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The effect of Feedback and Feedforward on Formation ISS

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Abstract

A new type of stability of leader follower formations is defined, based on input-to-state stability (ISS) properties of cascade interconnections. Formation ISS links leader input to internal state of the formation and characterizes the way this input affects performance. The effect of feedforward and feedback inter-agent communication is then investigated in this framework and it is indicated how the structure of interconnections and the amount of available information can affect stability performance.

1 Introduction

Recent advances on communication and computation have enabled the development of multi-agent robotic systems. Methods for analyzing interconnected systems are therefore necessary. Such methods find applications in automated highway systems [1, 2, 3], mobile robot reconnaissance [4], formation flight control [5, 6] and sattelite clustering [7].

Existing methods are based mainly on three different approaches to interconnection architecture. In the behavior based approach [4, 8, 9] each agent is thought of being able to exhibit a number of primary behaviors. The group behavior emerges as a weighted sum of the independent behaviors of its agents. In [4] behavior - based schemes are implemented on formations of unmanned ground vehicles and different formation types are tested. In [9] elementary behavior strategies for maintaining a circular formation are developed with the use of potential field methods. Another approach focuses on maintaining a certain group configuration and forces each agent to behave as a particle in a rigid virtual structure [10, 11]. In [11] the agents try to maintain a virtual structure defined around an artificial reference agent called the virtual leader, using a centralized potential-field control scheme. The leader-follower approach [6, 12, 13, 14] distinguishes a designated leader which the other agents follow either directly or indirectly. In [14] feedback linearizing controllers are developed for the control of mobile robot formations in which each agent is required to follow one or two leaders. Reference [13] investigates the conditions under which a set of formation constraints can be satisfied given the dynamics of the agents and consider the problem of obtaining a consistent group abstraction for the whole formation.

Stability properties of interconnected systems is investigated using the notion of string stability [2, 3]. String stability actually requires that internal errors attenuate as they propagate through the interconnections. For this to be possible, inter-agent communication and (exponential) stability of the unforced system of each agent is typically required.

The approach presented in this paper is based on input-to-state stability [15]. We define *formation input-to-state stability (ISS)*, that relates the leader input to the internal state of the formation. Formation ISS stability requires only state feedback information from the preceeding agent. By exploiting the fact that ISS is preserved in cascade interconnections [16, 17], it is possible to propagate ISS properties from a pair of leader-follower to the whole formation and obtain gain functions that constraint internal errors based on the formation leader input. In this framework, different formation types can be characterized according to their stability properties. Then, the influence of additional feedback and feedforward information on stability performance is investigated.

The outline of the paper is as follows: section 2 introduces the formation dynamics considered in this paper and defines formally the notion of input-to-state stability for formations. In section 3 the ISS properties of a leader-follower interconnection within the considered formation are investigated. Section 4 describes how the leader-follower ISS gains can be used to calculate the gains of the whole formation. Section 5 examins the effect of additional feedforward information on formation stability. Example cases are presented in section 7 and section 8 summarizes the results.

2 Formation Input-to-State Stability

Consider a collection of n agents, the kinematics of which are represented by an equation of the form:

$$\dot{x}_i = u_i, \quad i \in \mathcal{N} \triangleq \{1, \ldots, n\} \tag{1}$$

where $x_i \in \mathbb{R}^n$ denotes the state of agent i in absolute coordinates and u_i its input.

A formation is constructed by defining feedback control laws for the agents:

$$u_i = K_i(x_j - x_i) \tag{2}$$

where K_i is positive definite, implying that agent i follows agent j. One agent, $L \in \mathcal{N}$, is assigned to be the leader of the formation. The leader does not follow any agent so that no law is defined for u_L.

The formation errors are defined as:

$$z \triangleq [z_{ij}], \text{ where } z_{ij} \triangleq x_j - x_i \tag{3}$$

By plugging (2) into (1), in view of (3) one obtains:

$$\dot{z}_{ij} = -K_i z_{ij} + \dot{x}_j \tag{4}$$

Our aim is to investigate the stability properties of the formation error kinematics with respect to the input of the formation leader, u_L. We thus need to define the kind of stability in terms of which the formation will be analyzed:

Definition 2.1 (Formation ISS). *A formation is called input-to-state stable iff there is a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that for any initial formation error $z(0)$ and for any bounded inputs of the formation leader $u_L(\cdot)$ the evolution of the formation error satisfies:*

$$\|z(t)\| \leq \beta(\|z(0)\|, t) + \gamma\left(\sup_{\tau \leq t} \|u_L\|\right) \tag{5}$$

Input to state stability thus establishes a relationship between the amplitudes of the formation leader input and the formation errors. The relationship provides ways to compare formation interconnections in terms of internal stability.

