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Advanced agricultural robots: kinematics and dynamics of multiple mobile manipulators handling non-rigid material

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Abstract

The equations of motion for a system of multiple mobile manipulators that handle a deformable object during an agricultural task are developed. The model is based on Kane's approach. The imposed kinematic constraints are included and incorporated into the dynamics. Sufficient conditions for avoiding tipping over of the mechanisms are also provided. The deformable nature of the object can easily accommodate a variety of agricultural products and the analysis allows for the inclusion of specific handling limitations in order for the objects not to be damaged during manipulation.

Keywords: Cooperation; Dynamic models; Manipulators; Mobile robots

1. Introduction

Stationary robots are used for sheep hearing, cow milking and the use of manipulators in slaughterhouses has also been proposed. Mobile robots are used both in outdoor environment (Papadopoulos and Sarkar, 1997) and inside a greenhouse (Mandow et al., 1996). Robots have also been employed in the inspection and harvesting of cucumbers (van Kollenburg-Crisan et al., 1998). Many robotic and mobile robotic applications in agriculture have been reported (American Society of Agricultural Engineering, 1989).

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Mobile manipulators have also been applied in agriculture (Figs.1 and 2). The University of Western Australia developed two robots (the ORACLE and the SM) for sheep shearing. The robots consist of one or more manipulators, which can translate on a horizontal platform. In Israel, Greentech Ltd has developed a greenhouse harvester, a mobile robot which moves around growing rows identifying fruit ripe for picking. Greenhouse spraying can be performed by the AURORA mobile robot (Mandow et al., 1996). The robot is capable of navigating autonomously along the corridors of a greenhouse while performing spraying tasks. Other mobile manipulators have been built to operate under field conditions and collect grapes (Kondo, 1995), oranges (Fujiura et al., 1990), watermelons (Iida et al., 1996) etc. The Center for Intelligent Machines at McGill University has developed an articulated forestry machine (Papadopoulos and Sarkar, 1997). The application is very promising for the employment of multiple mobile manipulator systems. The use of such systems for packaging and palletizing in food industry has also been considered (Dreyer, 1994; Masinick, 1994).

The mobile base of a mobile manipulator is subject to motion constraints called nonholonomic (Campion and Bastin, 1991). Many of the models proposed for mobile manipulator systems do not include them (Khatib et al., 1995; Tarn et al., 1996). Others include them, using classic Euler–Lagrange (Yamamoto, 1994) and Newton–Euler formulations (Chen and Zalzala, 1995), but do not consider all constraints imposed in such systems. Recently, Thanjavur and Rajogopalan (1997) modeled an AGV using Kane's equations.

Previous approaches to non-rigid material handling focus mainly on formulating general continuous dynamic equations for the object. Sun et al. (1996) followed Terzopoulos' (Terzopoulos and Fleischer, 1988) hybrid approach to deformable objects. Kosuge et al. (1995) used finite elements but ignored the dynamics of the object.

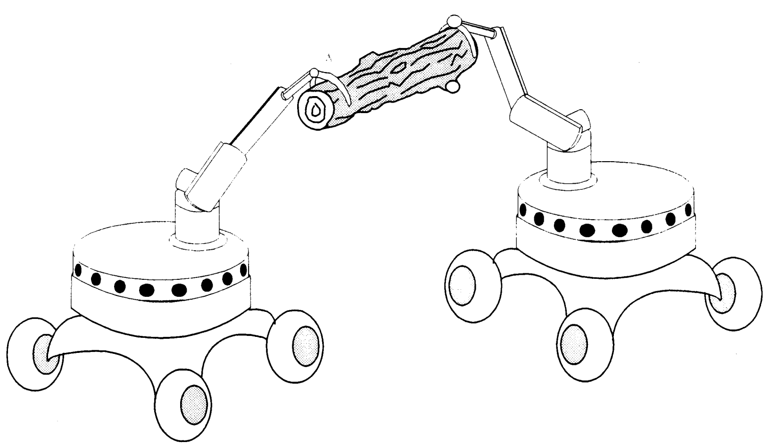

Fig. 1. Mobile manipulators in forestry.

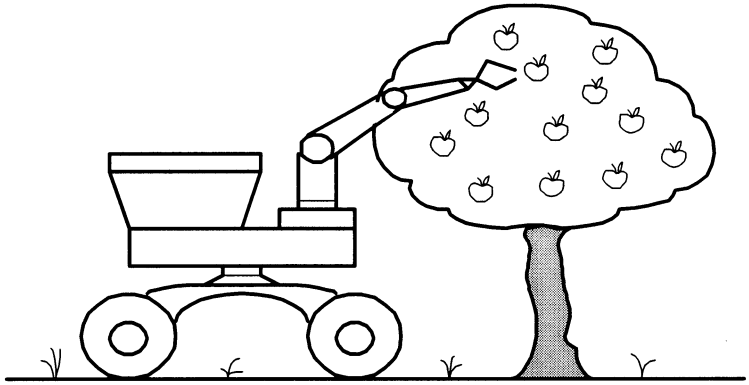

Fig. 2. Apple harvesting mobile manipulator.

Due to space limitations mathematical derivations have been shortened. The interested reader may refer to Tanner (1999). The rest of the paper is organized as follows: In Section 2, Kane's methodology is used to model a single mobile manipulator. Section 3 is devoted to modeling the deformable object. The combined model for the system of mobile manipulators handling a deformable object is constructed in Section 4. In Section 5, the dynamic constraints imposed are analyzed. Section 6 summarizes the conclusions from the present work and highlights future research directions.

2. Modeling of a mobile manipulator

Consider a mobile manipulator numbered k and a fixed inertial frame, $\{I\}$ as in Fig. 3. Its $\mathbf{x}$ and $\mathbf{y}$ axes lie on the horizontal plane. On the mobile platform of mobile manipulator, a frame $\{v\}$ at its mass center is attached with axis $\mathbf{z}^v$ parallel to $\mathbf{z}^I$. At the same point, the platform's central principal inertial frame, $\{v_{\mathrm{cpi}}\}$, is assigned. The point of the manipulator in contact with the platform is named mm and coincides with the platform point $\overline{mm}$. The manipulator frames are assigned according to Denavit and Hartenberg (1955). Finally, a frame is assigned at the manipulator end effector. The right superscript of any quantity will denote the rigid body it refers to. The left superscript will denote the reference frame with respect to which the quantity is expressed. When omitted, frame $\{I\}$ is assumed.

