

INFORMATION SURFING FOR RADIATION MAP BUILDING

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Abstract

We develop a control scheme for a group of mobile sensors to map radiation over a given planar polygonal region. The advantage of this methodology is that it provides quick situational awareness regarding radiation levels, which is being updated and refined in real-time as more measurements become available. The control algorithm is based on the concept of information surfing, where navigation is done by following information gradients, taking into account sensing performance and the dynamics of the observed process. It is tailored to cases of weak radioactivity, where source signals may be buried in background. We steer mobile sensors to locations which are critical points of a function that quantifies the information content of the measured signal, while

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the time-asymptotic properties of the selected information metric facilitate the stability of the group motion. Information surfing allows for reactive mobile sensor network behavior and adaptation to environmental changes, as well as human retasking. Computer simulations and experiments are conducted to verify the asymptotic behavior of the robot group, and its distributed sensing and mapping capabilities.

Key words: Sensor networks, cooperative control, radiation sensing

1 Motivation

This work is motivated by the emerging threat of contamination from a malicious attack or accidental release of radioactive material. In such a case, a radiation map can be a valuable tool for rescue, response, and cleanup efforts. Although our focus is on an unexpected release of nuclear contaminants, our methods can also be applied to a wide range of other problems such as nuclear forensics and non proliferation, where robots could investigate the possibility that fissile material has been processed, by searching for small specks of that material.

Existing technology in radiation detection is not well suited for the type of scenario described. Currently, searching for radiation sources is usually either done manually, by operators waving radiation counters in front of them as they walk, or by stationary portal monitors. The latter technology is used to detect radioactive sources in cargo or shipping containers at places such as ports of entry.

Handheld radiation detectors do not provide any visual or statistical data map of the area in question. If human operators are used in the nuclear forensics problem, it is unlikely that their counters register any measurement

at all, coming from a small amount of radioactive material or shielded special nuclear material (SNM). Portal monitor systems on the other hand, while able to address the problem of scanning cargo for radioactive sources, lack the mobility needed when measurements are taken over a large area.

2 Related Research

Existing control design methods for multiple mobile robot coordination apply to problems ranging from topological mapping to formation and flocking tasks, as well as reconfigurable sensor networks.

2.1 Sensor network deployment

Reference [1] employs potential fields to reconfigure a mobile sensor network. In [2] and [3] a gradient climbing algorithm is used to distribute sensor platforms in a geometrically optimal fashion over a given polygonal area. In [4], another gradient climbing method is used for the control of a sensor network that does not split the area among the team members. The authors of [5] extend the coverage algorithm of [3] to account for tracking of moving targets. Their approach involves time-varying coverage functions—similarly to this paper—but the stability analysis follows a completely different route, since it does not involve information theory.

Scalable mobile sensor coordination requires decentralized control schemes, similar in nature to those that have recently developed for flocking and formation control. In [6] a navigation function with Voronoi partitions is used to move the group of agents from one region to another *goal* region in the area while keeping formation. Reference [7] addresses a type of *pursuit evasion* game, where the group of robots tries to keep a formation

and enclose the *evader*. In [8], robots fall into formation by selecting a leader of the group and having each agent maintain a certain position and orientation with respect to its neighbor.

In hazardous environments mobile robots could maneuver, take measurements, and build a map without human intervention. Utilizing large numbers of mobile robots speeds up the map building task and makes the map available faster. Mobile robots can be equipped with a variety of sensors, so both topological (obstacles vs. free space) and spatial distribution maps can be created. Spatial distributions of interest include temperature, gas, as well as radiation.

2.2 Active sensing and source seeking

The idea of active sensing has been explored recently in relation to its applications in environmental monitoring, search and rescue, and source seeking problems. In one of the early approaches [9] a gradient descent algorithm is used to cooperatively estimate the state of dynamic targets in an optimal fashion. The work in [10] extends [9] by implementing the control algorithm in a distributed fashion.