Definition 2.2 (Formation ISS Measure). *Consider a formation that is input-to-state stable with gain functions $\beta(r,t)$ and $\gamma(r)$. Assume $\gamma(r) \in C^1$ and let $U \subseteq \mathbb{R}^n$ be a compact neighborhood of the origin containing all $u_L \in U$ that are of interest. Then*

$$P_{ISS} \triangleq \gamma^{-1}(1)$$

will be called the ISS measure of the formation.

In that sense, if the norms are taken as Euclidean, the ISS measure is the upper bound for the leader input that guarantees that the formation error vector remains within a unit ball.

In the following sections we will show how the formation ISS gain functions β and γ can be computed from those of the individual agent interconnections.

Formation ISS does not require any inter-agent communication: the feedback laws (2) can be constructed by means of position sensing. Furthermore, each agent is required to have information only for its immediate leader. Contrary to string stability which investigates the behavior of errors as they propagate in the formation chain, formation ISS focuses in characterizing the dependence of formation stability to leader input.

3 ISS of Agent Interconnection

Agent interconnections are represented in graph notation form. Agents are denoted by vertices. A directed edge from vertice j to vertice i implies that agent i follows agent j using feedback information. By abuse of notation we denote the exchange of additional feedforward information by a dashed directed edge.

Consider the agent interconnection error (4):

$$\dot{z}_{ij} = -K_i z_{ij} + \dot{x}_j$$

A Lyapunov function candidate could be $V_{ij} = \frac{1}{2} z_{ij}^T z_{ij}$. Then for $c_1 = c_2 = \frac{1}{2}$ it holds that $c_1 \|z_{ij}\| \leq V_{ij} \leq c_2 \|z_{ij}\|$, and the derivative of V_{ij} satisfies:

$$\begin{aligned} \dot{V}_{ij} &= -z_{ij}^T K_i z_{ij} + z_{ij}^T \dot{x}_{ij} \\ &\leq -2\lambda_m^i \|z_{ij}\|^2 + \|z_{ij}\| \cdot \|\dot{x}_{ij}\| \end{aligned}$$

where λ_m^i is the minimum eigenvalue of K_i. By taking any $\theta \in (0,1)$ and defining $c_3 \triangleq 2\lambda_m^i(1-\theta)$,

$$\dot{V}_{ij} \leq -c_3 \|z_{ij}\|^2, \quad \forall \ \|z_{ij}\| \geq \frac{\|\dot{x}_j\|}{2\lambda_m^i \theta}$$

Moreover, for $c_4 = 2\lambda_M^i$, where λ_M^i is the largest eigenvalue of K_i, it holds that:

$$\left\| \frac{\partial V_{ij}}{\partial z_{ij}} \right\| \leq c_4 \|z_{ij}\|$$

From stability of perturbed systems [lemma 5.2, [16]]:

$$\|z_{ij}(t)\| \leq \|z_{ij}(0)\| \, \mathrm{e}^{2\lambda_m^i(1-\theta)t} + \frac{\lambda_M^i}{\theta \lambda_m^i} \sup_{\tau \leq t} (\|\dot{x}_j\|) \tag{6}$$

showing that the interconnection kinematics are input-to-state stable with respect to the leader's velocity.