At the center of each wheel j, $j = 1, \ldots, w$, frame $\{w_j\}$ is attached (Fig. 3). Its $\mathbf{z}^{w_j}$ axis remains parallel to the vehicle $\mathbf{z}^v$ axis, but the frame can rotate around this axis. Each frame is related to the fixed frame through a rotation matrix $\mathbf{R}$. This rotation matrix will be written as, e.g. ${}_{w_j}^{I}\mathbf{R}$, to indicate transformation of free vectors expressed in frame $\{w_j\}$, to the inertial frame. In the sequel it is assumed that the motion of all vehicles is restricted on the horizontal plane. The coefficient of static friction between each wheel and the ground is η, assumed the same for all wheels.

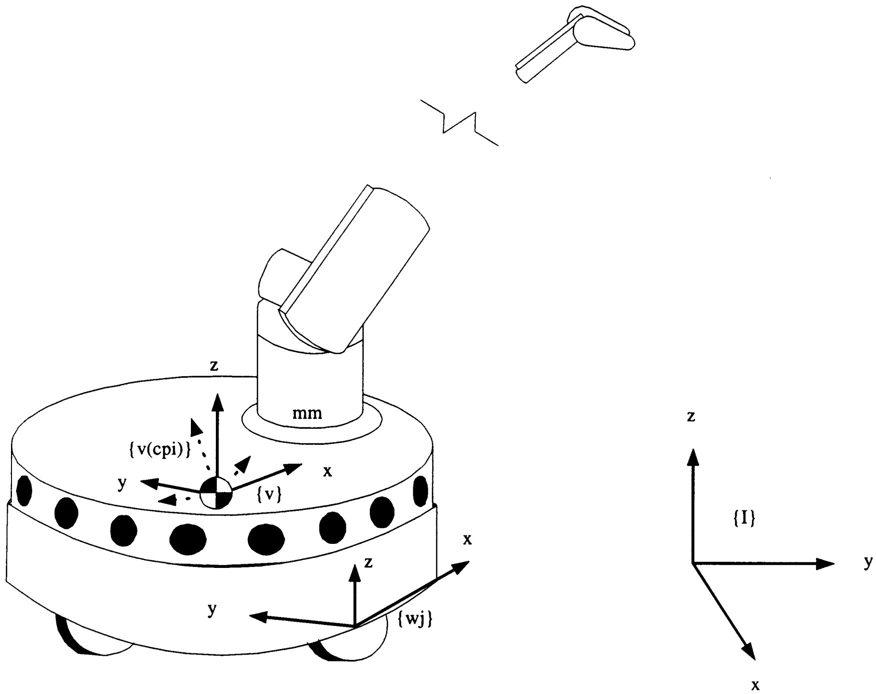

Fig. 3. Frame assignment on mobile manipulator k.

The manipulator has $n-4$ rotational joints. The system's generalized coordinates are

$$\mathbf{q} = [x^v \ \ y^v \ \ \theta \ \ \phi \ \ q^1 \ldots q^{n-4}]^{\mathrm{T}}$$

where x, y are the vehicle planar coordinates in coordinate system $\{I\}$, θ its orientation corresponding to the direction of its linear velocity, ϕ the steering angle of the vehicle and the rest correspond to the manipulator joints. The generalized speeds are defined as:

$$\mathbf{u} = [\dot{x}^v \quad \dot{y}^v \quad \dot{\theta} \quad \dot{\phi} \quad \dot{q}^1 \quad \ldots \quad \dot{q}^{n-4} \quad \ldots]^{\mathrm{T}}$$

2.1. The dynamic equations of the vehicle

A wheel j can have two degrees of freedom. A rolling angle about $\mathbf{y}^{w_j}$ axis (Fig. 3), controlled by input torque $\tau_d^{w_j}$ and steering angle ϕ^{w_j}, controlled by input torque $\tau_s^{w_j}$. All steering angles are kinematically coupled. Let ϕ be a particular steering angle, and assume $\phi^{w_j} = \phi^{w_j}(\phi)$.

If the *no skidding* condition for the wheels is used:

$$u_3 = \frac{u_2 \cos(\phi^{w_j} + \theta) - u_1 \sin(\phi^{w_j} + \theta)}{{}^v\ell_y^{w_j} \sin\phi^{w_j} + {}^v\ell_x^{w_j} \cos\phi^{w_j}} \quad \text{for any } j \in \{1, \ldots, w\} \tag{1}$$

where $\ell^{w_j} = [\ell_x^{w_j} \quad \ell_x^{w_j} \quad \ell_z^{w_j}]^T$ is the position vector from frame $\{v\}$ to the contact between the wheel and the ground, and $\mathbf{x}^{w_j}$ is a unit axis vector on the wheel (Fig. 3), then the partial velocities of the center of the wheel can be derived as:

$$\mathbf{v}_1^{w_j} = \cos(\phi^{w_j} + \theta)\mathbf{x}^{w_j} \quad \mathbf{v}_2^{w_j} = \sin(\phi^{w_j} + \theta)\mathbf{x}^{w_j}$$
$$\mathbf{v}_3^{w_j} = ({}^v\ell_x^{w_j} \sin\phi^{w_j} - {}^v\ell_y^{w_j} \cos\phi^{w_j})\mathbf{x}^{w_j} \tag{2}$$

If r^{w_j} is the wheel w_j radius, the *no slipping* constraint can be expressed as:

$$\mathbf{v}^{w_j} = \boldsymbol{\omega}^{w_j} \times r^{w_j}\mathbf{z}^{w_j} \tag{3}$$

From Eq. (3) the partial angular velocities of a wheel can be derived:

$$\boldsymbol{\omega}_1^{w_j} = \frac{\cos(\phi^{w_j} + \theta)}{r^{w_j}}\,\mathbf{y}^{w_j} \qquad \boldsymbol{\omega}_2^{w_j} = \frac{\sin(\phi^{w_j} + \theta)}{\tau^{w_j}}\,\mathbf{y}^{w_j}$$
$$\boldsymbol{\omega}_3^{w_j} = \frac{({}^v\ell_x^{w_j} \sin\phi^{w_j} - {}^v\ell_y^{w_j} \cos\phi^{w_j})}{r^{w_j}}\,\mathbf{y}^{w_j} + \mathbf{z}^{w_j} \qquad \boldsymbol{\omega}_4^{w_j} = \frac{\partial\phi^{w_j}}{\partial\phi}\,\mathbf{z}^{w_j}$$
$$\text{for each } j \in \{1, \ldots, w\} \tag{4}$$

Partial angular velocities of wheel w_j for $r = 5, \ldots, n$ are zero.