Active sensing is not limited to target estimation. In [11], a source seeking problem for an autonomous vehicle modeled as a nonholonomic unicycle is addressed, whereas [12–14] look into the problem of source seeking when direct gradient information of the measured quantity is unavailable. Reference [12] tackles chemical plume source localization by constructing a source likelihood map based on Bayesian inference methods. The work in [13] induces source seeking behavior without direct gradient information by mimicking *E. Coli* bacteria. Finally, the authors of [14] propose a hy-

brid control strategy to locate a radiation source utilizing radiation intensity measurements only. Their work concentrates on control design, and does not include the statistics of radiation measurement.

2.3 Spatial distributions and topological maps

Most of the recent work in the area of mapping focuses on obtaining a detailed topological or metric map of the environment in which the robots are deployed. Accurate two-dimensional and three-dimensional maps are constructed for static [15] and dynamic environments [16]. To build detailed metric maps, laser-range finders [17] or sonar sensors [18, 19] are typically used. Establishing line of sight between sensors and environmental features is a critical aspect of topological mapping.

Mapping spatial distributions can provide insight to the short or long term effects of variation in temperature, pressure, water current, and may warn against threats from pollutants. In [20] gas concentrations maps are created by maneuvering a robot over a predefined path that covers the entire area. An approach to search for ocean features is proposed in [21], using virtual bodies and artificial potentials to coordinate multiple robots into gradient climbing, in order to locate and track ocean features such as fronts and eddies. In [22] active learning techniques are used to map water currents to understand the hydrodynamics present in Lake Wingra.

2.4 Entropy-based approaches in sensor networks

Entropy-based metrics quantifying uncertainty or information gain are not new, and have been used in robotic exploration, localization and mapping. In [23] entropy is used among other utility measures, to evaluate the benefit

of visiting different locations in the context of exploration. However, that entropy describes the uncertainty over a certain location being reachable; it is not directly associated with the quality of the model nor is it linked with the statistics of measurement. Mutual information is used as a metric of significance of different discrete locations containing features or targets of interest, in an application where mobile robots and unmanned aerial vehicles (UAVs) localize target features in their environment [24]. The Fisher information is another metric used to describe control actions for general information acquisition tasks [25] as well as motion coordination tasks [26].

The most closely related work to the one presented here is probably that of [27] and [28]. In [27] the authors derive an entropy-based metric for determining optimal sensing poses for mobile robot agents that create a detailed three dimensional model of their environment. The main difference between [27] and our approach is that the underlying statistics are completely different. Here, radiation measurement statistics results in closed loop controllers which are both state and time dependent, making the stability analysis considerably more difficult.

In [28] the problem of maximizing information while minimizing communication cost between wireless sensor nodes is addressed in an area coverage setting. Although similar to our approach, [28] proposes a “one time” solution for a static sensor coverage problem, whereas we are addressing a dynamic sensor coverage problem.

3 Nuclear Search and Mapping

Sequential nuclear search allows one to quickly verify the existence of microscopic specks of radioactive material. When a ray of radiation emitted from

a source reaches a sensor, the latter is said to register a *count*. Radiation intensity is measured in counts per second, assuming that all emitted rays are detected and registered. Low-rate counting of radiation from nuclear decay is described by the Poisson statistics. Classical sequential testing theory [29] suggests the stopping rules, that is, when one knows with certain confidence that a source exists at a given point.

In [30] and [31] a procedure to automate nuclear search using a strategy based on the classical sequential testing theory in [29] is described. This strategy, however, is a detection strategy, not a mapping technique. Even as a detection tool, for which the technique is optimal in terms of completion time, there is an important caveat. The algorithm is sensitive to the assumed strength of the source that is to be detected. If the source intensity is underestimated, the method will give a false negative by default.

