\mathbf{z}} - \bar{\mathbf{z}}_f\|^2}{[\|\bar{\mathbf{z}} - \bar{\mathbf{z}}_d\|^{2k} + (x^2 + \varepsilon)\beta(\mathbf{z})]^{1/k}}$$

where  $\varepsilon$  a very small positive constant and  $\bar{\mathbf{z}}$  is defined as  $\bar{\mathbf{z}} \triangleq (x, y, \mathbf{q})^T$ .

#### 3.2 Motion Planning Scheme

Given a navigation function describing the robot workspace, we propose the following discontinuous feedback law:

$$v = -\text{sgn}\left(\frac{\partial V}{\partial x} \cos \theta + \frac{\partial V}{\partial y} \sin \theta\right) \cdot \left\{ k_v \left[ \left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 \right] + k_z (x^2 + y^2) \right\} \quad (2a)$$

$$\omega = k_\theta (\theta_d - \theta) \cdot \text{sgn}\left(-\frac{\partial V}{\partial \theta} (\theta_d - \theta)\right) \quad (2b)$$

$$u_1 = -\frac{\partial V}{\partial q_1} \quad (2c)$$

⋮

$$u_m = -\frac{\partial V}{\partial q_m} \quad (2d)$$

where  $V$  is the navigation function,  $k_v$ ,  $k_z$  and  $k_\theta$  are positive control gains. In particular, a large value for  $k_z$  keeps  $v$  from vanishing in configurations away from the origin where  $\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0$ . On the other hand,  $k_v$  amplifies the influence of the potential field on the system evolution.  $\theta_d$  is defined as:

$$\theta_d \triangleq \arctan 2(\text{sgn}(x) \frac{\partial V}{\partial y}, \text{sgn}(x) \frac{\partial V}{\partial x})$$

while the sign function is defined as

$$\text{sgn}(z) \triangleq \begin{cases} 1 & , z \geq 0 \\ -1 & , z < 0 \end{cases}$$

Stability for the system can be obtained via Lyapunov's direct method, utilizing features from nonsmooth analysis.

*Proposition 1.* Under the control law (2), the system (1) converges asymptotically to the origin.

**PROOF.** Consider a smooth navigation function as a Lyapunov candidate. It is positive definite by construction.

Then

$$\dot{V} = \bigcap_{\xi \in \partial V} \xi^T K \begin{bmatrix} v \cos \theta \\ v \sin \theta \\ \omega \\ \mathbf{u} \end{bmatrix}.$$

and since  $V$  is smooth,

$$\begin{aligned} \dot{V} &= \nabla V^T K \begin{bmatrix} v \cos \theta \\ v \sin \theta \\ \omega \\ \mathbf{u} \end{bmatrix} \subset \nabla V^T \begin{bmatrix} K[v] \cos \theta \\ K[v] \sin \theta \\ K[\omega] \\ \mathbf{u} \end{bmatrix} \\ &= -K \left[ \text{sgn} \left( \frac{\partial V}{\partial x} \cos \theta + \frac{\partial V}{\partial y} \sin \theta \right) \right. \\ &\quad \left. \left( \frac{\partial V}{\partial x} \cos \theta + \frac{\partial V}{\partial y} \sin \theta \right) \right. \\ &\quad \cdot \left\{ k_v \left[ \left( \frac{\partial V}{\partial x} \right)^2 + \left( \frac{\partial V}{\partial y} \right)^2 \right] + k_z (x^2 + y^2) \right\} \\ &\quad + \frac{\partial V}{\partial \theta} (\theta_d - \theta) K \left[ \text{sgn} \left( -\frac{\partial V}{\partial \theta} (\theta_d - \theta) \right) \right] \\ &\quad - \left( \frac{\partial V}{\partial q_1} \right)^2 - \dots - \left( \frac{\partial V}{\partial q_m} \right)^2 \\ &= - \left| \frac{\partial V}{\partial x} \cos \theta + \frac{\partial V}{\partial y} \sin \theta \right| \cdot \\ &\quad \left\{ k_v \left[ \left( \frac{\partial V}{\partial x} \right)^2 + \left( \frac{\partial V}{\partial y} \right)^2 \right] + k_z (x^2 + y^2) \right\} \\ &\quad - \left| \frac{\partial V}{\partial \theta} (\theta_d - \theta) \right| - \left( \frac{\partial V}{\partial q_1} \right)^2 - \dots - \left( \frac{\partial V}{\partial q_m} \right)^2 \leq 0 \end{aligned}$$

