Flocks of Autonomous Mobile Agents

Herbert G. Tanner, Ali Jadbabaie and George J. Pappas
Department of Electrical and Systems Engineering
University of Pennsylvania

***Abstract*—The motion of a group of nonholonomic mobile agents is synchronized using local control laws. This synchronization strategy is inspired by the early flocking model proposed by Reynolds [17] and following work [22, 8]. The control laws presented ensure that all agent headings and speeds converge asymptotically to the same value and collisions between the agents are avoided. The stability of this type of motion is closely related to the connectivity properties of the underlying interconnection graph. Proof techniques are based on LaSalle's invariant principle and algebraic graph theory and the results are verified in numerical simulations.**

***Keywords*—Cooperative control, multi-agent systems, flocking motion.**

I. Introduction

Recent technological advances offer more efficient computation and less expensive communication. The ability to compute locally and share information has facilitated the development of new multi-agent systems. Such type of systems promise increased performance, efficiency and robustness, at a fraction of the cost compared to their centralized counterparts, utilizing distributed coordination sensing and actuation. The question arising is how to achieve coordination in multi-agent systems.

Nature is abundant in marvelous examples of coordinated behavior. Across the scale, from biochemical cellular networks, up to ant colonies, schools of fish, flocks of birds and herds of land animals, one can find systems that exhibit astonishingly efficient and robust coordination schemes [1, 15, 23, 7, 5]. At the same time, several researchers in the area of statistical physics and complexity theory have addressed flocking and schooling behavior in the context of non-equilibrium phenomena in many-degree-of-freedom dynamical systems and self organization in systems of self-propelled particles [22, 21, 20, 13, 11, 18, 9]. Similar problems have become a major thrust in systems and control theory, in the context of cooperative control, distributed control of multiple vehicles and formation control; see for example [10, 2, 16, 4, 12, 6, 19, 8, 14, 3]. The main goal of the above papers is to develop a decentralized control strategy, such that a global objective, such as a tight formation with fixed pairwise inter vehicle distances is achieved.

In 1986 Craig Reynolds [17] made a computer model of coordinated animal motion such as bird flocks and fish schools. He called the generic simulated flocking creatures "boids". The basic flocking model consists of three simple steering behaviors which describe how an individual boid maneuvers based on the positions and velocities its nearby flockmates:separation, alignment, and cohesion. In 1995, a similar model was proposed by Vicsek *et al.* [22]. Under an alignment rule, a spontaneous development of coherent collective motion is observed, resulting in the headings of all agents to converge to a common value. A proof of convergence for Vicsek's model (in the noise-free case) was given in [8].

In this paper provide a system theoretic justification for the flocking phenomenon in [17]. In our flocking model, we consider dynamic mobile agents steered by local control laws. The mobile agents are described by nonholonomic differential equations of the form:

$$\dot{x}_i = v_i \cos\theta_i \tag{1a}$$

$$\dot{y}_i = v_i \sin\theta_i \tag{1b}$$

$$\dot{\theta}_i = \omega_i \tag{1c}$$

$$\dot{v}_i = a_i, \tag{1d}$$

for i ranging from 1 to N, which is the total number of agents in the group. In the above, $r_i = (x_i, y_i)^T$ is the position vector of vehicle i, θ_i its orientation (Figure 1), v_i its translational speed and a_i, ω_i its control inputs. The relative positions between the vehicles are denoted $r_{ij} \triangleq r_i - r_j$.

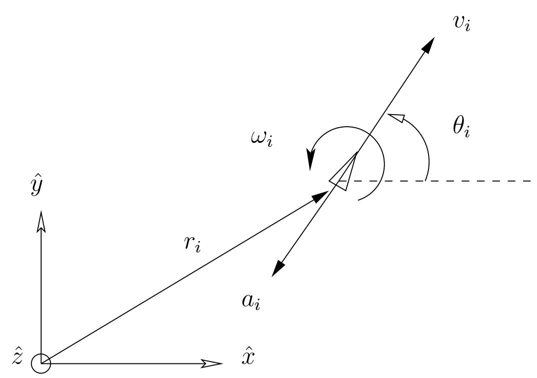

Fig. 1. Agent i and its control inputs.