The authors in [32] develop an algorithm which calculates and updates, in real-time, the belief about local radiation levels, thus creating a radiation map of the area in question. The variance of the local radiation level distribution is used as a metric to assess the accuracy of the radiation map. Two methods for navigating to obtain the radiation measurements are developed, the *sequential-based Bayesian search* and the *gradient-based Bayesian search*. Both methods use Bayes rule to update the radiation distribution over the area, but differ in the approach to navigation.

4 Modeling Radiation Distributions

Low-rate counting of radiation from nuclear decay is described by a homogeneous Poisson stochastic process. The probability to register n counts in

t seconds, from a source emitting an average of μ counts per second is:

$$P(n, t) = \frac{(\mu \cdot t)^n}{n!} e^{-(\mu \cdot t)}$$

In [33] the radiation measurement mechanism of stationary sensor receiving counts from a moving source is modeled. Similarly, we analyze the case of a stationary source detected by a moving sensor, and express the expected number of source counts per second μ as:

$$\mu = \chi \cdot \alpha \int_0^t \frac{1}{r^2(\tau)} d\tau \quad (1)$$

where χ is the cross sectional area of the sensor, α is the activity of the source measured, and $r(t)$ is the instantaneous distance of the source to the sensor. Poisson statistics suggests a probability density function (PDF) associated with the random variable expressing the total number of counts c recorded by the moving sensor to be of the form:

$$f(c) = \frac{(\mu)^c}{c!} \cdot e^{-(\mu)} \quad (2)$$

where μ is given by (1). In fact, the expected number of counts per second μ in (1) is conditioned on the source having activity α , the cross sectional area of the sensor being χ , and the distance between the source and sensor being $r(t)$. Therefore the PDF associated with c is actually $f(c) = f(c|\alpha, \chi, r(t))$.

4.1 Radiation Maps

Bayes' rule allows us to update our knowledge of the distribution of radiation based on new radiation measurements taken by the mobile robot sensors as:

$$f(\alpha|c, \chi, r(t)) = \frac{f(\alpha) \cdot f(c|\alpha, \chi, r(t))}{f_c(c)} \cdot \sigma \quad (3)$$

In the above, $f(\alpha)$ is the PDF of a source with activity α being at location p , $f(c|\alpha, \chi, r(t))$ is the PDF of registering c counts from a source of intensity

α with a sensor of cross sectional area χ positioned a distance r away from p , $f_c(c)$ is the marginal density function (MDF) associated with registering c counts from a source with activity α , and σ is a scaling constant. The conditional PDF of α is what we call the radiation map.

Without any knowledge about the radiation distribution, we assume initially that radiation levels are distributed uniformly, from source activity α_1 to a source activity α_2 . Using the uniform distribution allows us to search and map radiation levels from an arbitrary source with activity between a background radiation level of α_1 , and a nuclear material of activity α_2 . In general, $f(\alpha)$ is a function of position too (in the case of a uniform distribution, position is irrelevant). The PDF expressing our initial guess about the source activity can now be expressed as:

$$f(\alpha) = \begin{cases} \frac{1}{\alpha_2 - \alpha_1}, & \text{if } \alpha_1 < \alpha < \alpha_2 \\ 0, & \text{otherwise} \end{cases}$$

If prior knowledge is known about the radiation distribution, $f(\alpha)$ can be set accordingly. Function $f_c(c)$ is the marginal density function of $f(c)$ and is evaluated for the particular case as:

$$f_c(c) = \int_{\alpha_1}^{\alpha_2} \frac{(\chi \cdot \alpha \int_0^t \frac{1}{r^2(\tau)} d\tau)^c}{c!} \cdot e^{-(\chi \cdot \alpha \int_0^t \frac{1}{r^2(\tau)} d\tau)} d\alpha$$

Equation (3) can be used in a recursive fashion to improve our model of the radiation distribution. When new measurements are recorded, we improve the radiation map by narrowing down on the source activity that is producing the number of registered counts. The more accurate the prior distribution the algorithm starts with, the faster the map is completed.

Although this development applies to a single radiation sensor, the radiation mapping task can be accelerated by using multiple coordinated sensors

to distribute the work load.