Input torques contribute to the generalized active forces by:

$$(F_r)_w = \boldsymbol{\omega}_r^{w_j} \cdot \mathbf{T}_s^{w_j} + \boldsymbol{\omega}_r^{w_j} \cdot \mathbf{T}_d^{w_j} \quad r \in \{1, \ldots, n\},\ j \in \{1, \ldots, w\}$$

Let m^{w_j} be the mass of wheel j, which is being modeled as a disk. If $\mathbf{I}^{w_j}$ is the wheel's central inertial dyadic (Kane and Levinson, 1985), the inertial moment exerted on the wheel will be:

$$\mathbf{T}^{*w_j} = -[\alpha_x^{w_j} I_x^{w_j} - \omega_y^{w_j}\omega_z^{w_j}(I_y^{w_j} - I_z^{w_j})]\mathbf{x}^{w_j} - [\alpha_y^{w_j} I_y^{w_j} - \omega_z^{w_j}\omega_x^{w_j}(I_z^{w_j} - I_x^{w_j})]\mathbf{y}^{w_j}$$
$$- [\alpha_z^{w_j} I_z^{w_j} - \omega_x^{w_j}\omega_y^{w_j}(I_x^{w_j} - I_y^{w_j})]\mathbf{z}^{w_j}$$

where $\alpha_{x,y,z}^{w_j}$, $\omega_{x,y,z}^{w_j}$ and $I_{x,y,z}^{w_j}$ denote the components of angular acceleration, velocity and central principle moment of inertia of the wheel in frame $\{w_j\}$, respectively.

The generalized inertial forces of all wheels are computed as (Kane and Levinson, 1985):

$$(F_r^*)_w = \sum_{j=1}^{w} \boldsymbol{\omega}_r^{w_j} \cdot \mathbf{T}^{*w_j} + \sum_{j=1}^{w} \mathbf{v}_r^{w_j} \cdot \mathbf{F}^{*w_j}$$

Consider the vehicle mass center and the point $\overline{mm}$ which coincides with the base of the manipulator. The vehicle mass center c_v, where frame $\{v\}$ is located, translates with velocity $\mathbf{v}^{c_v} = u_1\mathbf{x}^I + u_2\mathbf{y}^I$. The partial velocities for the vehicle mass center and the point $\overline{mm}$ will be:

$$\mathbf{v}_1^{c_v} = \mathbf{x}^I \quad \mathbf{v}_2^{c_v} = \mathbf{y}^I \quad \mathbf{v}_1^{\overline{mm}} = \mathbf{x}^I \quad \mathbf{v}_2^{\overline{mm}} = \mathbf{y}^I \quad \mathbf{v}_3^{\overline{mm}} = \mathbf{z}^I \times {}^v\mathbf{r}^{\overline{mm}}$$

Frame's $\{v\}$ angular velocity is $\boldsymbol{\omega}^v = u_3\mathbf{z}^I$, and the only partial angular velocities of the vehicle frame and of the vehicle point $\overline{w_j}$ coinciding with the wheel center are:

$$\boldsymbol{\omega}_3^v = \mathbf{z}^I \qquad \boldsymbol{\omega}_3^{\overline{w_j}} = \mathbf{z}^I \tag{5}$$

The *reaction torques by the wheels* contribute to the generalized active forces by:

$$(F_r)_{\overline{w_j}} = \boldsymbol{\omega}_r^{\overline{w_j}}(-\mathbf{T}_s^{w_j}) + \boldsymbol{\omega}_r^{\overline{w_j}}(-\mathbf{T}_d^{w_j}) \Rightarrow (F_3)_{\bar{w}} = -\sum_{j=1}^{w} \tau_s^{w_j} \tag{6}$$

If $\mathbf{F}^{\overline{mm}}$ and $\mathbf{T}^{\overline{mm}}$ are the *force and torque exerted by the attached manipulator* on the vehicle, respectively, their contribution to the generalized active forces will be:

$$(F_1)_{\overline{mm}} = \mathbf{x}^I \cdot \mathbf{F}^{\overline{mm}} \quad (F_2)_{\overline{mm}} = \mathbf{y}^I \cdot \mathbf{F}^{\overline{mm}} \quad (F_3)_{\overline{mm}} = (\mathbf{z}^I \times {}^v\mathbf{r}^{\overline{mm}})\cdot \mathbf{F}^{\overline{mm}} + \mathbf{z}^I \cdot \mathbf{T}^{\overline{mm}} \tag{7}$$