For this system of mobile agents we show that all agent headings converge to the same value, and pairwise relative speeds are stabilized. This results in an overall motion in which the group maintains its shape and moves uniformly in a particular direction. An artificial potential function ensures that collisions between the agents are avoided and cohesion in the group is maintained. Our analysis is based on Lyapunov stability and algebraic graph theory. Central to this analysis is the connectivity of the graph that represents control interactions between the agents, a property which allows propagation of information. While the proof techniques are totally different from those in [8], the end result is similar, suggesting that addition of cohesion and separation forces in addition to alignment as well as addition of dynamics, does not affect the stability of the flocking motion.

II. Coordination Strategy

Let us now assume a fixed control interconnection topology on the group of vehicles, where interconnections express control dependences. The agents that are interconnected with i, are called *neighbors* of i. The set of neighbors of i is denoted $\mathcal{N}_i$. We can represent the interconnection topology by means of an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ is the set of vertices indexed by the agents in the group, and $\mathcal{E}$ is the set of edges, with one edge between each pair of interconnected agents.

For a pair of neighboring agents, $(i,j) \in \mathcal{E}$ we define an artificial potential function V_{ij} that depends on the distance between these two agents. An example of such function, given here for illustration purposes, is the following:

$$V_{ij}(\|r_{ij}\|) = \frac{1}{\|r_{ij}\|^2} + \log \|r_{ij}\|^2 .$$

The potential of an agent i, is now defined as the sum of all artificial potentials associated with every one of its neighbors:

$$V_i \triangleq \sum_{j \in \mathcal{N}_i} V_{ij}(\|r_{ij}\|).$$

The neighboring relations of each agent determine its control law:

$$\begin{aligned} a_i = & - k \sum_{j \in \mathcal{N}_i} (v_i - v_j) - [(\nabla_{r_i} V_i)_x \cos\theta_i \\ & + (\nabla_{r_i} V_i)_y \sin\theta_i] \left| \sum_{j \in \mathcal{N}_i} (v_i - v_j) \right| \quad \text{(2a)} \\ \omega_i = & - k \sum_{j \in \mathcal{N}_i} (\theta_i - \theta_j) - [(\nabla_{r_i} V_i)_y \cos\theta_i \\ & - (\nabla_{r_i} V_i)_x \sin\theta_i] \left| \sum_{j \in \mathcal{N}_i} (\theta_i - \theta_j) \right| \quad \text{(2b)} \end{aligned}$$

with k being a constant positive parameter (control gain). In (2a)-(2b) the first terms are responsible for stabilization of relative headings and speeds, while the second terms reduce the potential of each boid. For the closed loop system we have the following:

Proposition II.1 *Consider the system of N mobile agents with dynamics* (1). *Then, for a sufficiently large control gain k, the agent headings and speeds converge to the same value while collisions between the agents are avoided.*

Proof: Consider the positive semi-definite function

$$V = \frac{1}{2}\theta^T L_c \theta + \frac{1}{2} v^T L_c v, \quad (3)$$

where θ and v are the stack vectors of headings and speeds, respectively and L_c is the Laplacian of the complete graph with N vertices. Taking the derivative of V we get:

$$\begin{aligned}\dot{V} =& -\theta^T L_c (kL\theta + \text{diag}\{(\nabla_{r_i} V_i)_\perp\}\,|L\theta|) \\ & - v^T L_c (kLv + \text{diag}\{(\nabla_{r_i} V_i)_\parallel\}\,|Lv|) \\ =& -kN\theta^T L\theta - \theta L_c \text{diag}\{(\nabla_{r_i} V_i)_\perp\}\,|L\theta| \\ & - kNv^T Lv - vL_c \text{diag}\{(\nabla_{r_i} V_i)_\parallel\}\,|Lv|\,,\end{aligned}$$

where $(\nabla_{r_i} V_i)_\parallel$ and $(\nabla_{r_i} V_i)_\perp$ are the components of each $\nabla_{r_i} V_i$ along the direction of the velocity of agent i and its orthogonal direction, respectively. It turns out that components of v and θ that are along the direction of the vector of ones, $\mathbf{1}$, preserve the value of V. To this end, decompose these vectors as $\theta = \theta^1 \oplus \theta^{1^\perp}$ and $v = v^1 \oplus v^{1^\perp}$, where superscripts 1 and $1^\perp$ identify the components of the vectors along the direction of the vector of ones and its orthogonal, respectively. Then, the derivative of V becomes