4 Propagation of ISS

4.1 Cascade interconnection

Consider the leader-follower configuration of section 3 and assume that in addition, agent j is assigned to follow agent k (Figure 1):

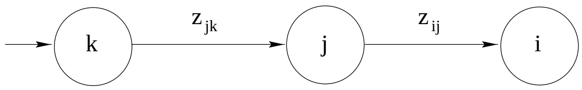

Figure 1: Cascade interconnection of agents. Solid arrows denote feedback information flow

Based on the assummed available feedback information, the agent control laws are formed as follows:

$$u_i = K_i(x_j - x_i)$$
$$u_j = K_j(x_k - x_j)$$

The results of section 3 establish the ISS of each pair:

$$\|z_{ij}(t)\| \leq \|z_{ij}(0)\| \, \mathrm{e}^{-\alpha_i(\theta)t} + \frac{\lambda_M^i}{\theta\lambda_m^i} \sup_{\tau \leq t} (\|\dot{x}_j\|) \quad (7a)$$

$$\|z_{jk}(t)\| \leq \|z_{jk}(0)\| \, \mathrm{e}^{-\alpha_j(\theta)t} + \frac{\lambda_M^j}{\theta\lambda_m^j} \sup_{\tau \leq t} (\|\dot{x}_k\|) \quad (7b)$$

where

$$\alpha_i(\theta) \triangleq 2\lambda_m^i(1-\theta) \qquad \alpha_j(\theta) \triangleq 2\lambda_m^j(1-\theta)$$

However, since $\dot{x}_j = K_j z_{jk}$, from (7) we get:

$$\|z_{ij}(t)\| \leq \|z_{ij}(0)\| \, \mathrm{e}^{-\alpha_i(\theta)t} + \frac{\lambda_M^i\lambda_M^j}{\theta\lambda_m^i} \sup_{0\leq\tau\leq t} (\|\dot{x}_k(\tau)\|)$$

Define the leader j-follower i ISS gains as:

$$\beta_{ij}(\|z_{ij}(0)\|, t) \triangleq \|z_{ij}(0)\| \, \mathrm{e}^{-\alpha_i(\theta)t} \quad (8a)$$

$$\gamma_{ij}(\sup(\|\dot{x}_k\|)) \triangleq \frac{\lambda_M^i\lambda_M^j}{\theta\lambda_m^i} \sup_{0\leq\tau\leq t} (\|\dot{x}_k(\tau)\|). \quad (8b)$$

The gains for the leader k- follower j pair are defined accordingly.

Based on the ISS property of the cascade interconnection of two ISS systems [16, 17], the stability property of the agent interconnection can be propagated to the new construction. Define the composite error:

$$z_{ik} \triangleq \begin{bmatrix} z_{jk}^T & z_{ij}^T \end{bmatrix}^T$$

It has been shown [16, 17] that the cascade interconnection of two input-to-state stable systems:

$$\dot{x}_1 = f_1(t, x_1, x_2, u)$$
$$\dot{x}_2 = f_2(t, x_2, u)$$

is itself input-to-state stable with gain functions:

$$\begin{aligned} \beta(r,t) =& \beta_1(2\beta_1(r, \tfrac{t}{2}) + 2\gamma_1(2\beta_2(r,0)), \tfrac{t}{2}) \\ &+ \gamma_1(2\beta_2(r, \tfrac{t}{2})) + \beta_2(r,t), \\ \gamma(r) =& \beta_1(2\gamma_1(2\gamma_2(r) + 2r), 0) \\ &+ \gamma_1(2\gamma_2(r) + 2r) + \gamma_2(r). \end{aligned}$$

A direct application of the above to (3) and setting $z_0 \triangleq \|z_{ik}\|$, $v_k \triangleq \sup_{0\leq\tau\leq t}\{\|\dot{x}_k(\tau)\|\}$, yields:

$$\begin{aligned} \beta_{ik}(z_0,t) = \beta_{ij} \Bigg(& 2\beta_{ij}(z_0, \tfrac{t}{2}) + \beta_{jk}(z_0,t) \\ & + 2\gamma_{ij}(2\beta_{jk}(z_0,0)), \tfrac{t}{2} \Bigg) + \gamma_{ij}(2\beta_{jk}(z_0, \tfrac{t}{2})), \end{aligned} \quad (9a)$$

$$\begin{aligned} \gamma_{ik}(v_k) =& \gamma_{ij}(2\gamma_{jk}(v_k) + 2v_k) \\ &+ \beta_{ik}(2\gamma_{ij}(2\gamma_{jk}(v_k) + 2v_k), 0) + \gamma_{jk}(v_k). \end{aligned} \quad (9b)$$

4.2 Parallel Interconnection

The parallel interconnection is the configuration where both agents i and j are assigned to follow agent k based on feedback information about the state of k (Figure 2):