Then the contribution of the vehicle body to the generalized active forces would be:

$$(F_r)_b = (F_r)_{\overline{mm}} + (F_r)_{\overline{w_j}} \tag{8}$$

The acceleration of the mass center of the vehicle is $\mathbf{a}^{c_v} = \dot{u}_1\mathbf{x}^I + \dot{u}_2\mathbf{y}^I$, and if the mass of the vehicle body is m^v then the inertial force and moment will, respectively, be:

$$\mathbf{F}^{*v} = -m^v(\dot{u}_1\mathbf{x}^I + \dot{u}_2\mathbf{y}^I) \qquad \mathbf{T}^{*c_v} = -\boldsymbol{\alpha}^v \cdot \mathbf{I}^v - \boldsymbol{\omega}^v \times \mathbf{I}^v \cdot \boldsymbol{\omega}^v$$

where $\boldsymbol{\alpha}^v$ is the angular acceleration of the vehicle, $\boldsymbol{\alpha}^v = \dot{u}_3\mathbf{z}^I{}_3$, and $\mathbf{I}^v$ the inertial dyadic. In the central principal inertial frame, the inertial dyadic (Kane and Levinson, 1985) is diagonal with entries I_x^v, I_y^v, I_z^v. In the central principal inertial frame, the inertial moment is expressed as:

$$\begin{aligned}\mathbf{T}^{*v} = &-[\alpha_x^v I_x^v - \omega_y^v\omega_z^v(I_y^v - I_z^v)]\mathbf{x}^{v_{\mathrm{cpi}}} - [\alpha_y^v I_y^v - \omega_z^v\omega_x^v(I_z^v - I_x^v)]\mathbf{y}^{v_{\mathrm{cpi}}} \\ &- [\alpha_z^v I_z^v - \omega_x^v\omega_y^v(I_x^v - I_y^v)]\mathbf{z}^{v_{\mathrm{cpi}}}\end{aligned}$$

The vehicle inertial forces and moments contribute to the generalized inertial forces by:

$$(F_r^*)_b = \boldsymbol{\omega}_r^v \cdot \mathbf{T}^{*v} + \mathbf{v}_r^{c_v} \cdot \mathbf{F}^{*v}$$

The vehicle dynamic equations are derived as:

$$\sum\{(F_r^*)_v + (F_r^*)_b\} + \sum\{(F_r^w)_v + (F_r^w)_b\} = 0$$

The reduced model will be obtained in Section 2.3.

Example 1. The dynamic equations of the mobile platform have been derived for the case of a mobile robot.

The mobile robot considered is depicted in Fig. 4. This vehicle has motion in the front wheel only (wheel 1). Rotation of the wheel results in change of the steering angle ϕ, while traction is implemented by the traction torque τ_d. For this model, the following parameter values were used:

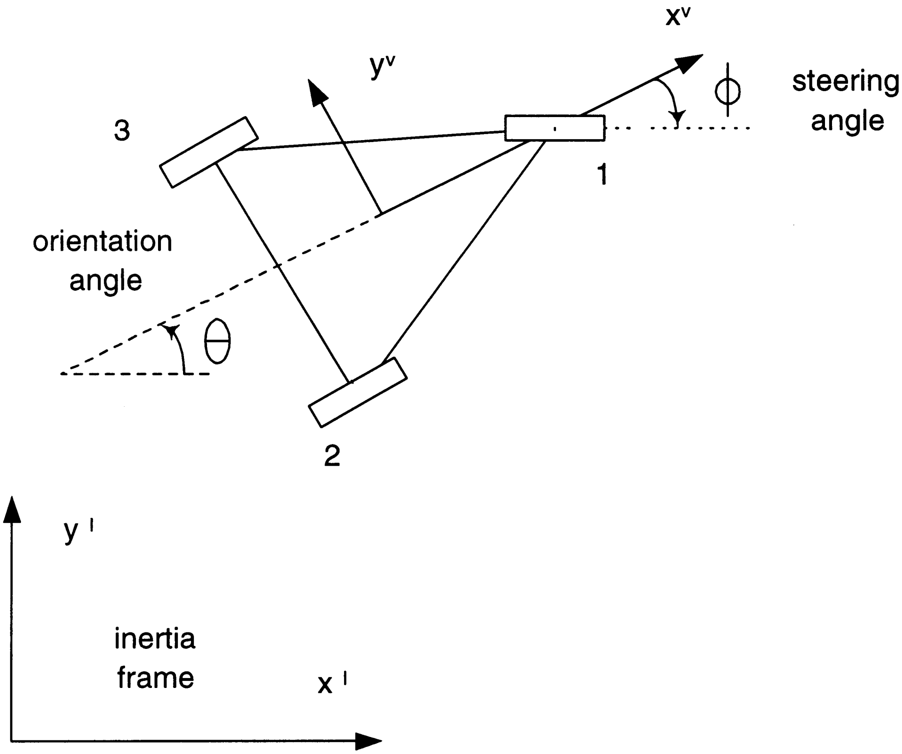

Fig. 4. The mobile robot considered in Example 1.

$$m^v = 41.3 \quad I_z = 4.37 \quad m^w = 2 \quad \ell_x^1 = 0.16 \quad \ell_y^1 = 0$$
$$\ell_x^2 = -0.08 \quad \ell_y^2 = -0.139 \quad \ell_x^3 = -0.08 \quad \ell_y^3 = 0.139 \quad r^w = 0.15$$

For this example we assumed a constant traction torque, while for the steering torque a simple PD controller has been applied. Simulations were conducted in MATLAB™. Fig. 5 shows a circular trajectory with zero initial conditions resulting from $\tau_d = 0.1$, $\tau_s = -0.1(\phi - (\pi/6)) - 0.01\dot{\phi}$.

Fig. 6 shows the motion with input torques $\tau_d = 0.1$, $\tau_s = -0.1(\phi - (\pi/7)\sin t) - 0.01\dot{\phi}$.

2.2. The manipulator

Link 0 is the manipulator base, attached to the mobile platform. Let *mm* be the manipulator point where the manipulator is attached to the vehicle. The interaction forces and torques exerted on the vehicle by the manipulator are of great importance but do not appear in the final expressions. Their expressions are necessary for deriving mechanical stability conditions. Thus, we introduce additional generalized speeds in order to bring them into evidence. Assume that point *mm* can translate and rotate, so that its partial velocities and partial angular velocities are:

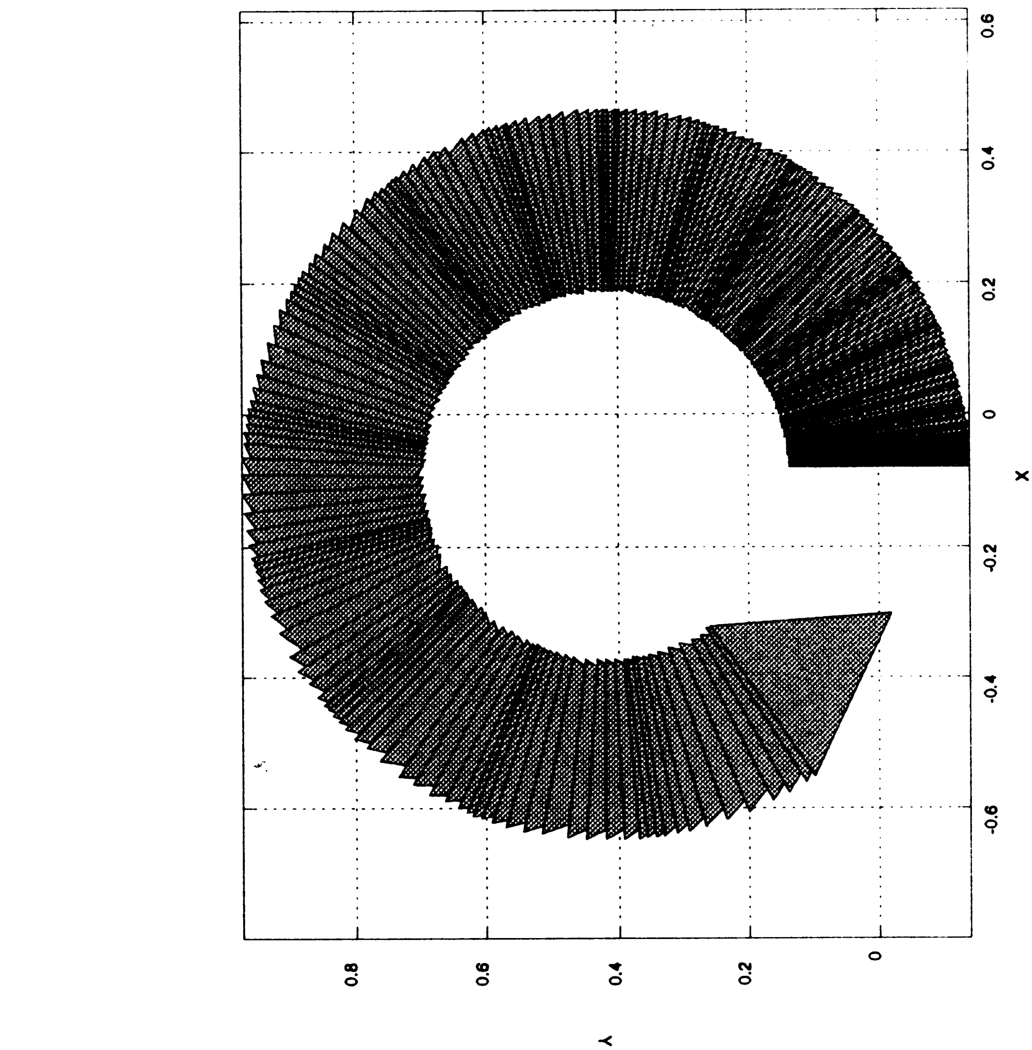

Fig. 5. Circular trajectory of the system in example 1.

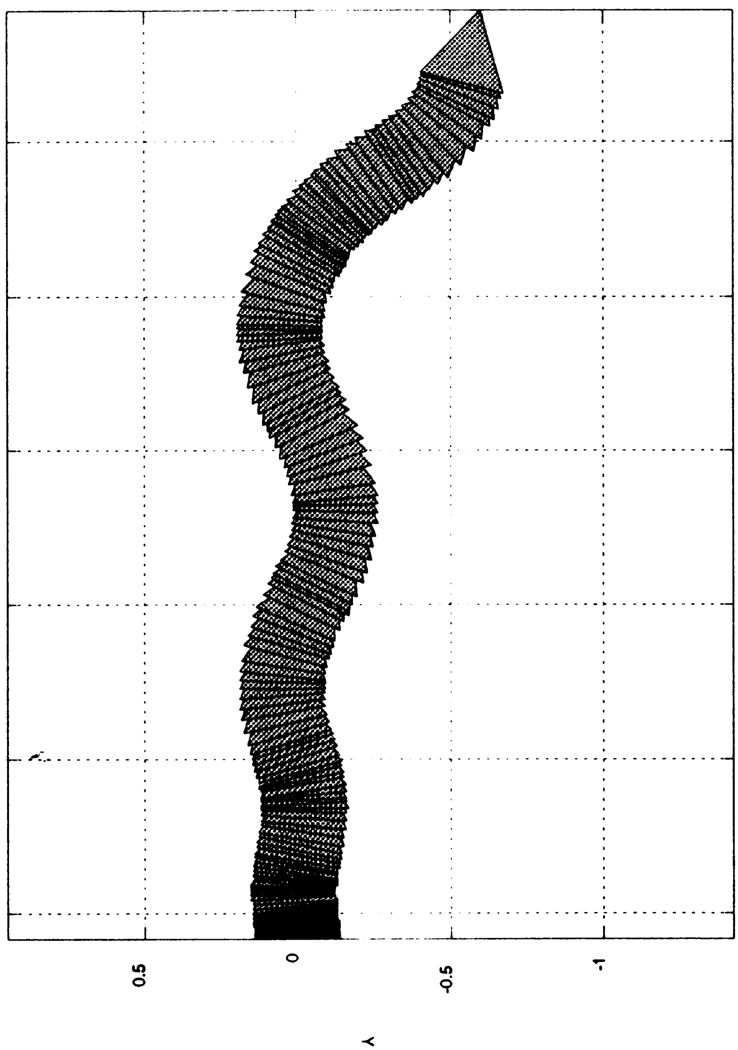

Fig. 6. Meandering trajectory of the system in example 1.