$$\begin{aligned}\dot{V} =& -k(\theta^{1^\perp})^T (NL\theta^{1^\perp} + L_c \text{diag}\{(\nabla_{r_i} V_i)_\perp\}|L\theta^{1^\perp}|) \\ & - (v^{1^\perp})^T (kNLv^{1^\perp} + L_c \text{diag}\{(\nabla_{r_i} V_i)_\parallel\}|Lv^{1^\perp}|) \\ \leq & -kN\lambda_2 \|\theta^{1^\perp}\|^2 \\ & + \|\theta^{1^\perp}\|\,\|L_c\|\,\|\text{diag}\{(\nabla_{r_i} V_i)_\perp\}\|\,\sqrt{N}\|L\theta^{1^\perp}\| \\ & - kN\lambda_2 \|v^{1^\perp}\|^2 \quad (4) \\ & + \|v^{1^\perp}\|\,\|L_c\|\,\left\|\text{diag}\{(\nabla_{r_i} V_i)_\parallel\}\right\|\,\sqrt{N}\|Lv^{1^\perp}\| \\ \leq & -N(k\lambda_2 - N\sqrt{N} f_{max})\|\theta^{1^\perp}\|^2 \\ & - N(k\lambda_2 - N\sqrt{N} f_{max})\|v^{1^\perp}\|^2 \quad (5) \\ & \quad (6)\end{aligned}$$

where f_{max} is the magnitude of the maximum potential force. Choosing

$$k \geq \frac{N\sqrt{N} f_{max}}{\lambda_2},$$

ensures that V will be decreasing. Any level set of V is invariant and from (3) all speed and heading differences have to remain bounded. Given that $v^{1^\perp} = \theta^{1^\perp} = 0$ is an equilibrium configuration for (1), application of LaSalle's invariant principle shows that this configuration is asymptotically stable. ■

III. Numerical Validation

Consider a group of 10 mobile agents. At first assume that the interconnection graph is complete, that is every agent has every other agent as a neighbor. Let the agents be initiated with (x, y) positions selected randomly in the interval $[-5, 5]$ m, speeds in the interval $[-1, 1]$ m/s and headings in $[-\pi, \pi]$. Control gain k is set to 1.

We depict the evolution of a set of $N - 1 = 9$ heading differences that span the relative headings space in Figure 2. The Figure shows that the heading differences have converged exponentially to zero after approximately 10 simulation seconds, resulting in all agents moving in the same direction.

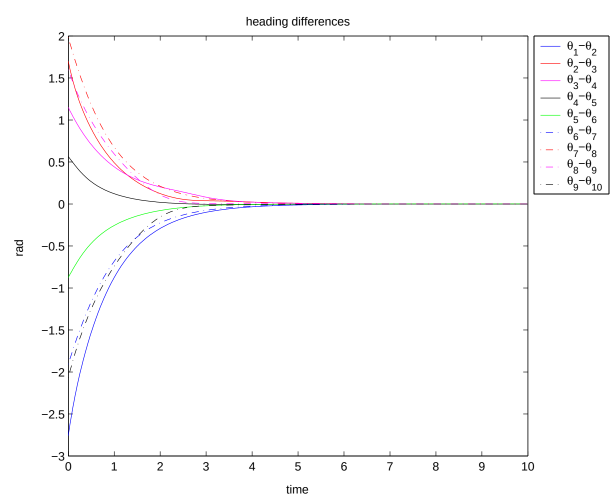

Fig. 2. Convergence of headings when the interconnection graph is complete.

Figure 3 shows the convergence of speeds. In Figure 3 the (absolute) speeds of the 10 agents converge exponentially to the same value, which is approximately 0.5m/s.