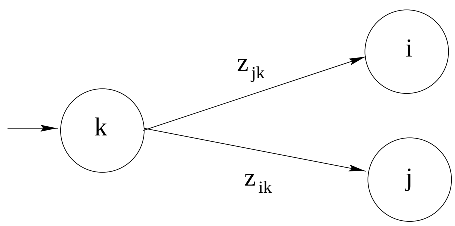

Figure 2: Parallel interconnection of agents. Solid arrows denote feedback information flow

It can easily be shown that the ISS gains for the parallel interconnection:

$$\begin{aligned} \dot{x}_1 =& f_1(t, x_1, u) \\ \dot{x}_2 =& f_2(t, x_2, u) \end{aligned}$$

are formed as

$$\begin{aligned} \beta(r,t) =& \beta_1(r,t) + \beta_2(r,t) \\ \gamma(r) =& \gamma_1(r) + \gamma_2(r) \end{aligned}$$

and for the linear case after defining:

$$z_{ijk} \triangleq \begin{bmatrix} z_{jk} \\ z_{ik} \end{bmatrix}, \quad z_0 \triangleq z_{ijk}(0), \quad v_k \triangleq \sup_{0\leq\tau\leq t} \{\|\dot{x}_k(\tau)\|\}$$

the gain function become:

$$\beta_{ijk}(z_0,t) = \beta_{ik}(z_0,t) + \beta_{ij}(z_0,t) \quad (10a)$$
$$\gamma_{ijk}(v_k) = \gamma_{ik}(v_k) + \gamma_{jk}(v_k) \quad (10b)$$

Using (9)-(10) recursively, the ISS gains of a group of agents can be calculated. This procedure finally yields the ISS gain functions of the whole formation.

The advantage of the linear structure of (4) is that one has constructive ways of obtaining the original ISS gains (8) for each individual pair of leader-follower. Note that the linear character of the input gain γ is preserved through each propagation. The transient term β becomes a sum of decaying exponentials, where each of them has a different rate of decrease; all of them though can be ultimately bounded by the slowest term.

5 The Effect of Feedforward

Suppose that for a specific pair of leader-follower, feedforward information from the leader to the follower is also available. The control law can be formed as:

$$u_i = K_i(x_j - x_i) + \dot{x}_j \tag{11}$$

and since K_i is assumed positive definite, the closed loop dynamics of the pair become exponentially stable:

$$\dot{z}_{ij} = -K_i z_{ij} \tag{12}$$

meaning that $\|z_{ij}(t)\| \leq \|z_{ij}(0)\| \, \mathrm{e}^{2\lambda_m^i(1-\theta)t}$. Thus, when feedforward information from the leader to the follower can be used, the input gain vanishes. In all cases examined the ISS bounds on both transient and steady state components are relaxed, a fact that reveals the stabilizing effect of using additional feedforward information in inter-agent control laws.

5.1 Feedforward in Second Cascade Link

Assume two leader-follower pairs: agent i following agent j and agent j following k and that feedforward information about agent j is available to agent i (Figure 3). This case appears when the link that where feedforward information is used is located at the end of the formation chain.

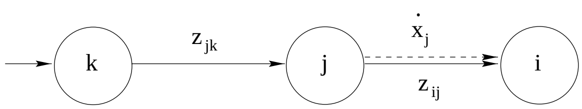

Figure 3: Cascade interconnection of agents with feedforward. Solid arrows denote feedback information flow; dashed arrows denote feedforward information.

The control laws can be defined now as:

$$\begin{aligned} u_i &= K_i(x_j - x_i) + \dot{x}_j \\ u_j &= K_j(x_k - x_j) \end{aligned}$$

Then, given $\gamma_{ij} = 0$, and setting $z_0 \triangleq \|z_{ik}(0)\|$, $v_k \triangleq \sup_{0\leq\tau\leq t}\{\|\dot{x}_k(\tau)\|\}$ the ISS gains simplify to:

$$\beta_{ik}(z_0, t) = \beta_{ij}\left(2\beta_{ij}(z_0, \frac{t}{2}), \frac{t}{2}\right) + \beta_{jk}(z_0, t), \tag{13a}$$

$$\gamma_{ik}(v_k) = \gamma_{jk}(v_k) \tag{13b}$$

5.2 Feedforward in First Cascade Link

In this case the feedforward information concerns agent k and is available to agent j (Figure 4). The link that uses feedforward information is an indermediate link in the formation chain.