Then, by LaSalle's invariant principle (Shevitz and Paden, 1994), every trajectory converges to the largest invariant set which is included in  $S \triangleq \{\mathbf{z} | 0 \in \dot{V}\}$ .  $\dot{V}$  vanishes only when:

$$\begin{cases} \frac{\partial V}{\partial x} \cos \theta + \\ \frac{\partial V}{\partial y} \sin \theta = 0, \\ \text{or} \\ \frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = \\ x = y = 0 \end{cases} \quad \text{and} \quad \begin{cases} \frac{\partial V}{\partial \theta} = 0, \\ \text{or} \\ \theta = \theta_d \end{cases}, \quad \frac{\partial V}{\partial \mathbf{q}} = 0$$

In an invariant set inside  $S$  it is also required

$$\theta = \theta_d \quad \text{and} \quad \frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = x = y = 0$$

In this case,  $\theta_d = 0$  so  $\theta = 0$  and the invariant set reduces to the origin, where the closed loop system trajectories must converge.  $\square$

If conventional potential functions are used, then  $\theta_d$  will be discontinuous at the origin. This implies that the system would approach the set  $\{\mathbf{z} | \bar{\mathbf{z}} = \mathbf{0}\}$  with arbitrary orientation and reorient itself once being there. This is mathematically acceptable for a unicycle, but it is quite difficult for mobile robots with other steering mechanisms. With the use of

dipolar navigation functions, however,  $\theta_d \rightarrow 0$  as  $(x, y) \rightarrow (0, 0)$ . Following the flow lines of the potential field, the system converges uniformly to the origin.

#### 4. IMPLEMENTATION ISSUES

Typical to discontinuous control laws is the appearance of *chattering*. Chattering is characterized by a high frequency switching in the control signals. Under law (2) chattering occurs primarily due to the change in the sign of  $v$  when the platform velocity is almost tangent to an isopotential surface. In particular, chattering will occur at positions where:

$$\frac{\partial V}{\partial x} \cos \theta + \frac{\partial V}{\partial y} \sin \theta = 0 \quad \text{or} \quad \frac{\partial V}{\partial \theta} = 0$$

Chattering can have an impact on convergence time and common techniques for alleviating the effect are employed *hysteric switching* (Slotine and Sastry, 1983). It should be noted however that chattering triggering configurations are non-attractive under (2) and the phenomenon appears in relatively difficult circumstances. In fact, it can be shown that aligning  $\theta$  with  $\theta_d$  makes  $\frac{\partial V}{\partial x} \cos \theta + \frac{\partial V}{\partial y} \sin \theta \neq 0$ , for every  $(x, y, \theta, \mathbf{q}) \neq \mathbf{0}$ .

Another issue is the need for  $\theta$  to converge fast to  $\theta_d$ . Successful implementation depends on the ability of the system to track to flow lines of the potential field. The convergence rate for  $\theta$  can be adjusted by tuning the control gain  $k_\theta$ .

#### 5. SIMULATIONS

The simulation setup considered consists of three tree-shaped obstacles and a six degree of freedom mobile manipulator. The mobile manipulator has five main components as depicted in Figure 1. Its mobile base can translate horizontally and rotate around the vertical axis by means of steering. The onboard manipulator has three rotational joints. The first two links of the manipulator rotate around horizontal axes while the end effector rotates on a plane normal to the link on which it is mounted.