Figures 4-5 refer to the case where the interconnection graph is not complete, i.e. it does

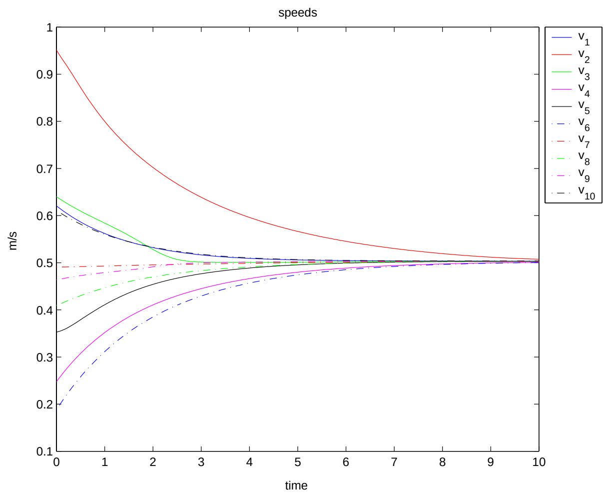

Fig. 3. Convergence of speeds when the interconnection graph is complete.

not have all possible edges, but it is nevertheless connected. The evolution of same set of heading differences as in the previous case is now depicted in Figure 4. The Figure shows that the headings still converge to the same value, however the rate of convergence is smaller and it takes almost 15 simulation seconds to reach steady state. This is due to the fact that in this case, the second eigenvalue of the graph Laplacian, λ_2 is smaller. Recall from (6) that λ_2 is related to the convergence rate of the Lyapunov-like function.

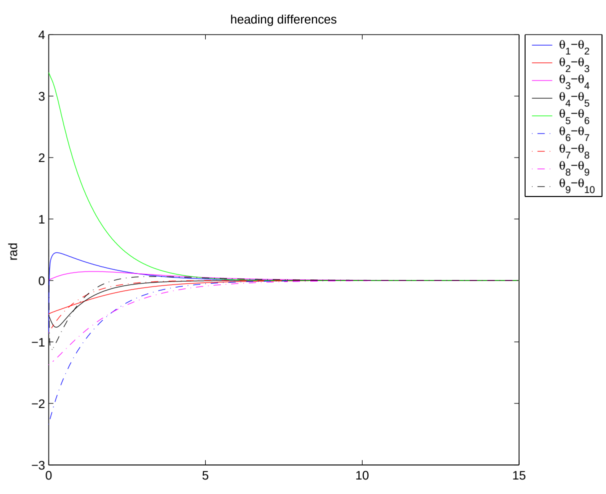

Fig. 4. Convergence of speeds when the interconnection graph is incomplete but connected.

The evolution of the agent speeds is given in Figure **??**. Speeds converge exponentially to a value close to 0.7m/s. Note once again that due to the reduced value of λ_2, the agent speeds take longer to reach their steady state compared to the case of the complete interconnection graph.

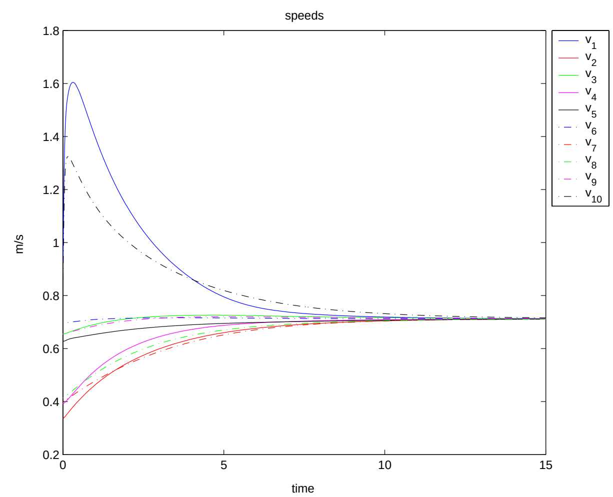

Fig. 5. Convergence of speeds when the interconnection graph is connected but not complete.

IV. Conclusions

In this paper we show how a group of nonholonomic mobile agents can synchronize their headings and speeds using local control laws. These control laws are constructed as a combination of a synchronization term that drives each agent speed and heading to the average over its set of neighbors and a sector bounded artificial potential term that steers the agent towards a direction that minimizes its potential and keeps it from colliding with its neighbors. Stability analysis makes use of Lyapunov theory and known results from algebraic graph theory. This is where the topology of interconnection is reflected on the stability and robustness properties of the group. The dependence of convergence speed to the degree of connectivity in the interconnection graph becomes evident in simulation examples.