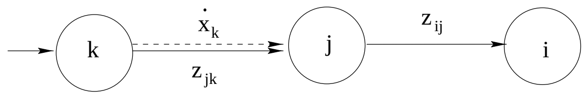

Figure 4: Cascade interconnection of agents with feedforward. Solid arrows denote feedback information flow; dashed arrows denote feedforward information.

The control laws have the form:

$$\begin{aligned} u_i &= K_i(x_j - x_i) \\ u_j &= K_j(x_k - x_j) + \dot{x}_k \end{aligned}$$

and result in $\gamma_{jk} = 0$. Setting $z_0 \triangleq \|z_{ik}(0)\|$ and $v_k \triangleq \sup_{0\leq\tau t}\{\|\dot{x}_k(\tau)\|\}$ the composite ISS gains can be expressed as

$$\begin{aligned} \beta_{ik}(z_0, t) = \beta_{ij}&\left(2\beta_{ij}(z_0, \frac{t}{2}) + 2\gamma_{ij}(2\beta_{jk}(z_0, 0)), \frac{t}{2}\right) \\ &+ \beta_{jk}(z_0, t) + \gamma_{ij}(2\beta_{jk}(z_0, \frac{t}{2})), \end{aligned} \tag{14a}$$

$$\gamma_{ik}(v_k) = \beta_{ik}\left(2\gamma_{ij}(2v_k), 0\right) + \gamma_{ij}(2v_k) \tag{14b}$$

5.3 Parallel Link with Feedforward

This is the case where one of the parallel links uses feedforward information about the leader (Figure 5).

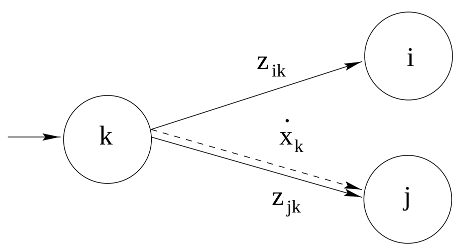

Figure 5: Parallel interconnection of agents with feedforward. Solid arrows denote feedback information flow; dashed arrows denote feedforward information.

The control laws are then given as:

$$u_i = K_i(x_k - x_i)$$
$$u_j = K_j(x_k - x_j) + \dot{x}_k$$

Setting $z_0 \triangleq z_{ijk}(0)$, $\sup_{0\leq\tau t}\{\|\dot{x}_k(\tau)\|\}$, the ISS gains for this interconnection become:

$$\beta_{ijk}(z_0,t) = \beta_{ik}(z_0,t) + \beta_{ij}(z_0,t) \tag{15a}$$
$$\gamma_{ijk}(v_k) = \gamma_{ik}(v_k) \tag{15b}$$

6 The Effect of Additional Feedback

Consider the leader-follower interconnection (3). For agent j, given (1) and (2) it holds that $\dot{x}_j = K_j z_{jk}$ and (3) can be rewritten:

$$\dot{z}_{ij} = -K_i z_{ij} + K_j z_{jk} = -(K_i + K_j)z_{ij} + K_j z_{ki}$$

The above implies that additional feedback information from the leader of an agent's leader can have a stabilizing effect just as feedforward information from the leader itself. The feedback law:

$$u_i = K_i z_{ij} + K_j z_{ki}$$

can transform the cascade interconnection to a cascade with feedforward (Figure 6: if $u_j = K_j z_{kj}$, then

$$\dot{z}_{ij} = -(K_i + K_j)z_{ij}$$
$$\dot{z}_{kj} = -K_j z_{kj} + \dot{x}_k$$

and the composite ISS gains are as given by (13)

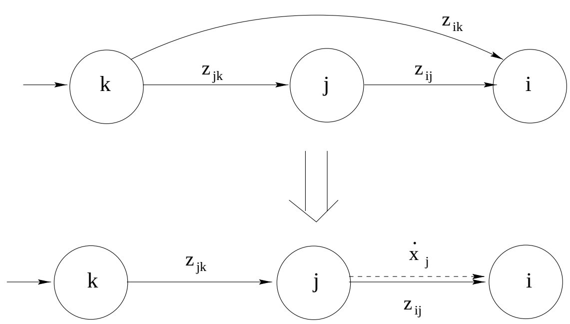

Figure 6: Additional feedback can substitute for feedforward.