The tree environment is implemented in the form of a plane on which three tree-shaped obstacles reside (Figure 2). The mobile manipulator maneuvers amongst the trees and reaches a desired point on tree with a specified orientation. The number of the system degrees of freedom allows only a finite number of configurations that realize the desired grasp. In systems with more degrees of freedom some form of redundancy resolution must be employed for the determination of the goal configuration. During the motion, the proposed

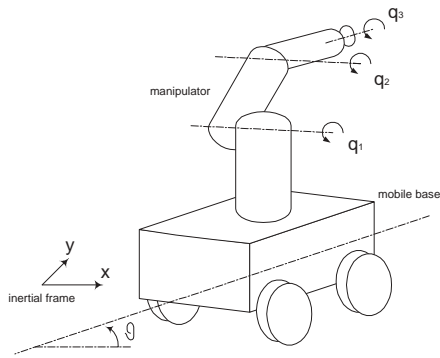


Fig. 1. The mobile manipulator

scheme is capable of resolving redundancy so that additional objective requirements can be satisfied (Tanner and Kyriakopoulos, 2000).

The simulation environment is depicted in Fig. 2. The objective for the robot end effector is to reach a specific point on the tree in the middle. This configuration corresponds to the position  $(x, y, \theta) = (0, 0, 0)$  for the mobile base and  $(q_1, q_2, q_3) = (35^\circ, 63^\circ, 0^\circ)$  for the manipulator. The robot is initially positioned at  $(x, y, \theta, q_1, q_2, q_3) = (10, 8, 0^\circ, 0^\circ, 0^\circ, 0^\circ)$  (Fig. 2).

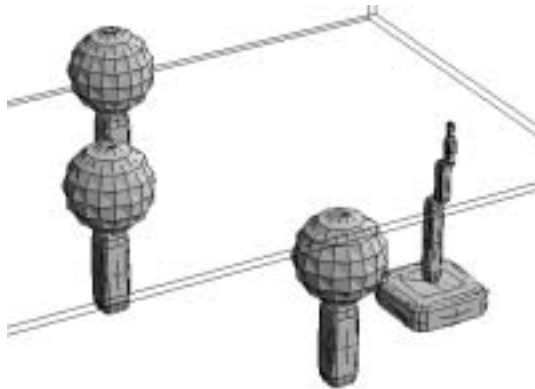


Fig. 2. Initial position

Screenshots of the motion of the robot are shown in Fig. 3-6. In Fig. 3 the mobile base starts to move trying to reorient itself in order to be able to approach the middle tree. The manipulator links do not move significantly yet since this would hindered the motion of the robot.

The robot maneuvers to fix the position and orientation of its mobile base Fig. 4. During this time the manipulator links move very slowly.

At the next screen shot (Fig. 5), the robot has sufficiently approached the desired position for its mobile base and begins to reorienting the manipulator links.

Finally, Fig. 6 shows the mobile manipulator at the desired final configuration. Small maneuvers for the mobile base take place while the manipulator is fine tuning its position.

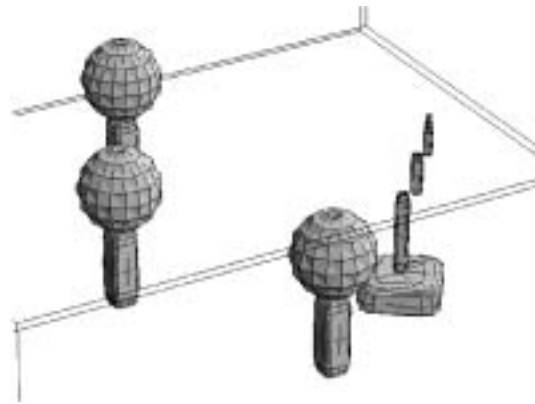


Fig. 3. Beginning of motion (at 20 iterations)