References

[1] C. M. Breder. Equations descriptive of fish schools and other animal aggergations. *Ecology*, 35:361–370, 1954.

[2] Jorge Cortes, Sonia Martinez, Timur Karatas, and Francesco Bullo. Coverage control for mobile sensing networks. *IEEE Transactions on Robotics and Automation*, 2002. (submitted).

[3] J. P. Desai, J. P. Ostrowski, and V. Kumar. Modeling and control of formations of nonholonomic mobile robots. *IEEE Transactions on Robotics and Automation*, 17(6):905–908, 2001.

[4] J. Alexander Fax and Richard M. Murray. Graph Laplacians and stabilization of vehicle formations. *15th IFAC Congress, Barcelona, Spain*, 2002.

[5] G. Flierl, D. Grunbaum, S. Levin, and D. Olson. From individuals to aggregations: the interplay between behavior and physics. *Journal of Theoretical Biology*, 196:397–454, 1999.

[6] V. Gazi and K. M. Passino. Stability analysis of swarms. *IEEE Transactions on Automatic Control*, 48(4):692–696, April 2003.

[7] D. Grunbaum and A. Okubo. Modeling social animal aggregations. *Frontiers in Theoretical Biology*, 100 of Lecture Notes in Biomathematics:296–325, 1994.

[8] A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *Accepted for publication in IEEE Transactions on Automatic Control*, July 2002.

[9] M. Kardar, G. Parisi, and Y.-C. Zhang. Dynamic scaling of growing interfaces. *Physical Review Letters*, 56:889–892, 1986.

[10] N. Leonard and E. Friorelli. Virtual leaders, artificial potentials and coordinated control of groups. *IEEE Conference on Decision and Control*, 2001.

[11] H. Levine and W. J. Rappel. Self organization in systems of self-propelled particles. *Physical Review E*, 63:208–211, 2001.

[12] Y. Liu, K. M. Passino, and M. M. Polycarpou. Stability analysis of m-dimensional asynchronous swarms with a fixed communication topology. *IEEE Transactions on Automatic Control*, 48(1):76–95, February 2003.

[13] A. S. Mikhailov and D. Zannette. Noise induced breakdown of collective coherent motion in swarms. *Physical Review E*, 60:4571–4575, 1999.

[14] P. Ögren, M. Egerstedt, and X. Hu. A control Lyapunov function approach to multiagent coordination. *IEEE Transactions on Robotics and Automation*, 18, October 2002.

[15] A Okubo. Dynamical aspects of animal grouping: swarms, schools, flocks, and herds. *Advances in Biophysics*, 22:1–94, 1986.

[16] R. Olfati and R. M. Murray. Distributed structural stabilization and tracking for formations of dynamic multi-agents. In *IEEE Conference on Decision and Control*, Las Vegas, NV, 2002.

[17] C. Reynolds. Flocks, birds, and schools: a distributed behavioral model. *Computer Graphics*, 21:25–34, 1987.

[18] N. Shimoyama, K. Sugawa, T. Mizuguchi, Y. Hayakawa, and M. Sano. Collective motion in a system of mobile elements. *Physical Review Letters*, 76:3879–3873, 1996.

[19] P. Tabuada, G. J. Pappas, and P. Lima. Feasible formations of multi-agent systems. In *Proceedings of the American Control Conference*, Arlington, VA, 2001.

[20] J. Toner and Y. Tu. Long range order in a two dimensional xy model: How birds fly together. *Physical Review Letters*, 75:4326–4329, 1995.

[21] J. Toner and Y. Tu. Flocks, herds, and schools: A quantitative theory of flocking. *Physical Review E.*, 58:4828–4858, 1998.

[22] T. Vicsek, A. Czirok, E. Ben Jacob, I. Cohen, and O. Schochet. Novel type of phase transitions in a system of self-driven particles. *Physical Review Letters*, 75:1226–1229, 1995.

[23] K. Warburton and J. Lazarus. Tendency-distance models of social cohesion in animal groups. *Journal of Theoretical Biology*, 150:473–488, 1991.