4.2 Distributed Sensors

We think of our n mobile robot sensor platforms as pieces of a single “shattered” sensor. We can now define the expected number of source counts per second μ for a group of n distributed radiation sensors in the following way:

$$\mu = \chi \cdot \alpha \int_0^t \frac{1}{\min_i \|q - p_i(\tau)\|^2} d\tau \quad (4)$$

where χ is the cross sectional area of the sensor, α is the activity of the source measured, and $\min_i \|q - p_i(\tau)\|$ is the minimum distance from all of the sensors p_i to a radiation source at location q . Notice the only difference in (1) and (4) is the way we define distance between the source and sensor. Defining the distance as the minimum distance between any robot sensor platform p_i and source location q also suggests how to allocate the mapping task among the n robot sensor platforms.

Following [3], the area in which radiation is to be mapped Q is assumed to be a simple convex polygon in \mathbb{R}^2 including its interior. Let P be a set of n distinct points $\{p_1, \dots, p_n\}$ that reside in the interior of Q . The Voronoi Partition of Q , generated by P is the set of all points in Q such that all points in the Voronoi cell $V_i(P)$ are closer to p_i than any other point in Q :

$$V_i(P) = \{q \in Q \mid \|q - p_i\| \leq \|q - p_j\|, \forall p_j \in P\}$$

5 Radiation Measurement Uncertainty

Based on the number of counts registered by a radiation sensor, and the measurement statistics, we derive the differential entropy and mutual information associated with this radiation measurement. If a radiation sensor is

viewed as a channel between the world and the system, then mutual information quantifies the information gained by each measurement.

Information theory defines the conditional differential entropy of continuous random variables A and C , associated with the transmitted signal, (here: the radiation source activity α), and with the received signal, (here: the number of counts registered by our sensors c), respectively, as follows:

$$h(A|C) = - \int_{\alpha_1}^{\alpha_2} f(\alpha|c) \cdot \log_2 f(\alpha|c) d\alpha \quad (5)$$

It is known that continuous differential entropy cannot be directly associated with information gain or uncertainty —contrary to the discrete case [34]. Mutual information $I(X; Y)$, however, which captures how knowledge of random variable Y , reduces our uncertainty about the random variable X , does carry over, and enjoys the following useful property:

Lemma 5.1 ([35]). *$I(X; Y) \geq 0$ with equality iff X and Y are independent.*

For the problem at hand, mutual information is expressed as $I(A; C) = h(A) - h(A|C)$, where $h(A) = (\alpha_2 - \alpha_1) \left[\frac{1}{\alpha_2 - \alpha_1} \cdot \log \left(\frac{1}{\alpha_2 - \alpha_1} \right) \right] \triangleq K$, and K is a constant. With $h(A|C)$ as in (5), mutual information reduces to:

$$I(A; C) = K - h(A|C) \quad (6)$$

Equation (6) expresses how knowing the number of radiation counts, reduces our uncertainty regarding the presence of the source A .

6 How to Maximize Information Gain

6.1 Objective and assumptions

A *performance function* $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a non-increasing and piecewise differentiable map with finite jump discontinuities [3]. We use the performance

function as a quantitative model of the signal-to-noise ratio (SNR) of our radiation sensor, which drops as the distance between sensor and source increases [36]. Our performance function is a smooth function based on the constructions in [37], which models an exponential decrease in signal quality as the distance from the sensor increases, no useful signal beyond the sensor's detection range R , and ideal (perfect) sensing at the footprint of the sensor given as a fraction ϵ of the range R :

$$f(\|q - p_i\|) = \begin{cases} \frac{\exp\left(\frac{-1}{R^2 - \|q - p_i\|^2}\right)}{\exp\left(\frac{-1}{R^2 - \|q - p_i\|^2}\right) + \exp\left(\frac{-1}{\|q - p_i\|^2 - \epsilon^2 R^2}\right)} & , \epsilon R < \|q - p_i\| < R, \\ 1 & , 0 \leq \|q - p_i\| \leq \epsilon R, \\ 0 & , R \leq \|q - p_i\| \end{cases} \quad (7)$$

The sensor detection range, denoted R , is considered constant, and $\|q - p_i\|$ is the Euclidean distance from the sensor to the source. Function f is identically equal to one for part of the sensing range. This limit is determined by the geometry of the particular sensor; in our case, a perfect reading can be obtained along the whole length of the sensor, and not just at a particular point. Function f may be tuned, if needed, by introducing a scalar coefficient in the numerator of the exponentials in the first branch of (7).