$$\left.\begin{array}{llll} \mathbf{v}_1^{mm} = \mathbf{x}^I & \mathbf{v}_{n+1}^{mm} = \mathbf{x}^v & \boldsymbol{\omega}_1^0 = 0 & \boldsymbol{\omega}_{n+4}^0 = \mathbf{x}^v \\ \mathbf{v}_2^{mm} = \mathbf{y}^I & \mathbf{v}_{n+2}^{mm} = \mathbf{y}^v & \boldsymbol{\omega}_2^0 = 0 & \boldsymbol{\omega}_{n+5}^0 = \mathbf{y}^v \\ \mathbf{v}_3^{mm} = \mathbf{z}^I \times {}^v\mathbf{r}^{\overline{mm}} & \mathbf{v}_{n+3}^{mm} = \mathbf{z}^v & \boldsymbol{\omega}_3^0 = \mathbf{z}^I & \boldsymbol{\omega}_{n+6}^0 = \mathbf{z}^v \end{array}\right\} \tag{9}$$

The contribution of the reaction forces by the platform to the generalized active forces is:

$$(F_r)_{mm} = \mathbf{v}_r^{mm} \cdot (-\mathbf{F}^{\overline{mm}}) + \boldsymbol{\omega}_r^0 \cdot (-\mathbf{T}^{\overline{mm}}) \quad r = 1, \ \ldots, \ n+6 \tag{10}$$

Due to Eq. (9) the weight of link 0 contributes. Let ${}^0\mathbf{r}^{c_0}$ be the position vector of the mass center in frame {0} and ${}^0\mathbf{r}^{mm}$ the position vector of point mm in the same frame. Then the partial velocities of the mass center of link 0 can be expressed as:

$$\left.\begin{array}{lll} \mathbf{v}_1^{c_0} = \mathbf{x}^I & \mathbf{v}_{n+1}^{c_0} = \mathbf{x}^v & \mathbf{v}_{n+4}^{c_0} = \mathbf{x}^v({}^0\mathbf{r}^{c_0} - {}^0\mathbf{r}^{mm}) \\ \mathbf{v}_2^{c_0} = \mathbf{y}^I & \mathbf{v}_{n+2}^{c_0} = \mathbf{y}^v & \mathbf{v}_{n+5}^{c_0} = \mathbf{y}^v({}^0\mathbf{r}^{c_0} - {}^0\mathbf{r}^{mm}) \\ \mathbf{v}_3^{c_0} = \mathbf{z}^I({}^v\mathbf{r}^{\overline{mm}} + {}^0\mathbf{r}^{c_0} - {}^0\mathbf{r}^{mm}) & \mathbf{v}_{n+3}^{c_0} = \mathbf{z}^v & \mathbf{v}_{n+6}^{c_0} = \mathbf{z}^v({}^0\mathbf{r}^{c_0} - {}^0\mathbf{r}^{mm}) \end{array}\right\} \tag{11}$$

The link weight $\mathbf{G}^0 = m^0 g \mathbf{z}^I$ contributes to the generalized active forces by:

$$(F_r)_{G0} = m^0 g \mathbf{z}^I \cdot \mathbf{v}_r^{c_0} \quad r = 1, \ \ldots, \ n+6$$

The contributions of the manipulator joint torques can be expressed as:

$$(F_r)_{\text{joint}i} = \frac{\partial(\boldsymbol{\omega}^i - \boldsymbol{\omega}^{i-1})}{\partial u_r} \cdot \tau_i \mathbf{z}^{i-1} \tag{12}$$

where $\mathbf{z}^{i-1}$ is the axis of rotation of link i relative to link $i-1$.

The contribution of the base of the manipulator to the generalized active forces is:

$$(F_r)_0 = (F_r)_{mm} + (F_r)_{G0} \tag{13}$$

The inertial force is $\mathbf{F}^{*0} = -m^0 \mathbf{a}^{c_0}$, where $\mathbf{a}^{c_0}$ is the acceleration of the mass center of the link. The inertial moment in principle inertial frame is:

$$\mathbf{T}^{*0} =$$

$$[-(\alpha_x^0 I_x^0 - \omega_y^0 \omega_z^0 (I_y^0 - I_z^0)) \quad -(\alpha_y^0 I_y^0 - \omega_z^0 \omega_x^0 (I_z^0 - I_x^0)) \quad -(\alpha_z^0 I_z^0 - \omega_x^0 \omega_y^0 (I_x^0 - I_y^0))]^{\mathrm{T}}$$

where $\omega_{x,y,z}^0$ and $\alpha_{x,y,z}^0$ denote the components of the link's angular velocity and acceleration.

The contribution of the inertia forces and moments of link 0 can be calculated as:

$$(F^*)_0 = \mathbf{v}_r^{c_0} \cdot \mathbf{F}^{*0} + \boldsymbol{\omega}_r^0 \cdot \mathbf{T}^{*0} \quad r = 1, \ \ldots, \ n+6 \tag{14}$$

For link i, the *input torque's* contribution is given by Eq. (12), whereas the *weight's* of link i by:

$$(F_r)_{Gi} = \frac{\partial \mathbf{v}^{c_i}}{\partial u_r} \cdot m^i \mathbf{g}$$

where $\mathbf{v}^{c_i}$ is the velocity of link i mass center. Its contribution to generalized active forces is:

$$(F_r)_i = (F_r)_{\text{joint}i} + (F_r)_{Gi} \tag{15}$$

Inertial force and inertial moment on link i are, respectively:

$$\mathbf{F}^{*i} = m^i \mathbf{a}^{c_i} \qquad \mathbf{T}^{*i} = -\boldsymbol{\alpha}^i \cdot \mathbf{I}^i - \boldsymbol{\omega}^i \times \mathbf{I}^i \cdot \boldsymbol{\omega}^i$$

where $\mathbf{a}^{c_i}$ is the linear acceleration of the mass center of link i, $\boldsymbol{\alpha}^i$ is the angular acceleration of the link and $\mathbf{I}^i$ is the central inertia dyadic of i. Their contribution to generalized inertia forces is:

$$(F_r^*)_i = \frac{\partial \mathbf{v}^{c_i}}{\partial u_r} \cdot \mathbf{F}^{*i} + \frac{\partial \boldsymbol{\omega}^i}{\partial u_r} \cdot \mathbf{T}^{*i}, \quad r = 1, \ldots, n+6 \tag{16}$$