Therefore, the unavailability of feedforward information can be compensated by additional feedback from further up the formation hierarchy.

7 Examples

Suppose that the objective is to control a one dimensional platoon of vehicles using only feedback information, such that each vehicle maintains a certain distance d from its neighboring vehicles. For that purpose, three interconnection options, depicted in Figure 7 are considered.

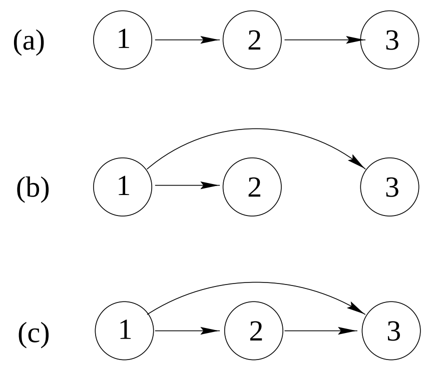

Figure 7: Three formation interconnection options.

The closed loop kinematics in case (a) is given by:

$$\dot{x}_1 = u$$
$$\dot{x}_2 = k(d + x_1 - x_2)$$
$$\dot{x}_3 = k(d + x_2 - x_3)$$

where x_i, $i = 1, 2, 3$ is each agent absolute coordinate, u is the leader's speed, and k is a given constant gain of the feedback control law. The dynamics of the formation errors will then be:

$$\dot{z}_{12} = -kz_{21} - u$$
$$\dot{z}_{32} = -kz_{32} + kz_{21}$$

By simple arguments it can be established that

$$\gamma_{21} \triangleq \frac{\sup\{|u|\}}{k\theta} \qquad \gamma_{32} \triangleq \frac{\sup_{0\leq\tau\leq t}\{|z_2(\tau)|\}}{\theta}$$

Application of (9) yields the formation ISS input gain: $\gamma_a(\sup|u|) = \frac{6+6k\theta+\theta}{k\theta^2}\sup\{|u|\}$ and a formation ISS measure: $P^a_{ISS} = \frac{6+6k\theta+\theta}{k\theta^2}$

The closed loop kinematics in case (b) is

$$\dot{x}_1 = u$$
$$\dot{x}_2 = k(d + x_1 - x_2)$$
$$\dot{x}_3 = k(2d + x_2 - x_3)$$

and lead to a formation ISS input gain: $\gamma_b(\sup|u|) = \frac{2}{k\theta}\sup\{|u|\}$ and an formation ISS measure: $P^b_{ISS} = \frac{2}{\theta k}$

In case (c), with $u_i = kz_{32} + kz_{31}$ the formation error kinematics become:

$$\dot{z}_{12} = -kz_{12} + u$$
$$\dot{z}_{23} = -2kz_{23}$$

yielding a formation ISS input gain: $\gamma_c(\sup\{|u|\}) = \frac{1}{\theta k}\sup\{|u|\}$ and a formation ISS measure: $P^c_{ISS} = \frac{1}{\theta k}$

Since

$$\gamma_a = \frac{6 + 6k\theta + \theta}{k\theta^2} \geq \frac{6\theta + 6k\theta + \theta}{k\theta^2}$$
$$= \frac{6k\theta + 7\theta}{k\theta^2} = \frac{6k+7}{k\theta} \geq \frac{7}{k\theta} \geq \frac{2}{k\theta} = \gamma_b \leq \gamma_c$$

according to the performance measures it is: $P^a_{ISS} \leq P^b_{ISS} \leq P^c_{ISS}$ implying that formation (c) outperforms (a) and (b).

8 Conclusion

The paper presents a new type of stability defined for leader-follower formations which is based on input-to-state stability of interconnected systems. The new stability notion gives rise to a performance measure by which different interconnections can be compared in terms of stability.

Preliminary analysis based on this tool indicates that the depth of the formation has an adverse effect on its internal stability with respect to the leader's input. Furthermore, the fact that additional information can in gerenal improve performance can now be formally expressed. Also, there seems to be a close link between feedforward and feedback links in interconnected systems in the sense that under some conditions, feedback links can replace feedforward links and vice versa.

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