The authors in [3] maximize the sum of the all the functions \mathcal{H}_i :

$$\mathcal{H}(P) = \sum_{i=1}^n \int_{V_i(P)} f(\|q - p_i\|) \phi(q) \, dq$$

where $f(\|q - p_i\|)$ is the sensing performance function and $\phi(q)$ a density function. To formulate the problem at hand, the static density function ϕ is replaced with mutual information (6):

$$\mathcal{W}(P) = \sum_{i=1}^n \int_{V_i(P)} f(\|q - p_i\|) I(q, P, t) \, dq$$

where $f(\|q - p_i\|)$ is given in (7). We thus substitute a time and configuration varying quantity $I(q, P, t)$ for a static density function, $\phi(q)$. Due to the dynamic nature of $I(q, P, t)$, control and stability analysis are inherently more difficult. Note that contrary to [5], the density function here (mutual information) is also dependent on the robot group configuration. $I(q, P, t)$ is time dependent because the mutual information depends on the number of radiation counts collected. Comparing equation (6) to equations (5), (3), (2), and (1), we see that $A = \alpha(q)$ (distribution of sources) and $C = c(t)$ (radiation counts with respect to time). In the following section we exploit the asymptotic properties of the mutual information to establish stability.

6.2 Control design

Assume that the kinematics of each robot i are:

$$\dot{p}_i = u_i \quad (8)$$

where u_i is the control input, designed as:

$$u_i = \frac{\partial \mathcal{W}(P)}{\partial p_i} = \left[\int_{V_i(P)} \frac{\partial f(\|q - p_i\|)}{\partial p_i} I(q, P, t) dq + \int_{V_i(P)} f(\|q - p_i\|) \frac{\partial I(q, P, t)}{\partial p_i} dq \right] \quad (9)$$

The following Lemmas are important for establishing stability:

Lemma 6.1. *Let $I(q, P, t)$ be the mutual information of the (virtual) radiation sensor information channel. Then, $\lim_{t \rightarrow \infty} \frac{\partial I(q, P, t)}{\partial t} = 0$.*

Proof. See [38]. □

Lemma 6.2. *If the mutual information of the (virtual) radiation sensor information channel $I(q, P, t)$ converges to a constant, then for the system defined by (8)–(9), $\dot{p}_i \rightarrow 0$ as $t \rightarrow \infty$ for all $i = 1, \dots, n$.*

Proof. This follows directly from [3], because in the case where $I(q, P, t)$ becomes constant we are left with a static density function which has been shown to converge. \square

The following proposition states that (8)-(9) stabilizes in configurations where the information flow to the sensor from its environment is locally maximized. The robots stop when no new information can be gained from the environment.

Proposition 6.3. *Consider the gradient field defined by (8) – (9). Then the system stabilizes at configurations that are (locally) critical points of the information flow from each robot, as expressed by the product $I(q, P, t)f(\|q - p_i\|)$, for $i = 1, \dots, n$.*