Let torque $\mathbf{T}^{\text{ee}}$ and force $\mathbf{F}^{\text{ee}}$ represent the end effector load, applied at the end effector frame $\{\text{ee}\}$, coincident with $\{n\}$. The load contributes to the generalized active forces by:

$$(F_r)_{\text{ee}} = \frac{\partial \mathbf{v}^{\text{ee}}}{\partial u_r} \cdot \mathbf{F}^{\text{ee}} + \frac{\partial \boldsymbol{\omega}^{\text{ee}}}{\partial u_r} \cdot \mathbf{T}^{\text{ee}} \tag{17}$$

2.3. Mobile manipulator k dynamic equations

Kane's dynamic equations for mobile manipulator k can be summarized in:

$$(F_r)_k + (F_r^*)_k = 0 \quad r = 1, \ldots, n+6 \tag{18}$$

where $(F_r)_k$ and $(F_r^*)_k$ are the sum of generalized active forces and inertial forces, respectively.

The model can easily be brought (Tarn et al., 1996) into the classical form:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}_e = \tau \tag{19}$$

The nonholonomic constraints are expressed as $\mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = 0$. Let $\mathbf{S}(\mathbf{q})$ be the matrix the columns of which span the null space of $\mathbf{A}$. $\mathbf{S}$ implies the existence of variables $\eta \in \mathscr{R}^{n-2}$ such that (Campion and Bastin, 1991) $\dot{\mathbf{q}} = \mathbf{S}\boldsymbol{\eta}$. Then Eq. (19) can be transformed to:

$$\mathbf{Q}(\mathbf{q})\dot{\boldsymbol{\eta}} + \mathbf{f}(\mathbf{q}, \boldsymbol{\eta}) + \mathbf{g}(\mathbf{q}) = \mathbf{B}(\tau - \mathbf{F}_e) \tag{20}$$

where $\mathbf{Q} = \mathbf{S}^{\mathrm{T}}\mathbf{M}\mathbf{S}$, $\mathbf{f} = \mathbf{S}^{\mathrm{T}}\{\mathbf{M}(\partial/\partial \mathbf{q})[\mathbf{S}\boldsymbol{\eta}]\}\mathbf{S}\boldsymbol{\eta} + \mathbf{S}^{\mathrm{T}}\mathbf{C}$, $\mathbf{g} = \mathbf{S}^{\mathrm{T}}\mathbf{G}$ and $\mathbf{B} = \mathbf{S}^{\mathrm{T}}$

Consider the operational point of the mobile manipulator, $\mathbf{p}$. Using the relation: $\dot{\mathbf{p}} = \mathbf{J}(\mathbf{q})\boldsymbol{\eta}$, and Eq. (20) the dynamics of the operational point of mobile manipulator k can be described as:

$$\boldsymbol{\Lambda}_k(\mathbf{q}_k)\ddot{\mathbf{p}}k + \boldsymbol{\mu}_k(\mathbf{q}_k, \boldsymbol{\eta}_k) + \boldsymbol{\nu}_k(\mathbf{q}_k) = \mathbf{F}_k \tag{21}$$

where $\boldsymbol{\Lambda}(\mathbf{q}) = [\mathbf{J}\mathbf{Q}^{-1}\mathbf{J}^{\mathrm{T}}]^{-1}$, $\boldsymbol{\mu}(\mathbf{q}, \boldsymbol{\eta}) = \bar{\mathbf{J}}^{\mathrm{T}}\mathbf{f} - \boldsymbol{\Lambda}\mathbf{h}$, $\boldsymbol{\nu}(\mathbf{q}) = \bar{\mathbf{J}}^{\mathrm{T}}\mathbf{g}$, $\mathbf{F} = \bar{\mathbf{J}}^{\mathrm{T}}(\mathbf{B}\tau - \mathbf{F}_e)$ and $\bar{\mathbf{J}}$ (Khatib, 1995) is the pseudoinverse of $\mathbf{J}(\mathbf{q})$: $\bar{\mathbf{J}}(\mathbf{q}) = \mathbf{Q}^{-1}(\mathbf{q})\mathbf{J}^{\mathrm{T}}(\mathbf{q})\boldsymbol{\Lambda}(\mathbf{q})$.

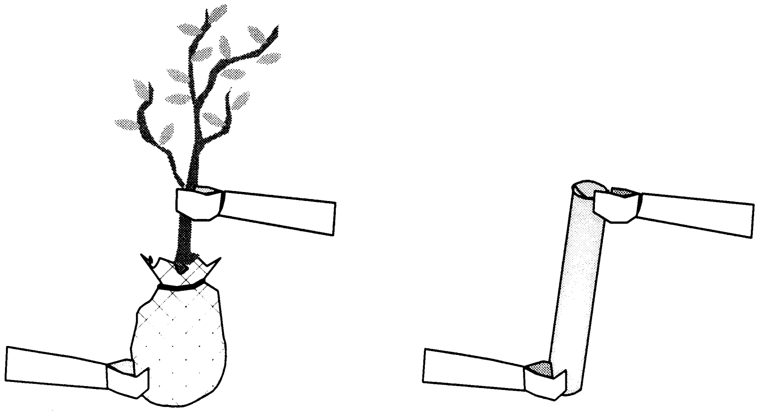

Fig. 7. Approximating the deformable object.

3. Modeling of the deformable object

We discretize the deformable object using finite elements. For each element, the displacements $\mathbf{U}$ within its volume can be expressed in terms of the displacements at its nodes, $\mathbf{U} = \mathbf{N}\mathbf{q}$, where $\mathbf{N}$ is the matrix of shape functions (Rao, 1989). The deformations, $\boldsymbol{\varepsilon}$, displacements, $\mathbf{U}$, and stresses, $\boldsymbol{\sigma}$ are related as follows $\boldsymbol{\varepsilon} = \mathbf{D} \cdot \mathbf{U}$ $\boldsymbol{\sigma} = \mathbf{E} \cdot \boldsymbol{\varepsilon}$. Substituting yields $\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{q}$.