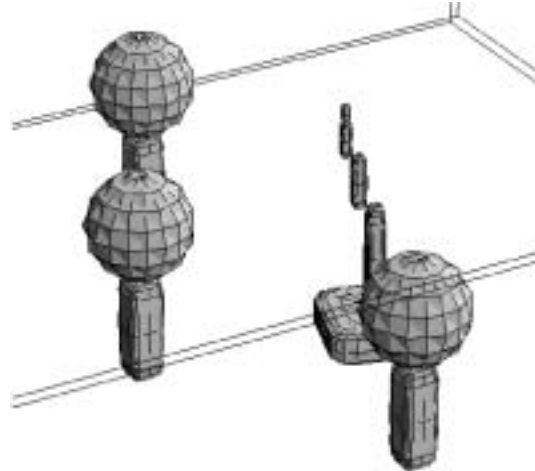


Fig. 4. Maneuvering (at 160 iterations)

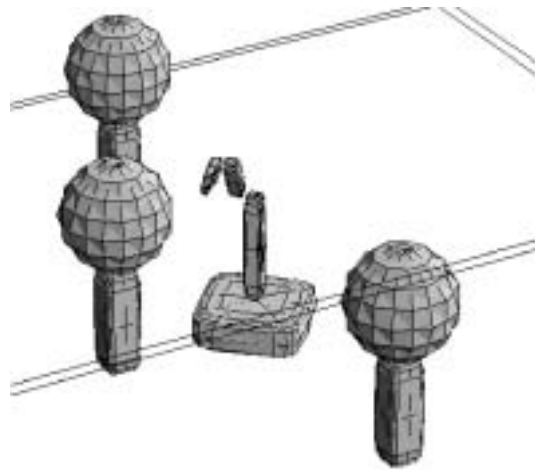


Fig. 5. Moving the manipulator (at 360 iterations)

## 6. CONCLUSION

In this paper the motion planning problem for a nonholonomic mobile manipulator navigating in an outdoor tree environment is investigated. The proposed motion planning scheme is based on a full state discontinuous feedback law which guarantees convergence to the desired final position and collision avoidance. All information about the

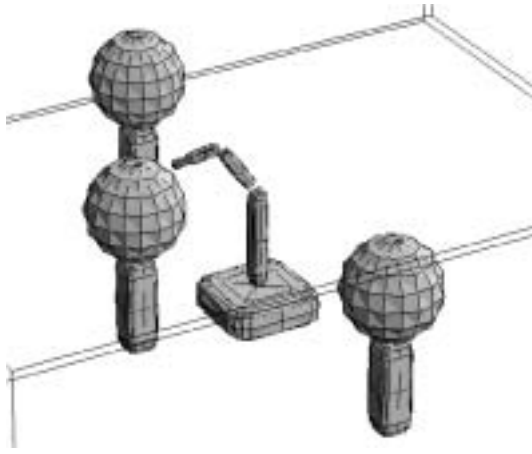


Fig. 6. Final configuration

robot and the environment geometry, constraints and objectives is integrated into a potential field function through which motion planning and coordination is achieved. The scheme merges path planning and trajectory generation and is suitable for real time implementation. The method can easily be combined with a tele-operated scheme allowing the user to specify only high-level tasks.

One of the intrinsic limitations of the potential-field approach is the requirement for a topological model of the robot's geometry and its environment. At this issue, however, potential-based methods are superior to classical findpath algorithms since the symbolic mathematical representation of the workspace theoretically allows a computationally cheap adaptation of the navigation function parameters as new (or revised) information comes along (Rimon and Koditschek, 1992).

Research is continued on the construction of new kind of dipolar navigation functions allowing faster convergence and easier grown obstacle representation. The method seems promising for coordinating the simultaneous motion of multiple robots under several configuration dependent constraints.

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