Proof. Notice that in the expression:

$$\frac{\partial I(q, P, t) f(\|q - p_i\|)}{\partial p_i} = \frac{\partial f(\|q - p_i\|)}{\partial p_i} I(q, P, t) + f(\|q - p_i\|) \frac{\partial I(q, P, t)}{\partial p_i}$$

we have $\frac{\partial f(\|q - p_j\|)}{\partial p_i} = 0, \forall i \neq j$, and $\frac{\partial I(q, P, t)}{\partial p_i} = 0, \forall q \notin V_i(P)$. Define $\mathcal{W}(P) \triangleq \sum_{i=1}^n \mathcal{W}_i(P)$. Taking time derivatives:

$$\begin{aligned} \dot{\mathcal{W}}(P) &= \sum_{i=1}^n \frac{\partial \mathcal{W}(P)}{\partial p_i} \cdot \dot{p}_i + \frac{\partial \mathcal{W}(P)}{\partial t} = \sum_{i=1}^n \left[\frac{\partial \mathcal{W}(P)}{\partial p_i} \right]^2 + \frac{\partial \mathcal{W}(P)}{\partial t} \\ &= \sum_{i=1}^n \left\| \frac{\partial \mathcal{W}_i(P)}{\partial p_i} \right\|^2 + \frac{\partial \mathcal{W}(P)}{\partial t} \end{aligned} \quad (10)$$

because $\frac{\partial \mathcal{W}_i}{\partial p_j} = 0$ for $i \neq j$.

We establish our main result by contradiction: assume that the system does not stabilize to configurations where $\frac{\partial I(q, P, t) f(\|q - p_i\|)}{\partial p_i} = 0$. This partial derivative is time varying because of $I(q, P, t)$. If $I(q, P, t)$ were to converge, the partial derivative would also converge to a constant, which we assume is

not zero. However, since Lemma (6.1) establishes that $\lim_{t \rightarrow \infty} \frac{\partial I(q, P, t)}{\partial t} = 0$ there exists a time instant T for which there will be an $\epsilon > 0$ such that $\left\| \frac{\partial I(q, P, t) f(\|q - p_i\|)}{\partial p_i} \right\| > \epsilon \quad \forall t > T$. Now note that (10) can be written:

$$\begin{aligned} \dot{\mathcal{W}}(P) &= \sum_{i=1}^n \left\| \frac{\partial \mathcal{W}_i(P)}{\partial p_i} \right\|^2 + \frac{\partial \mathcal{W}(P)}{\partial t} \\ &= \sum_{i=1}^n \left(\int_{V_i} \left\| \frac{\partial I(q, P, t) f(\|q - p_i\|)}{\partial p_i} \right\|^2 dq \right) + \frac{\partial \mathcal{W}(P)}{\partial t} \end{aligned}$$

since the assumed statement that $\left\| \frac{\partial I(q, P, t) f(\|q - p_i\|)}{\partial p_i} \right\| > \epsilon$ for $t > T$, implies that $\frac{\partial I(q, P, t) f(\|q - p_i\|)}{\partial p_i}$ maintains a constant sign after that time; without being able to cross zero, it has to be either constantly positive or constantly negative, and thus equality is preserved when the norm is taken inside the integral. It follows that:

$$\dot{\mathcal{W}}(P) > \int_{V_i} \epsilon^2 dq + \sum_{i=1}^n \int_{V_i(P)} f(\|q - p_i\|) \frac{\partial I(q, P, t)}{\partial t} dq$$

If we define the following functions:

$$\begin{aligned} W_3(\epsilon) &\triangleq \int_{V_i(P)} \epsilon^2 dq \quad (\text{positive constant}), \\ \beta(t) &\triangleq \sum_{i=1}^n \int_{V_i(P)} f(\|q - p_i\|) \frac{\partial I(q, P, t)}{\partial t} dq \end{aligned}$$

we can write $\dot{\mathcal{W}}(P) > W_3(\epsilon) + \beta(t)$. For $\beta(t)$, knowing that $\lim_{t \rightarrow \infty} \frac{\partial I(q, P, t)}{\partial t} = 0$, and noticing that the integration is over position only, we conclude that:

$$\lim_{t \rightarrow \infty} \int_{V_i(P)} f(\|q - p_i\|) \frac{\partial I(q, P, t)}{\partial t} dq = 0$$

Therefore, irrespectively of the sign of $\beta(t)$, there will be a time instant where $|W_3(\epsilon)| > |\beta(t)|$, thus $\dot{\mathcal{W}}$ will be strictly positive, and bounded away from zero. This states that $\mathcal{W}(P)$ will grow monotonically after a certain point in time with a nonvanishing rate of increase.