The dynamic equations of each element can be derived following Lagrange's formulation:

$$\mathbf{M}^e\ddot{\mathbf{q}}^e + \mathbf{C}^e\dot{\mathbf{q}}^e + \mathbf{K}^e\mathbf{q}^e = \mathbf{F}_i^e$$

The characteristic matrices appearing in this equation can be expressed as:

$$\mathbf{M}^e = \iiint \rho \mathbf{N}^{\mathrm{T}}\mathbf{N}\, \mathrm{d}V \quad \mathbf{K}^e = \iiint \mathbf{B}^{\mathrm{T}}\mathbf{E}\mathbf{B}\, \mathrm{d}V \quad \mathbf{C}^e = \iiint \mu \mathbf{N}^{\mathrm{T}}\mathbf{N}\, \mathrm{d}V$$

Element dynamic equations are assembled to form the object dynamic equations:

$$\mathbf{M}^0\ddot{\mathbf{q}}^0 + \mathbf{C}^0\dot{\mathbf{q}}^0 + \mathbf{K}^0\mathbf{q}^0 = \mathbf{F}_i^0 \tag{22}$$

4. Multiple mobile manipulator system

Suppose that there are m mobile manipulators grasping the object (Fig. 7).

The dynamics of the deformable object can be described in terms of all operational points.

Definition 1. The global operational point is a vector formed by the Cartesian product of the operational points of all manipulator systems grasping the deformable object:

$$\mathbf{P} \triangleq [\mathbf{p}^1 \quad \mathbf{p}^2 \quad \cdots \quad \mathbf{p}^k \quad \cdots \quad \mathbf{p}^m]^{\mathrm{T}} \in \mathscr{D}_{p^1} \times \mathscr{D}_{p^2} \times \cdots \times \mathscr{D}_{p^k} \times \cdots \mathscr{D}_{p^m}$$

where p^i is the operational point defined for each mobile manipulator, $\mathscr{D}_{p^i}$ the operational space of mobile manipulator i and m the total number of mobile manipulators.

In the framework of the *augmented object approach* (Khatib, 1995), only rigid objects have been considered. Subsystems can be integrated into a centralized dynamic model in the global operational space. Consider Eq. (22) and set: $\mathbf{q}^0 \triangleq \mathbf{P}$.

The dynamic equations of the deformable object are expressed as a function of the global operational point coordinates: $\mathbf{M}^0\ddot{\mathbf{P}} + \mathbf{C}^0\dot{\mathbf{P}} + \mathbf{K}^0\mathbf{P} = \mathbf{F}_i^0$. Now consider all m mobile manipulators in a single system with dynamic equations:

$$\boldsymbol{\Lambda}_{\oplus}\ddot{\mathbf{P}} + \boldsymbol{\mu}_{\oplus} + \boldsymbol{v}_{\oplus} = \mathbf{F}_{\oplus} \tag{23}$$

where $\boldsymbol{\Lambda}_{\oplus} = \mathbf{M}^0 + \mathrm{diag}\{\boldsymbol{\Lambda}_k\}$, $\mu_{\oplus} = \mathbf{C}^0 + \mathrm{diag}\{\boldsymbol{\mu}_k\}$, $\boldsymbol{v}_{\oplus} = \mathbf{K}^0 + \mathrm{diag}\{\boldsymbol{v}_k\}$ and $\mathbf{F}_{\oplus} = \mathbf{F}_i + \mathrm{diag}\{\mathbf{F}_k\}$

5. Dynamic constraints

The resultant of all but the ground forces exerted on the platform k is:

$$\bar{\mathbf{R}} = m^v(\mathbf{g} - \mathbf{a}^v) + \sum_{j=1}^{w^k} m^j(\mathbf{g} - \mathbf{a}^j) + \mathbf{F}^{\overline{mm}}$$

where j is used to denote each where, w^k is the number of wheels of mobile manipulator k, and the subscript k has been dropped for simplicity. Denote the reaction forces exerted to the wheels of platform k by $\mathbf{F}^j$. Then the set of constraint equations can be summarized as follows:

$$\bar{\mathbf{R}}\mathbf{z}^I = \sum_{j=1}^{w^k} \mathbf{F}^j\mathbf{z}^I \qquad 0 \leq \mathbf{F}^j\mathbf{z}^j \qquad \mathrm{r}^j\mathbf{F}^j\mathbf{x}^j = (\tau_d^j)$$

$$0 \geq \frac{1}{\sqrt{1+\eta}}\|\mathbf{F}^j\| - \mathbf{z}^j\mathbf{F}^j \qquad \left(\bar{\mathbf{R}} + \sum_{j=1}^{w^k} \mathbf{F}^j\right)(\boldsymbol{\omega}^v \times \mathbf{v}^{c_v}) = 0$$

Finally, the moments of all external forces exerted on mobile platform k are related as:

$$\begin{aligned} &-\boldsymbol{\alpha}^v \cdot \mathbf{I}^v - \boldsymbol{\omega}^v \times \mathbf{I}^v \cdot \boldsymbol{\omega}^v + \mathbf{F}^{\overline{mm}} \times {}^v\mathbf{r}^{\overline{mm}} + \mathbf{T}^{\overline{mm}} \\ &+ \sum_{j=1}^{w}[\mathbf{F}^j \times {}^v\ell^j - \mathrm{m}^j\mathbf{a}^j \times ({}^v\ell^j - r^j\mathbf{z}^j)] = 0 \end{aligned} \tag{24}$$

where $\mathbf{I}^v$ is the central inertia dyadic of the platform of mobile manipulator k.

6. Conclusion

In this work, a multiple mobile manipulator system handling a deformable object is modeled, introducing novelties an terms of systematic analysis and generality. Steps towards implementing such a system in agriculture have been taken (Mandow et al., 1996; Papadopoulos and Sarkar, 1997; van Kollenburg-Crisan et al., 1998). The model is based on Kane's methodology which has the inherent advantages of simplicity, physical insight, speed in simulation and control orientation and allows for the manipulations of a deformable object of arbitrary shape. It is pointed out that conventional velocity constraint equations are not sufficient to guarantee nonholonomic motion for a real system. A set of additional conditions is formed which ensures mechanical stability and nonholonomic motion.

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