This is a contradiction because $\mathcal{W}(P)$ is finite unless $I(q, P, t)$ goes to infinity, which can only happen when radiation activity can be directly and accurately measured, not inferred from measured radiation counts. \square

7 Implementation Issues

The radiation map is supposed to be obtained by applying (3) over time. Equation (3) gives the distribution of radiation activity, conditioned upon the measurements and the robots motion. There are, however, several reasons why the direct application of (3) may be impractical.

One issue is the numerical instability that can be observed when calculating the derivatives of mutual information in (6), due to the fact that the expressions involve incomplete gamma functions, evaluated over long time periods (implying large values for c). The incomplete gamma functions evaluate to very large numbers, sometimes causing numerical overflow. For this reason, we resort to a receding horizon type of approach, where we calculate these derivatives for an appropriately short time interval, update the radiation prior, and then repeat with new initial conditions.

Once such an approach is adopted, a second issue arises: the expressions for the marginal distributions and the mutual information derivatives are based on the assumption that the initial radiation prior is uniform. This assumption enables one to obtain closed form expressions for these derivatives, and subsequently for the control law. If $f(\alpha)$ is updated in real time, the radiation prior is no longer uniform when initializing the algorithm at the next step of the “receding horizon” method. One option is to evaluate the derivatives numerically; another, which we chose to take, is to discretize the radiation and mutual information maps in the form of a three-dimensional

bar chart, where each cell is assigned to a uniform distribution, but the distribution is different among different cells. The map can then be updated cell by cell, since within each cell a uniform prior is assumed.

8 Experiments

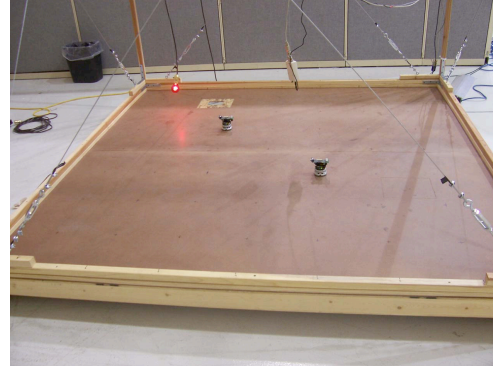
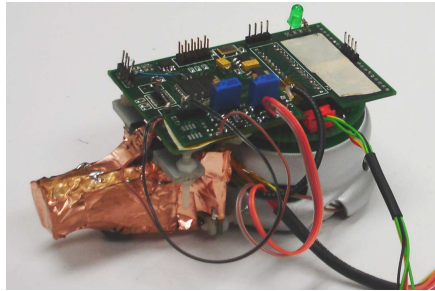
Rather than experimenting with actual radioactive material¹ we use a light source to emulate radiation emission intensity (Fig. 1(b)). Each robot takes its light intensity measurement and passes it through a Poisson distribution filter as the mean count rate at that particular cell. This filter returns a randomly distributed number from a Poisson distribution taken with a mean of the measured light intensity.

8.1 Experimental Snapshots

The area over which radiation is mapped is discretized into a 6×6 cell grid, and a uniform distribution between $\alpha_1 = 1$ and $\alpha_2 = 10$ counts/sec is assumed each cell's radiation intensity α . Localization of the robots is done through odometry and triangulation using distance measurements from Crossbow Crickets placed on the robots (Fig. 1(b)).

Fig. 2 shows the change in the information map and the updated radiation intensity map after two updates (each update takes 5 seconds). The completed radiation intensity map is shown in Fig. 3 and is obtained after 17 updates.

¹Preliminary experiments with a 10 nCi Na-22 radiation source, using the Khepera platform of Fig. 1(a) have been conducted by the first author at Los Alamos National Laboratory facilities.



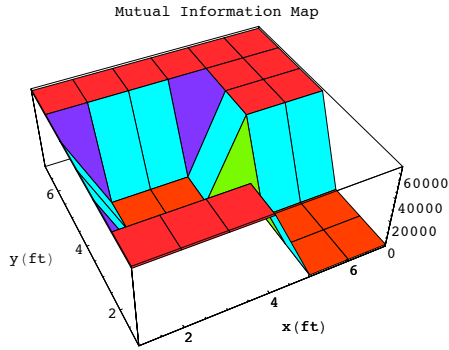
(a) A small radiation sensor — CsI(Tl) scintillator— covered with copper tape to reduce noise, and interfaced with a Khepera II robot for low-count radiation detection.

(b) The experimental test bed with two mobile robots, each carrying a cricket beacon. The origin of the coordinate system is at the near right corner of the platform, and the source in the far left corner.

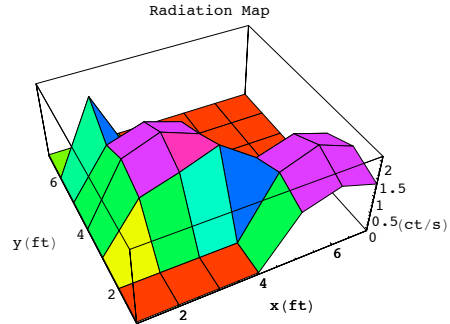
Figure 1: The mobile radiation sensor platform developed for mapping and the experimental testbed where the methodology was implemented. In our experiments, the radiation source was simulated with a red light bulb, exploiting the sensitivity of the onboard IR sensors to red light wavelength.

9 Summary

We developed a control scheme for a team of mobile robots with radiation sensing capabilities to build a radiation map in a distributed fashion, as an automated means of obtaining situational awareness in possibly radioactive environments. Although our control scheme can be applied to radiation “source seeking” type problems, the novelty in our approach is that the robots prioritize their measurement collection based on the information content of each prospective measurement. Robot controllers require position information from neighboring robots and need to share the mutual

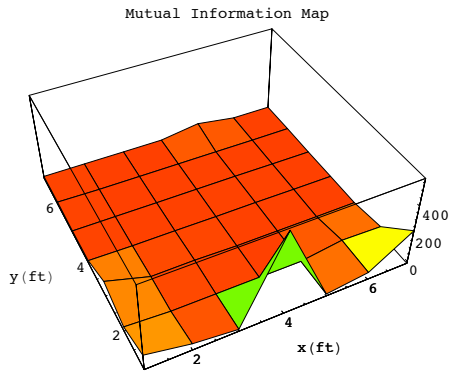


(a) Mutual information map.

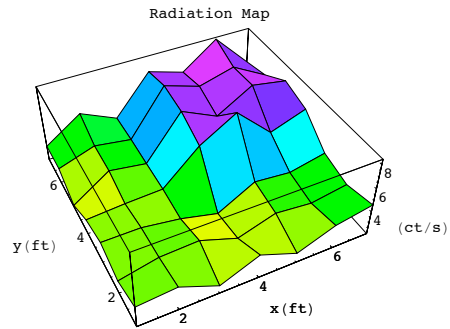


(b) Radiation intensity map

Figure 2: Map estimate after two updates.



(a) Mutual information map.



(b) Radiation intensity map

Figure 3: Map estimate after 17 updates.

information map, which is being updated on-line. We established the stability of the closed loop, time varying system by exploiting the asymptotic properties of the mutual information in radiation measurements.

Future research directions include incorporating multiple sensing behaviors by the robot team to further realize situational awareness. Multiple types of environmental information may aid in the speed of rescue, response,

and clean up. Another avenue of future research is in the connectivity of the robot team. We hope to incorporate some aspects of communication constraints to better model the real-time nature of the robot communication network.

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